WHY ARE STELLAR SYSTEMS ANISOTROPIC?

E. Bettwieser, K. J. Fricke, R. Spurzem
Universitätssternwarte, Geismarlandstraße 11
D-3400 Göttingen, Federal Republic of Germany

Spherical stellar systems show during their secular evolution the development of velocity anisotropy in their halo (cf. e.g. Hénon, 1971). The present study examines the general reasons for generation of anisotropy in stellar systems by means of a gaseous star cluster model including anisotropy. Moment equations of the Boltzmann equation are considered for spherical symmetry in coordinate space but not in velocity space closed in third order by a heat flux equation. The coefficient of heat conductivity is tailored to describe the flux of energy due to the cumulative effect of distant gravitative encounters and generalized to include effects of anisotropy and external gravitation by a massive central object (Bettwieser et al., 1984).

Anisotropy in velocity space leads to additional equations and terms compared to a usual gasdynamical approach (Heggie, 1984). Thus for example anisotropic hydrostatic equilibrium has the form

$$dp/dr = -\rho g - 2p_a/r \quad . \qquad (1)$$

Here the stellar mass density, gravitational acceleration and radius are denoted with ρ, g, r, resp. and the definitions $p=\rho\sigma_r^2$, $p_a=\rho(\sigma_r^2-\sigma_t^2)$ are used, where σ_r^2 and σ_t^2 denote the one-dimensional mean squared radial and tangential velocity dispersions. $p_a=0$ means isotropy. Note that in hydrostatics the anisotropy is another degree of freedom which is not a priori physically determined. The development of anisotropy is described by an anisotropy balance equation yielded from second order moment equations. It is given here in a simplified form for nearly isotropic systems ($p_a<<p$):

$$\frac{dp_a}{dt} = -\,2\rho\sigma_r^2\,\frac{d}{dr}\left(\frac{u}{r}\right) - \frac{4}{5}\,\frac{d}{dr}\left(\frac{F}{r}\right) \quad . \qquad (2)$$

u and F are the mean radial velocity and the energy flux, resp. Due to hydrostatic equilibrium in homogeneous cores of self-gravitating systems it is in first order $\nabla\sigma_r^2 \propto r$ and the coefficient of heat conductivity is constant, hence $F \propto r$. A similar argument can be given for u. So

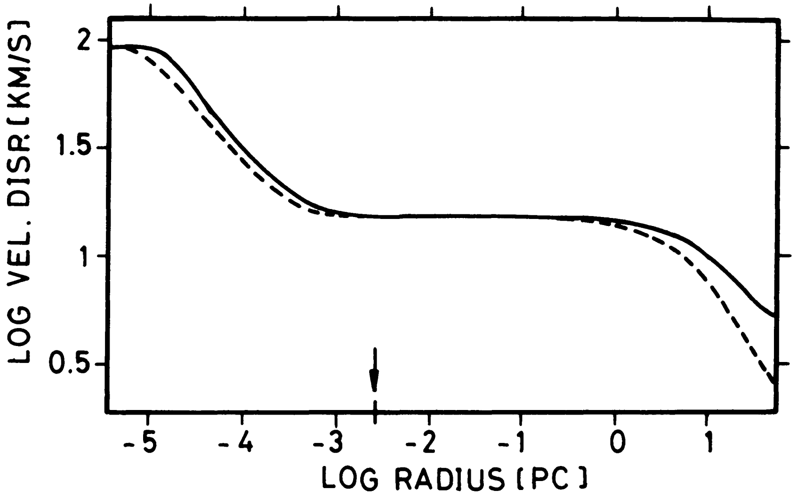
J. Goodman and P. Hut (eds.), Dynamics of Star Clusters, 301–303.

https://doi.org/10.1017/S0074180900147473 Published online by Cambridge University Press

there is no anisotropy generation. In general non-linear velocity or flux are a source of anisotropy according to equation (2) even for an initially isotropic configuration. The anisotropy is limited by a collision term proportional to p_a over stellardynamical relaxation time, which describes the isotropization of the velocity distribution by distant encounters. The general form of eq. (2) for appreciable degree of anisotropy ($p_a \approx p$) is more complex and does not allow for a simple discussion as in the special case of small anisotropy above.

We also considered the problem of anisotropy generation from the following point of view. The Boltzmann entropy is extremized by stationary solutions of the hydrostatic equilibrium. But, allowing for the anisotropy degree of freedom, is it in fact a local maximum? We know that it is not if the stationary configuration is a bound isothermal sphere with a density contrast above the critical value of 709. Besides this normal gravothermal runaway there exist different perturbations, which increase the total entropy. The present stage of our results is the following: we considered for a singular isothermal sphere (SIS, $\rho \propto r^{-2}$) perturbations in anisotropy δp_a but did not allow for redistribution of heat. Assuming $\delta p_a(r=0)=0$ as a boundary condition yields an increasing specific entropy for anisotropy perturbations $\delta p_a>0$. Thus even a perturbation which suppresses the gravothermal effects makes the hydrostatic solution unstable. We guess that if one considers the more realistic problem including the thermal perturbation effects of the anisotropic catastrophe will be enhanced.

Numerical simulations were carried out by using a fully implicit code solving the set of discretized time-dependent moment equations in a Eulerian grid. Effects of a massive central black hole, which is assumed to sit fixed at the very centre and accretes stars by tidal disruption, are included. Without central object and a regular Plummer's model as initial model we find that during the secular core collapse the halo develops more and more anisotropy, whereas the shrinking core remains strictly isotropic. The halo anisotropy increases outwards. SIS taken as another initial model developed in contrast to the regular one extreme anisotropy ($\sigma_r^2 >> \sigma_t^2$) in the post-collapse phase, which has a spatial maximum in the central region and decreases outwards. Note that the gravothermal instability of SIS (cf. Bettwieser and Sugimoto, 1984) is here accompanied by an anisotropy instability as it is proposed by the entropy considerations above. For simulations of the star cluster with star-accreting central black hole the accretion of bound stars in its vicinity and the related energy release are taken into account. The loss cone process was neglected (cf. for details Frank and Rees, 1976). After 16.1 Spitzer-Hart reference times the core contraction was halted due to the enhanced energy release by the star accretion and turned into reexpansion. The figure next page refers to a post-collapse model with an age of 71.4 reference times. Plotted are the r.m.s. radial (solid line) and tangential (dashed line) one-dimensional velocity dispersions in km/s versus radius in pc. There is appreciable anisotropy not only in the halo but also in the central velocity cusp surrounding the black hole. The radius inside which the hole's gravity dominates is marked.



The onset of anisotropy generation in the central region becomes evident from the anisotropy balance eq. (2): for an isotropic steady state under external gravitation the fluxes of energy and stars are source free ($F \propto r^{-2}$, $u \propto r^{-1/4}$). Hence the right hand side of eq. (2) causes the generation of anisotropy. Thus an external gravitational field makes anisotropy by non-linearity of fluxes. Under this viewpoint the anisotropy generation in the halo is explainable, too, because the core's gravity acts on it like an external gravitational field.

Conclusions: we have outlined the essential reasons for anisotropy to develop in stellar systems. External gravity causes non-linear odd-order moments. Hence e.g. the core region around a massive central black hole and the halo of a star cluster become anisotropic in the course of evolution. Anisotropy generation is considered to be a secular instability of such systems. Isotropy is unstable, too, for the singular isothermal sphere which even suffers from an anisotropic catastrophe. Not that also in this case there is a singular gravitational acceleration ($g \propto 1/r$) simulating the 'external' gravitational field. For real stellar systems containing a massive central black hole however, the loss-cone accretion has to be included and it will influence the anisotropy, too (cf. Cohn and Kulsrud, 1978).

REFERENCES

Bettwieser, E., Fricke, K.J., Spurzem, R.: 1984, to be subm. to Mon.Not.R.astr.Soc.

Bettwieser, E., Sugimoto, D.: 1984, Mon.Not.R.astr.Soc. **208**, 493.

Cohn, H., Kulsrud, R.: 1978, Astrophys.J. **226**, 1087.

Frank, J., Rees, M.: 1976, Mon.Not.R.astr.Soc. **176**, 633.

Heggie, D.C.: 1984, Mon.Not.R.astr.Soc. **206**, 179.

Hénon, M.: 1971, Astrophys. Space Sc. **13**, 284.

https://doi.org/10.1017/S0074180900147473 Published online by Cambridge University Press