Internal Structure of Uranus

J. J. MacFarlane and W. B. Hubbard
Department of Planetary Sciences
Lunar and Planetary Laboratory
University of Arizona
Tucson, Arizona 85721

ABSTRACT

We present an updated study of Uranus interior models using current information about the planet's gravity field and rotation rate. The most plausible model, both from the point of view of recent data and cosmogony, has a central core of iron and magnesium silicates, an outer envelope of liquid water, methane, and ammonia, and a deep "atmosphere" of almost four earth masses of hydrogen, helium, and methane. The "atmosphere" contains a gravitationally nonnegligible amount of methane -- about 40% by mass. All plausible models are most consistent with a rotation period of ∿15 to 16 hours.

I. INTRODUCTION

The goal of studies of the interior of Uranus is to achieve a synthesis of data on the planet's gravitational and magnetic fields, average density, atmospheric composition, heat flow, and various cosmogonical considerations. Recent years have seen a reduction in the number of possible interior models, although many uncertainties still remain. The purpose of this paper is to summarize a number of recent developments and to indicate the status of Uranus interior models in the context of a number of observational constraints, as of early 1981. A set of Uranus and Neptune models were presented by Hubbard and MacFarlane (1980; HM hereafter); this paper serves as an update on several important results since that time. Due to space limitations, this paper cannot also serve as a review paper and so we will be unable to discuss in any detail such important studies of Uranus' interior as Reynolds and Summers (1965), Podolak and Cameron

(1974), Podolak (1976), and Podolak and Reynolds (1981).

Although preferably one should first derive an interior model and then draw cosmogonical conclusions from it, the Uranus problem is still so ill-constrained that it is preferable to limit interior models to those consistent with a plausible cosmogony. Thus we consider a scenario for formation of Uranus (and Neptune) roughly as follows. Cooling of an initially hot solar-composition gas to temperatures below $\sim 1400^{\circ}K$ will result in the condensation of iron and magnesium silicates ("rock"). At still lower temperatures, say $\lesssim 150^{\circ}K$, various abundant species such as H_2O and then NH_3 and CH_4 will likewise form solid condensates ("ice"), leaving behind a gas phase composed principally of H_2 and He in solar proportions. Now it is unclear whether material in Uranus' formation zone was ever heated to temperatures high enough to vaporize silicates, although H_2 and He will of course never condense and thus would be expected to be fractionated with respect to condensibles. It is assumed (Mizuno 1980) that the condensed species in Uranus' formation zone will aggregate to form planetesimals and then a planetary core with a mass of at least several M_E (one earth mass = M_E). We would expect this core to be made up of "rock" and "ice", with the ratio of "ice" to "rock" (I/R) depending on the precise chemical state of the condensibles and on whether CH_4, NH_3, etc. have completely condensed. For solar composition and complete condensation of "ice" and "rock", I/R $\cong$ 3 (HM).

According to Mizuno's (1980) calculations, a small amount proportionally of H_2-He, $\sim$ few M_E, will then be captured by the protoplanetary core in Uranus' formation zone. Cosmogonically then, we expect that the hydrogen-rich atmosphere of Uranus is by mass only a small fraction of the planet, although this result may have benefited by hindsight from earlier interior models such as Reynolds and Summers (1965).

Since "ice" will condense *after* "rock", it seems most plausible that I/R $\leq$ 3 in Uranus' interior. Models with proportionally more "ice" than this limiting value would need to be produced by an initially chemically inhomogeneous nebula. In any case, we would

https://doi.org/10.1017/S0252921100082385 Published online by Cambridge University Press

normally expect "ice" and "rock" to be separated in the interior of Uranus. For an adiabatic temperature distribution in the planet, which seems most plausible because of the low thermal conductivity of the planetary material (Zharkov and Trubitsyn, 1972), we have typical interior temperatures ∿5000K at pressures of several megabars (HM). According to estimates by Hubbard (1981), iron and magnesium silicates will tend to be solid under these conditions but "ice" will be liquid. In view of the great density contrast between "ice" and "rock", it seems inevitable that a "rock" core will be formed.

The above considerations led HM to consider three-layer models for Uranus and Neptune, consisting of a central "rock" core, a mantle composed of "ice", and a deep atmosphere composed primarily of hydrogen and helium. These models were the most centrally condensed ones which had been proposed, having dimensionless moment of inertia factors $C/Ma^2 \cong 0.20$ (C = polar moment of inertia, M = planetary mass, a = equatorial radius at 1 bar pressure). In contrast, Jupiter has $C/Ma^2 \cong 0.26$ and for Saturn $C/Ma^2 \cong 0.23$. We must stress that there is no "similarity law" for the structure of Jovian planets. In Jupiter and Saturn, C/Ma^2 is basically determined by the structure of the deep hydrogen-rich envelope and by a relatively small core. The detailed structure of the core plays no role. For Uranus and Neptune, however, the structure of the core is as important as the structure of the hydrogen-rich envelope, and it makes a significant difference whether or not the "ice" and "rock" components are separated.

In the following, we will consider some new observational constraints and some resulting variations on the HM models.

II. CONSTRAINTS ON MODELS

The principle constraints on interior models are M, a, and the dimensionless zonal harmonics of the gravity field J_{2n}, defined by

$$V(r,\theta) = \frac{GM}{r} [1 - \sum_{n=1}^{\infty} J_{2n}(a/r)^{2n} P_{2n}(\cos\theta)], \quad (1)$$

where V is the planet's external gravitational potential as a function of distance r from the center of mass and angle θ from the rotation axis, G is the gravitational constant, and P_{2n} are Legendre polynomials. The best available value of a is 25,900 ± 300 km (Danielson, et al. 1972) obtained from an analysis of Stratoscope images. Occultation measurements give a' = 26,200 ± 100 km (Elliot, et al. 1981) for the equatorial radius at the occultation level which is much higher than the 1-bar level. In fact the above values of a and a' are consistent. The error bar in a' has been increased above the formal value of 30 km to allow for possible systematic errors. Although it is customary to define the J_{2n} by using the 1-bar equatorial radius a in Eq. (1), much of recent work on Uranus' gravity field uses a' instead. We will conform to this usage in this paper, so that all calculated and observed J_{2n} are normalized to a' = 26,200 km.

The J_{2n} provide integral constraints on the structure of Uranus via the following relationships:

$$J_{2n} = \Lambda_{2n} q^n + \Lambda'_{2n} q^{n+1} + \ldots, \qquad (2)$$

where

$$q = \omega^2 a^3 / GM \qquad (3)$$

is a dimensionless parameter and ω is the angular rotation velocity of the planet. The dimensionless response coefficients Λ_{2n}, together with the higher-order corrections Λ'_{2n}, . . . are functions of the planetary interior structure and serve as additional integral constraints (Zharkov and Trubitsyn 1978). For consistency in the following we will replace a with a' in Eq. (3), so that for Uranus and a rotation period of 15.5 hours we have q = 0.03935. Without knowledge of q, the J_{2n} themselves provide little or no information about interior structure.

From an analysis of the precession of the ε-ring, Nicholson, et al. (1978) found $J_2 = 3.4 \times 10^{-3}$. This result was made more precise by Elliot, et al. (1981) who found $J_2 = (3.354 \pm 0.005) \times 10^{-3}$. They also obtained $J_4 = (-2.9 \pm 1.3) \times 10^{-5}$.

Some disagreement still exists about the correct rotation period for Uranus. Brown and Goody (1977) obtained 15.6 hours, Trauger, et al. (1978) obtained 13.0 hours, Elliot, et al. (1981) obtained 15.5 hours, and Franklin, et al. (1980) obtained 16.6

hours. On the other hand, Trafton (1977) found 23 hours and Hayes and Belton (1977) found 24 hours. Our preliminary conclusion, based on considerations presented below, is that more plausible interior models are associated with a rotation period of 15 hours, but that a 24-hour period cannot be absolutely ruled out.

If the density of the H_2-He atmosphere is increased by enrichment of other constituents beyond their solar proportions, this density increase can become gravitationally significant. Therefore it is important to obtain bounds on such possible enhancements. A thorough discussion of this problem is given by Wallace (1980). From an analysis of Uranus spectra at wavelengths from the visible to the microwave region, he concludes that the number ratio of CH_4 to H_2 in the deep atmosphere is greater than 0.01 and probably less than 0.10. If CH_4 is the only species enhanced above solar abundance, then these limits correspond to 0.06 $< f_{CH_4} < 0.4$, where f_{CH_4} is the ratio of the mass of CH_4 in the deep atmosphere to the total mass.

III. VARIANTS ON THE HM MODEL

As initiated by Podolak (1976), it is useful to plot the optical oblateness ε versus J_2 for various interior models. Here ε is the difference between the equatorial and polar radii in units of the equatorial radius. We have, to lowest order in q,

$$\varepsilon = q(3\Lambda_2 + 1)/2, \tag{4}$$

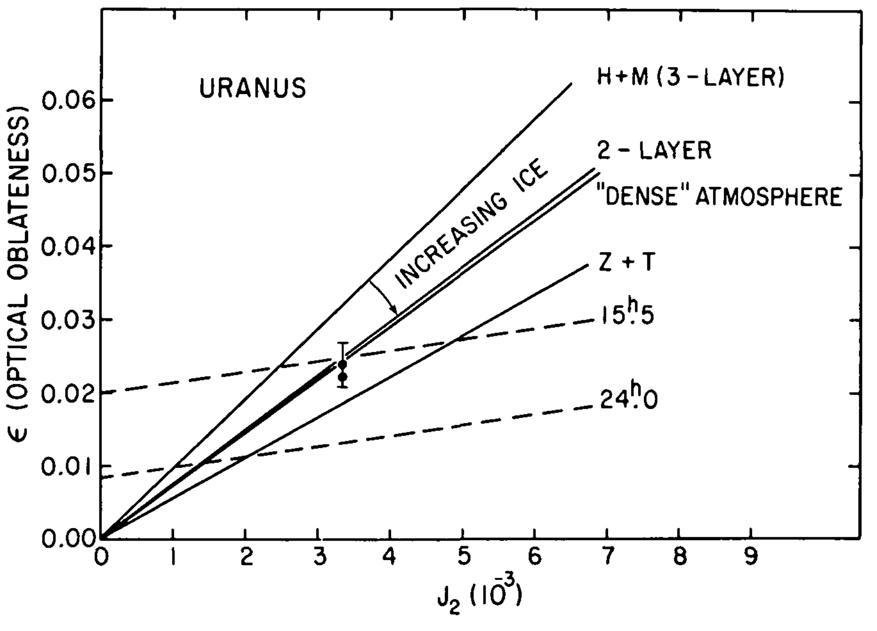
so that a given interior model plots as a straight line with slope $(3 + \Lambda_2^{-1})/2$. When allowance is made for higher-order terms in Eq. (2), the lines are actually curved, but the curvature is negligible to the present order of accuracy.

Fig. 1 shows a plot of several interior models together with rotation periods of 15.5 hours and 24.0 hours. The three-layer H+M model is taken from HM (1980), while the Z+T model is taken from Zharkov and Trubitsyn (1978). It is similar to the H+M model except that it has only two layers: the "ice" and "rock" are homogeneously mixed in the interior below the H_2-He atmosphere. Also, the Z+T model uses an older H_2O equation of state which is significantly

https://doi.org/10.1017/S0252921100082385 Published online by Cambridge University Press

"harder" (i.e., has a higher pressure at a given density) than the one which we now use and which is favored by shock data on H_2O (Mitchell and Nellis 1979, Ree 1976). The dramatic effect of this revision in the H_2O equation of state is shown by the line marked "2-layer" in Fig. 1. This model is essentially identical to the Z+T model except that the new H_2O equation of state is used. Since Uranus is about one-half water, there is a major change due to the revision; the water is more compressed, the model is therefore more centrally condensed, and Λ_2 is smaller. For comparison, we have plotted observational data points for J_2 and ε from Elliot, et al. (1981) and Franklin, et al. (1980). These data seem to favor the revised "2-layer" model, i.e., the Z+T model with the updated H_2O equation of state. However, for reasons given above, this model

Fig. 1. Optical oblateness ε as a function of J_2 for various Uranus models.



https://doi.org/10.1017/S0252921100082385 Published online by Cambridge University Press

seems cosmogonically less plausible since it assumes that the "rock" and "ice" can remain uniformly mixed. Therefore it is important to note that this model does not give a unique fit to the data. We can also bring the H+M three-layer model into agreement by increasing I/R, as shown. This has the effect of increasing the moment of inertia because of the smaller rock core and smaller H_2-He envelope and therefore increasing Λ_2. One needs I/R $\sim$ 6 in order to bring the three-layer model into agreement with the data points. Since this exceeds the cosmic I/R ratio, what has become of the missing "rock" during formation of the planet?

The most cosmogonically plausible way to adapt the three-layer model to fit the data points is to assume that the H_2-He envelope contains a non-negligible amount of denser material such as CH_4. Increasing the density of the deep atmosphere has a significant effect on the moment of inertia, leading to a larger Λ_2. This requires f_{CH_4} $\sim$0.4 to 0.5.

Is there any way to distinguish between the three models outlined above? For this purpose, we have computed the higher-order response coefficient Λ_4 using techniques which we will now describe.

IV. HIGHER-ORDER GRAVITY FIELD OF URANUS

Current data are not yet sufficiently accurate to permit Uranus models to be well-constrained by the higher $J_{2n}(n > 1)$. Nevertheless, there are prospects for improvement based on study of the motions of the Uranian rings and an eventual flyby by the Voyager 2 spacecraft. Therefore we have begun a study of the higher gravity components of Uranus models with the goal of determining how well these can discriminate between different models with essentially identical Λ_2's.

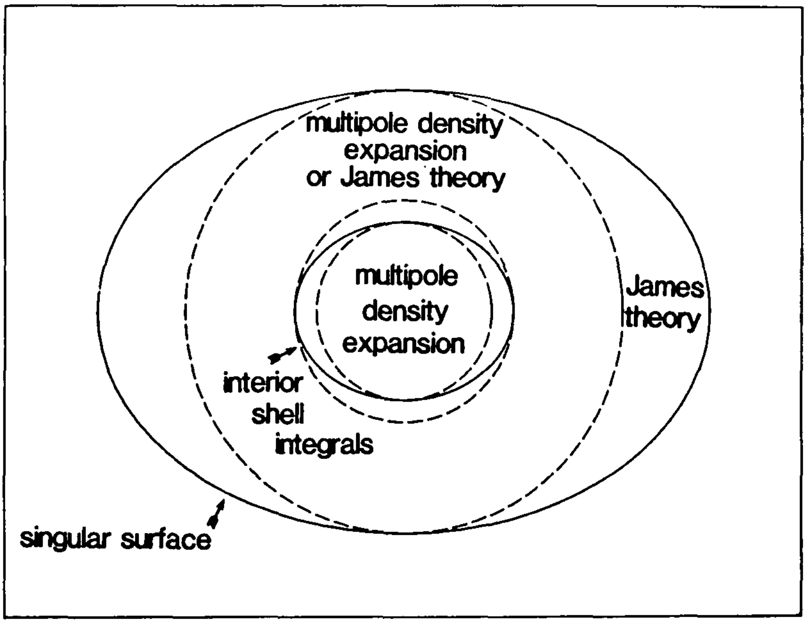
Our calculation makes use of an approach which is described by Hubbard, et al. (1975), Slattery (1977), and Hubbard, et al. (1980). In this approach, we carry out a multipole expansion of the mass density $\rho(r,\theta)$ in the form

$$\rho = \sum_{n=0}^{\infty} \rho_{2n}(r) P_{2n}(\cos\theta) \quad (4)$$

https://doi.org/10.1017/S0252921100082385 Published online by Cambridge University Press

over all spherical regions of the planet which do not contain discontinuities of ρ or singular points where derivatives of ρ do not exist. In Fig. 2, which schematically shows a rotating two-layer planet, expansion (5) may be used in the innermost region enclosed by a dashed line and it may also be used in the region between the outermost dashed line and the next dashed line. In the spherical region which contains the oblate surface of discontinuity between the core and the mantle, we use a set of shell integrals (defined by Hubbard, et al., 1975) to determine the contribution to the various Λ_{2n} from this region. In the region where expansion (5) is valid, the contribution to each Λ_{2n} is produced only by the corresponding $\rho_{2n}(r)$.

Fig. 2. Division of a rotating, oblate planet into various interior zones for calculation of gravitational harmonics.



The surface (the 1-bar surface, say) requires special treatment. For an adiabatic variation of temperature and near-solar composition, the pressure-density relation resembles a polytrope of index 2.5 as the density approaches zero. As is well known, various derivatives of the density with radius become infinite as $\rho \to 0$, and thus it is not possible to accurately locate the surface by means of a Maclaurin series expansion from any interior point. To avoid this problem, we place a grid of quadrature points in r and θ in the outer region of the planet which includes the $\rho = 0$ surface. This approach was employed by James (1964) and does not make use of any assumed analyticity of the density distribution. For Uranus, our theory typically yields J_2 to an accuracy of $\sim 10^{-2}\%$, and J_4 to $\sim 0.4\%$. Values for J_6 and J_8 are also calculated but currently play no role in constraining interior models.

Most interior models of Uranus were calculated for a rotation period of $15^h.5$. For each model we then calculated effective response coefficients by the relations

$$\Lambda_2 = J_2/q, \tag{6}$$

$$\Lambda_4 = -J_4/q^2. \tag{7}$$

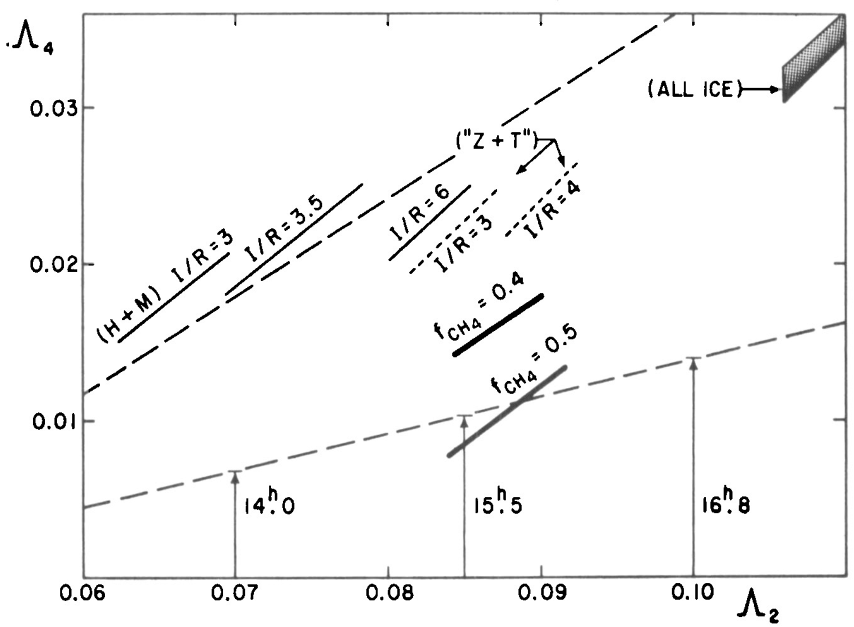
Comparing with Eq. (2), we see that this result is valid in the limit $q \to 0$. For finite q, Eqs. (6) and (7) thus include a very weak q-dependence in Λ_2 and Λ_4, but for our purposes this is negligible. The advantage of this representation for models is that the response coefficients are essentially independent of the adopted rotation period. Fig. 3 shows a plot of Λ_4 versus Λ_2 for a variety of Uranus interior models; all of these models were described above. The locus of points between the dashed lines is consistent with the results of Elliot, et al. (1981); their preferred rotation period is $15^h.5$. If the adopted rotation period is allowed to change, the observational constraints move as shown while the model values remain essentially fixed.

On the left we have three sequences of models of the H+M type, i.e., with "rock" cores, "ice" envelopes, and H_2-He atmospheres. For any given value of I/R, there is a range of models produced by varying the thickness of the H_2-He atmosphere. The range shown corresponds to a reasonable range in a about the observed value of 25,900 km.

The models indicated with short dashed lines are of the two-layer Z+T type, i.e., with H_2-He atmospheres and homogeneous "ice"-"rock" interiors. The updated H_2O equation of state is used, however.

Interestingly, the addition of substantial amounts of methane to the H_2-He atmosphere of an H+M model (models with heavier lines) can produce a substantial decrease in Λ_4 for fixed Λ_2. Since models of this type are the most cosmogonically plausible, it is interesting to see that they are also consistent with the observations. Note that Wallace sets a limit $f_{CH_4} < 0.4$ for the deep Uranus atmosphere. There is still a great variety of possible models.

Fig. 3. The response coefficient Λ_4 as a function of Λ_2 for various Uranus models. Current observational constraints on the gravity field limit models to the region between the dashed lines.



https://doi.org/10.1017/S0252921100082385 Published online by Cambridge University Press

Finally, we have considered the possibility of a Uranus model with a solid or liquid "surface" that is directly observationally accessible. Such a model would require an extremely thin hydrogen-rich atmosphere, extending only up to pressures $\sim$50 bar. Such an atmosphere would be gravitationally negligible, and the planet would basically have to consist of "rock" and "ice" alone. With this composition, the model turns out to be too dense; the best fit to the observed mean density is achieved with pure "ice" and no "rock" core at all, but such a model still ends up with a radius a $\approx$ 23,000 km, i.e., about 10-15% too small. One may argue that the equation of state of the "ice" component is sufficiently uncertain that such a model cannot be excluded. Allowing for such uncertainties, we estimate that suitable "all-ice" models would be found in the location indicated in Fig. 3.

V. CONCLUSIONS

Recent observations of parameters relevant to Uranus' interior are beginning to point toward a particular type of interior model. This model has a "rock" core, an "ice" layer, and an H_2-He atmosphere with copious amounts ($f_{CH_4} \sim 0.4$) of methane. The overall value of I/R is about 2.5. Apparently methane is soluble in the atmosphere although ammonia may not be because it is observed to be depleted (Wallace, 1980). Perhaps there is some stirring between the atmosphere and the "ice" layer since molecular diffusion times are very long. It is difficult to think of an alternative, cosmogonic way of producing the large methane enrichment.

On the other hand, before such a model is too enthusiastically adopted, we should mention two troublesome observational constraints. The first comes from spectroscopic studies of Uranus' atmosphere, which show that the deuterium to hydrogen ratio (D/H) there is approximately the same as that in Jupiter and Saturn and approximately primordial (Hubbard and MacFarlane, 1980). When methane or water condenses from a primordial solar nebula, temperatures are low enough to cause substantial fractionation of deuterium into H_2O and CH_4. Thus we would expect D/H to be at least

https://doi.org/10.1017/S0252921100082385 Published online by Cambridge University Press

four times larger than the primordial ratio in Uranus' atmosphere. The observations seem to imply that Uranus' atmosphere is not stirred with the interior. But in this case, why is methane so enriched in the atmosphere?

The second problem arises from the fact that there is evidence that CH_4 tends to dissociate at pressures above 200 kbar and temperatures above 2000 K, i.e., probable conditions within the "ice" layer (Ross and Ree, 1980). Apparently the methane decomposes into hydrogen and elemental carbon. If the carbon sinks and the hydrogen rises, this would tend to deplete methane in the atmosphere, contrary to observation. Therefore the decomposition of methane, if it occurs, either plays no role because there is no communication between the atmosphere and the interior, or else it occurs in a highly reversible manner.

It should also be pointed out that the interior temperature profile of Uranus (in an adiabatic model) can be significantly altered if H_2O is significantly enhanced in the deep atmosphere as well as CH_4. Calculations by Hunten (1981) indicate that internal temperatures are reduced by $\sim$10% if H_2O has a number abundance of 3% relative to H_2. This effect has not been included in models discussed here.

As discussed by HM and by Hubbard (1978), the structure of the Uranian atmosphere may play an important role in the heat balance of Uranus as well. A measurement of the actual intrinsic heat flow rather than the upper limit which is currently available, may play an important role in helping to understand the cosmogony of Uranus and its present fluid circulation patterns.

ACKNOWLEDGEMENTS

We thank D. M. Hunten, J. L. Elliot, and M. Ross for advance copies of papers relevant to this discussion. This research was supported by NASA Grant NSG-7045.

REFERENCES

Brown, R. A., and Goody, R. M. (1977). The rotation of Uranus. Astrophys. J. 217, 680-687.

Danielson, R. E., Tomasko. M. G., and Savage, B. D. (1972). High resolution imagery of Uranus obtained by Stratoscope II. Astrophys. J. 178, 887-900.

Elliot, J. L., French, R. G., Frogel, J. A., Elias, J. H., Mink, D., and Liller, W. (1981). Orbits of nine Uranian rings. Astron. J., 86, 444-455.

Franklin, F. A., Avis, C. C., Colombo, G., and Shapiro, I. I. (1980). The geometric oblateness of Uranus. Astrophys. J. 236, 1031-1034.

Hayes, S. H., and Belton, M. J. S. (1977). The rotation periods of Uranus and Neptune. Icarus 32, 383-401.

Hubbard, W. B. (1978). Comparative thermal evolution of Uranus and Neptune. Icarus 35, 177-181.

Hubbard, W. B. (1981). Constraints on the origin and interior structure of the major planets. Phil. Trans. Roy. Soc.,in press.

Hubbard, W. B., and MacFarlane, J. J. (1980). Structure and evolution of Uranus and Neptune. J. Geophys. Res. 85, 225-234.

Hubbard, W. B., and MacFarlane, J. J. (1980). Theoretical predictions of deuterium abundances in the Jovian planets. Icarus 41, in press.

Hubbard, W. B., MacFarlane, J. J., Anderson, J. D., Null, G. W., and Biller, E. D. (1980). Interior structure of Saturn inferred from Pioneer 11 gravity data. J. Geophys. Res. 85, 5909-5916.

Hubbard, W. B., Slattery, W. L., and DeVito, C. L. (1975). High zonal harmonics of rapidly rotating planets. Astrophys. J. 199, 504-516.

Hunten, D. (1981). In preparation.

James, R. A. (1964). The structure and stability of rotating gas masses. Astrophys. J. 140, 552-582.

Mitchell, A. C., and Nellis, W. J. (1979). Water Hugoniot measurement in the range 30-220 GPa. High Pressure Sci. Technol. 1, 428-434.

Mizuno, H. (1980). Formation of the giant planets. Progr. Theoret. Phys. 64, 544-557.

Nicholson, P. D., Persson, S. E., Matthews, K., Goldreich, P., and Neugebauer, G. (1978). The rings of Uranus: results from the 10 April 1978 occultation. Astron. J. 83, 1240-1248.

Podolak, M. (1976). Methane rich models of Uranus. Icarus 27, 473-477.

Podolak, M., and Cameron, A. G. W. (1974). Models of the giant planets. Icarus 22, 123-148.

Podolak, M., and Reynolds, R. T. (1981). On the structure and composition of Uranus and Neptune. Icarus, 46, 40-50.

Ree, F. (1976). Equation of state of water. Rep. UCRL-52190, Lawrence Livermore Lab., Livermore, Calif.

Reynolds, R. T., and Summers, A. L. (1965). Models of Uranus and Neptune. J. Geophys. Res. 70, 199-208.

Ross, M., and Ree, F. H. (1980). Repulsive forces of simple molecules and mixtures at high density and temperature. J. Chem. Phys. 73, 6146-6152.

Slattery, W. L. (1977). The structure of the planets Jupiter and Saturn. Icarus 32, 58-72.

Trafton, L. (1977). Uranus' rotational period. Icarus 32, 402-412.

Trauger, J. T., Roesler, F. L., and Munch, G. (1978). A redetermination of the Uranus rotation period. Astrophys. J. 219, 1079-1083.

Wallace, L. (1980). The structure of the Uranus atmosphere. Icarus 43, 231-259.

Zharkov, V. N., and Trubitsyn, V. P. (1972). Adiabatic temperatures in Uranus and Neptune. Izv. Akad. Nauk SSSR Fiz. Zemli 7, 120-127.

Zharkov, V. N., and Trubitsyn, V. P. (1978). Physics of Planetary Interiors (edited and translated by W. B. Hubbard), Pachart, Tucson.