The Calculus of Variations, by N. I. Akhiezer. Translated from the Russian by Aline H. Frink. Blaisdell Publishing Co. (A division of Random House, Inc.), New York - London, 1962. viii + 248 pages. $7.50.

According to the preface, it is the author's aim to acquaint the student with the problems and the main methods of the calculus of variations rather than to present a comprehensive treatise. While the book is suitable for advanced undergraduates as well as graduates the high degree of rigour maintained throughout is remarkable indeed.

The book is concerned mainly with non-parametric problems whose Lagrangians are of the form $f(x, y_i, y_i')$, where $y_i' = dy_i/dx$, and $i = 1, \ldots, n$. Parametric problems are only briefly discussed for the case $n = 1$. Chapter I is devoted to properties of extremals (i.e. solutions of the Euler equations), particular attention being paid to existence and uniqueness problems, including the theorem of S. N. Bernstein on the existence of global solutions when $n = 1$. Mayer fields are defined in Chapter II directly in terms of the independent integral of Hilbert. Since the latter concept appears for the first time in the formulation of this definition such a process is probably somewhat bewildering to the beginner, but it nevertheless leads directly to the Weierstrass Excess function and the derivation of sufficient conditions for strong or weak minima. This again gives rise to a discussion of the necessary conditions of Weierstrass and Legendre, while the condition of Jacobi is treated solely for the case $n = 1$. A very brief description of the Hamilton-Jacobi theory is given which, however, lacks a thorough geometrical interpretation.

In Chapter III generalizations of the fundamental problem are studied (variable end points, double integrals, Lagrangians depending on r^{th} order derivatives of y, the problem of Lagrange and the isoperimetric problem). In all these problems the author confines himself to the derivation of the relevant Euler equations and mostly also to the case $n = 1$.

Chapter IV represents the most outstanding feature of the book by virtue of an excellent survey of the direct methods of the calculus of variations. The existence of absolute minima for parametric problems is established by means of compactness and lower semi-continuity arguments which are based on the theorems of Hilbert and Tonelli. This chapter alone more than justifies the book. An appendix of 80 pages is devoted to exercises and supplementary topics not treated in the text, such as the theorem of Haar on regular functionals (double integrals) and the Sturm-Liouville problem.

The book can be warmly recommended to any one who seeks a brief but rigorous survey of the subject.

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