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Mathematical Thinking and Geometric Exploration in Africa and Elsewhere

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The editors asked me to contribute to the Benin symposium on the 'Encounter Between Rationalities' from the particular perspective of my research experience in ethnomathematics – the study of mathematical ideas and practices as embedded in their cultural contexts.[1]

As views of mathematics as 'culture-free' and 'universal' have been rather dominant in academia, ethnomathematics emerged relatively late. It took a practising teacher from New York City, Claudia Zaslavsky, to study mathematical aspects of various African cultures and publish the seminal book 'Africa Counts' in 1973, to overcome colonial prejudices. One aspect evidenced by ethnomathematical studies is that the mathematics practised in academia and in schools are not the only ones: each human being and each cultural group develops spontaneously certain mathematical methods (D'Ambrosio 1990); in all human societies there exists mathematical knowledge that is transmitted orally from one generation to the next (Carraher et al. 1987; Kane 1987). Some authors call some forms of mathematics indigenous (e.g. Gay and Cole 1967; Lancy 1978),[2] informal (Posner 1982), non-standard (Carraher et al. 1987), implicit or non-professional (Zaslavsky 1994). As part of an explanation of why academia did not recognize some forms of mathematics, Ascher and Ascher (1981, 158–9) stressed that 'Because of the provincial view of the professional mathematicians, most definitions of mathematics exclude or minimize the implicit and informal. It is, however, in the nature of any professional class to seek to maintain its exclusivity and to do this, in part, by recreating the past in terms of unilinear progress towards its own present.'

The paper tries to contribute to the understanding of mathematical reasoning, as embedded in cultural practices, by means of illuminating some complementary aspects of geometrical exploration in diverse cultural contexts. At the end of the paper I present a few comments on possible educational implications.

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Introductory example of geometric exploration

GiTonga is the main language spoken in the central coastal districts of Inhambane, a province in the Southeast of Mozambique. Because of their beauty and utility, baskets made by Tonga weavers – mostly women – are among the most appreciated products of Mozambican craft work. High-quality baskets with a lot of variation in their design are mostly produced on the peninsula of Linga-Linga, reachable by small boats. The peninsula lies in the district of Morrumbene, about 500km to the northeast of the country's capital Maputo. Some baskets are sold at nearby markets while most find their way to markets in Maputo and neighbouring countries.

Almost since Mozambique's Independence in 1975, I have been collecting Tonga baskets. The books by Gerdes and Bulafo (1994) and Gerdes (2003a) analyse the structure of Tonga handbags called *gipatsi* (plural: *sipatsi*). To produce these handbags the artisans use as basic weave the 'over two, under two' twill (notation: 2/2). By introducing, along bands of the texture, some systematic changes in the weave they create attractive strip patterns.[3]

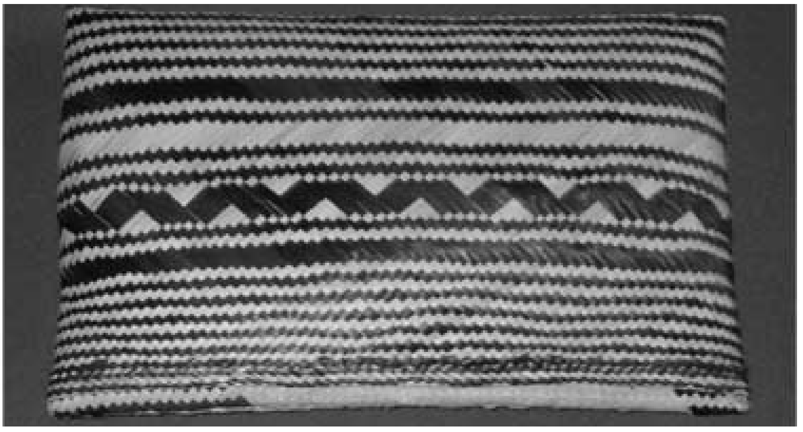


Photo 1. *Gipatsi* handbag with strip pattern

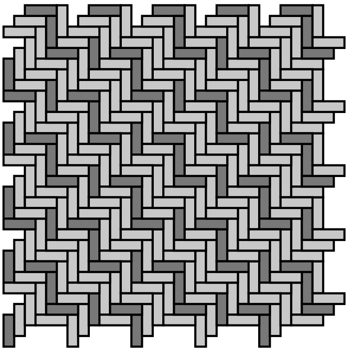
Photograph 1 presents an example of a *gipatsi*. In one weaving direction all the strands are coloured, while in the opposite direction all strands maintain their natural colour. In recent years the strip patterns have found their way into other plaited objects like hats and larger bags. The Tonga weavers are very creative in inventing new strip patterns. A catalogue of 362 different strip patterns is included in Gerdes (2003a), and an update with 58 more strip patterns in Gerdes (2003b).

Recently Tonga basket-weavers created an interesting series of plane patterns (Gerdes 2003c). In contrast with the strip patterns, these plane patterns are not the result of changes in the weaving. Throughout large parts of the texture the twill remains the same, with the 'over three, under three' twill (3/3) being the most common. The patterns, on the contrary, are produced by repeating in both weaving directions sets of a certain number of strands (let us say n strands), whereby the first strand of the set is coloured and the other strands maintain their natural colour. In both weaving directions, the plane patterns have the same periodicity (period = n). We will use the notation [m/m, n] to indicate the class of plane patterns, for which the m/m twill is used and that have period n. Photograph 2 presents an example of a bag decorated with two different [4/4, 4] patterns.

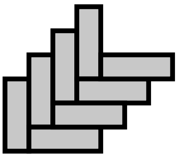
Photo 2. Bag with two [4/4, 4] patterns

After seeing for the first time Tonga baskets with plane patterns, I felt myself immediately, as a mathematician, stimulated and challenged to analyse which and how many different twill plaited plane patterns of the type [m/m, n] might be constructed for certain values of m and n.

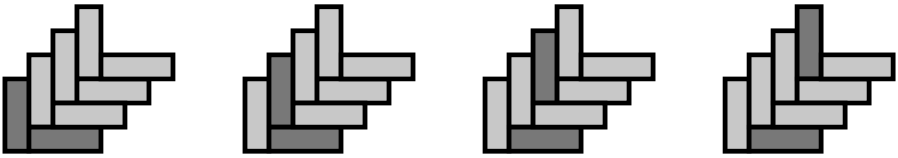
The plane pattern in fig. 1 belongs to the [3/3, 4] class. A basic weaving unit of period 4 is repeated (fig. 2). One may ask how many different plane patterns belong to the considered class. At first sight one might think that the number of distinct patterns [3/3, 4] is equal to 4, as four possibilities exist to introduce a coloured strand



1. Plaiting texture of a plane pattern belonging to the [3/3, 4] class



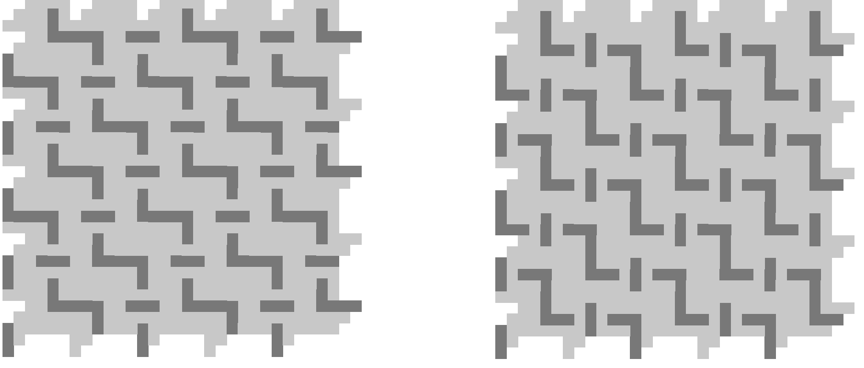
2. 3/3 basic weaving unit of period 4



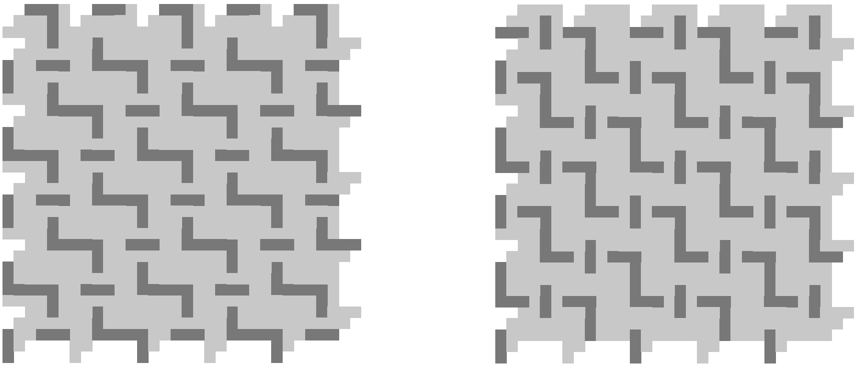
3. Four possibilities for introducing a coloured strand

in the vertical direction after having selected a coloured strand for the first horizontal strand of its basic weaving unit (see fig. 3).

Figure 4 presents the visual image of the respective plane patterns that the four basic weaving units generate. They are very similar. The first one can be transformed into the second by a rotation of 90 degrees followed by a reflection; the third can be transformed into the first by a translation, etc. The four patterns may be considered instances of the same plane pattern. In general, two plane patterns are considered instances of the same plane pattern if one can be transformed into the other by a (sequence of) rotation(s), translation(s), and/or reflection(s). Only one plane pattern belongs to the class [3/3, 4].



4. Visual images of the patterns generated by the four possible basic weaving units



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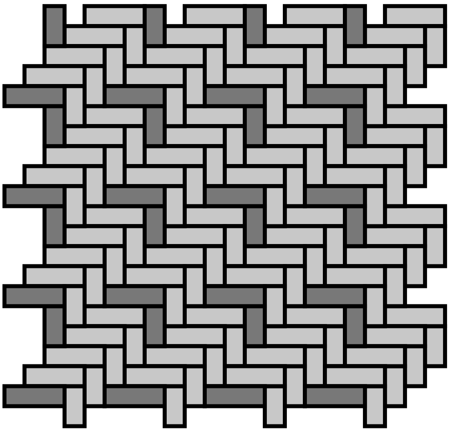
Almost as if some basket-makers had been competing with me I saw, during visits to marketplaces in the following weeks, that Tonga basket-makers had been creating more plane patterns. Moreover they had found all the possibilities for the values of m and n they considered. Within the conditions posed by the basket-weavers and taking into account the symmetries, they had discovered all the possible solutions.

The baskets with these plane patterns started to appear in Maputo in the second half of 2002. The same techniques, the same material and the same colours were used to produce them. Probably we are dealing with the creative imagination of one basket-weaver or of a small group of basket-weavers. Unfortunately, the variation in patterns will hardly be noticed by most potential clients and so far adds (little or) nothing to their commercial value. The invention of the plane patterns responds, however, to the intellectual, geometric and artistic pursuit of their creator(s).

A special basket reinforces our conclusion (photo 3). The basket-weaver used exceptionally a less balanced twill, in one direction 'over two, under three', and consequently 'under two, over three' in the other direction. It is the 2/3 twill. The artisan opted for period 5: in both directions, four natural strands follow each time after one coloured strand (fig. 5). The option for the 2/3 twill enabled the weaver to produce a plane pattern with hooks that displays an axial symmetry. This would have been impossible if the basket-weaver would have used a balanced m/m twill. The selection of the 2/3 twill resulted from a careful, rational analysis and it reflects a particular interest in symmetry by the pattern's inventor.

Photo 3. Bag with a [2/3, 5] pattern



5. Plaiting texture of a [2/3, 5] pattern

After noting the profoundly mathematical character of the artisans' work one can envisage, in the words of Maurice Bazin (2001), doing 'ethnomathematics for the People'. It would consist, at this point, in making contact with the school system in the region, organizing workshops with the teachers, in the local language, urging

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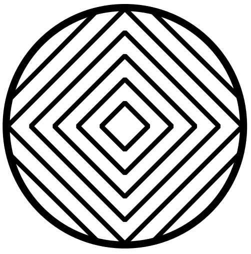
everyone to analyse 'mathematically' the locally produced baskets with the participation of the weavers themselves. In this way the formal study would revert to the inventors and to the enhancement of the culture that made it possible in the first place. The Tonga example illustrates also the strong relationship between African art forms and geometry discussed in Njock (1985).[4]

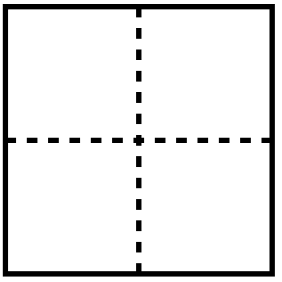
Example of similarity and cultural diversity in geometric exploration

The study of the awakening and development of geometrical thought in diverse cultural activities is a relatively new field of research and demands the development of adequate methods.[5] One methodological starting-point could be the following. The researcher may study first of all the usual production techniques (for example, weaving) of traditional useful labour products, such as mats, baskets, traps, etc., and at each stage of the fabrication process ask which aspects, of a geometrical nature, play a role in arriving at the next stage. This methodological starting-point proves to be fertile as the following example may illustrate.

In several continents male and female artisans have produced basket trays with a circular border and a twill-plaited bottom. Figure 6 shows the geometrical design of some of these circular basket trays seen from above.



6. Geometrical design structure with concentric squares



7. Middle lines of a square

The history of the discovery and invention of twill-plaited circular basket trays may be summarized as follows (Gerdes 2000a, 2003e): from the oldest plaiting technique 'over one, under one' (notation 1/1), artisans in various cultures advanced to weaving 'over two, under two' (2/2) or 'over three, under three' (3/3); crafts people conceived the idea of fastening a mat to a circular border; they learned that a square mat is more advantegeous than other rectangular mats in order to be able to produce a well-balanced basket; artisans discovered that it is easier to fasten a plaited square mat to the circular border if they make the middle lines of the square visible in one way or another (fig. 7); one way to make a middle line visible is by the introduction of a discontinuity line in the plaiting structure (fig. 8 shows an example); on the intersection of discontinuity lines sets of concentric toothed squares or crossings in the form of an X appear automatically (see the examples in fig. 9).

Structures with one or two discontinuity lines may be substituted by more com-

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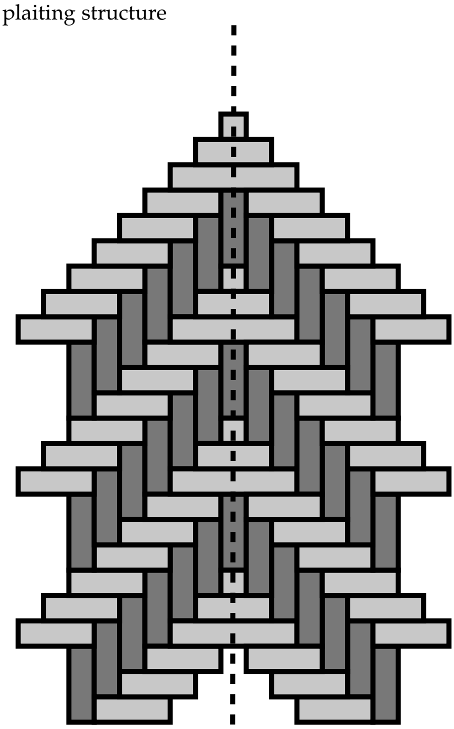
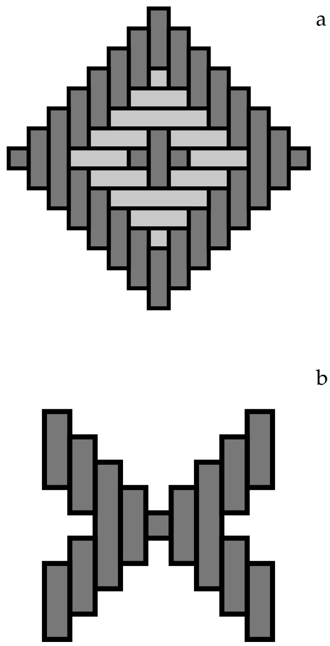
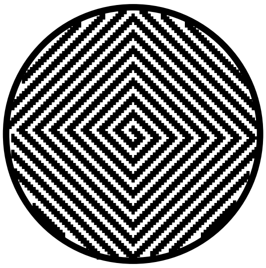
8. Example of a discontinuity line in the plaiting structure



9. Examples of structures of concentric toothed squares (a) and in the shape of an X (b)



10. Geometrical design structure with spirals



plex weaving structures resulting from the introduction of more discontinuity lines or by the variation of the colours of the strips, abstracting from the importance of the perpendicular discontinuity lines. Geometric forms, like the one in fig. 7, may gain value on their own account and on being transposed to other cultural contexts. The same may happen with the more complex patterns. The discovery of the importance of the perpendicular middle lines reflects the understanding of how the fabrication

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of the circular basket tray is facilitated. Once this importance is understood, the use of it may vary from one culture to another and from one period to another, discovering various structures of sets of concentric toothed squares and structures with the shape of an X. More cultural variability emerges in the subsequent phases of the introduction of more complex designs and the transposition to other cultural contexts, resulting from the greater freedom achieved by the artisans. Even in the phase of elaborating more complex designs, striking parallel developments may occur, such as the introduction of plaited spirals (see fig. 10) in 11th-century Arizona by Anasazi basket-weavers and by 19th/20th-century Makhuwa basket-weavers in northeast Mozambique (Gerdes, 2000).

The discovery and the development of circular basket trays exemplifies a more general process of similarity and diversity in geometric exploration, particularly in early cultural contexts.

Relative uniformity of basic geometric ideas

The relative uniformity of ideal structures reflects the unity of humankind, or more accurately, the unity of utensil manufacturing from natural sources: similar situations led generally to similar problems with comparable attempts to solve them, although possibly widely differing in detail. Corresponding societal activity, together with the general human constitution, led to equally elaborate basic geometric forms (for examples, see Gerdes, 1990a, 1992a, 2003d).

The multiplicity of forms in nature is so large that it becomes necessary to explain how humans gradually acquired the ability to perceive certain of these natural forms. There are no forms in nature that are a priori conducive to human observation. The capacity of human beings to recognize geometrical forms in nature and also in their own products was formed in productive activity.

The capacity to recognize order and regular spatial forms in nature has been developed through active labour. Regularity is the result of human creative labour and not its presupposition. It is the real, practical advantages of the invented regular form that lead to the growing awareness of order and regularity. The same advantages stimulate comparison with other products of labour and with natural phenomena. The regularity of the labour product simplifies its reproduction, and in that way the consciousness of its form and the interest in it become reinforced. With the growing awareness develops simultaneously a positive valuing of the discovered form: the form is then also used where it is not necessary; it is now considered beautiful.

The cylinder, cone, or other symmetrical shapes of vessels, the hexagonal patterns of baskets, hats and snowshoes, etc., may at first sight appear to be the result of instinctive impulses or of an innate feeling for these forms. Or they may appear as if generated by a collective ‘cultural spirit’ or ‘archetype’, or also, mechanically, as an imitation of natural phenomena, for example, of crystal structure or of honeycombs. In fact, however, humans create these forms in their practical activity to be able to satisfy their daily needs. They elaborate them. The understanding of these materially necessary forms emerges and develops further through interaction with the given material in order to be able to produce something useful: bows, boats, hand axes, baskets, pots, etc. In the recognition of these necessities, and of the thereby acquired possibilities of employing them to achieve certain aims, emerged the human freedom to make things that are useful and considered beautiful.

With the reflection in art and games of shapes formed in activity, early mathematical thinking started to liberate itself from material necessity: form becomes emancipated from matter, and thus emerges the concept and understanding of form. The way is made free for an intra-mathematical development.

In the interplay of the needs important to a society, material possibilities and experimental activity, certain shapes – for example, symmetrical forms – proved themselves to be optimal. Thinking in terms of order and symmetry does not need a mythical explanation. It reflects the societal experience of production. Once this experience has established itself to the degree that the regularity acquired an aesthetic value, then also new and, in a certain sense, ordered shapes could be created, without an immediate, inescapable material compulsion existing for them. In this process, geometrical thinking develops further – that is the capacity to create thinkable or imaginable forms.

Example of geometric exploration, reconstruction and potential

A very interesting example in Africa of the creation of imaginable forms is the emergence of 'sona' geometry. This tradition was developed principally among the Cokwe (Chokwe, Tshokwe) of Northeast Angola. The Cokwe culture is well known for its decorative art, including drawings made in the sand called sona (singular: lusona). Each boy learnt the meaning and execution of the easier sona. Only a few experts knew and developed the more complicated ones. They were storytellers using sona as illustrations. The drawings were swiftly executed in the following way.

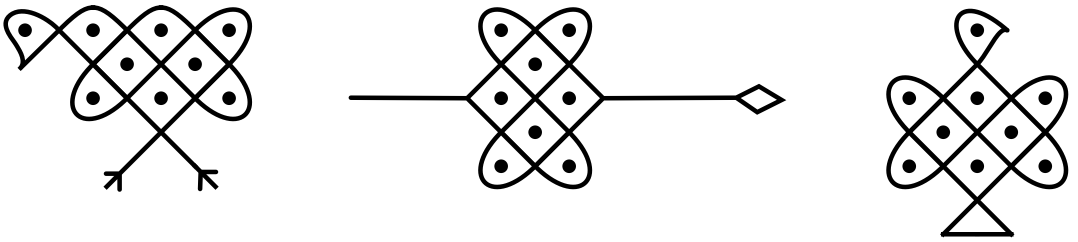
After cleaning and smoothing the ground, the drawing experts first set out with their fingertips a net of equidistant points and then they drew a line figure that embraces the points of the network. Once drawn, the designs were generally immediately wiped out. Slave trade, colonial penetration and occupation provoked the loss of a great deal of knowledge about sona.[6]

Figure 11 presents some examples of relatively easy sona. The algorithm for their construction seems to have been derived from mat plaiting. As the examples of sona in fig. 12 suggest, symmetry and monolinearity played an important role as cultural values: most drawings are symmetrical and/or monolinear. Monolinear means composed of only one (smooth) line; a part of the line may cross another part of the line, but never should a part of the line touch another part.

The drawing experts developed a whole series of geometric algorithms for the construction of monolinear, symmetrical designs. Figure 13 displays two monolinear sona belonging to the same class in the sense that, although the dimensions of the underlying grids are different, both sona are drawn applying the same geometric algorithm.

The drawing experts also invented various rules for building up monolinear sona.

11. Examples of sona



12. Examples of symmetrical, monolinear sona

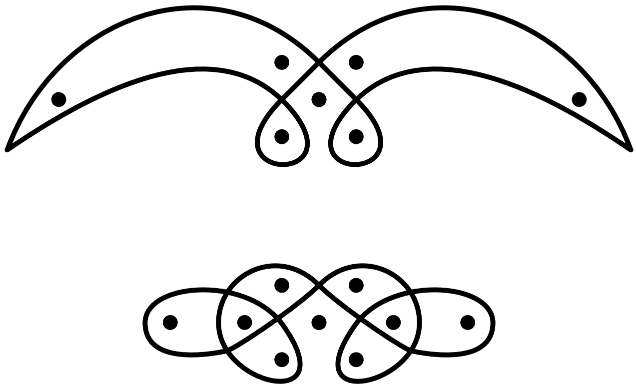
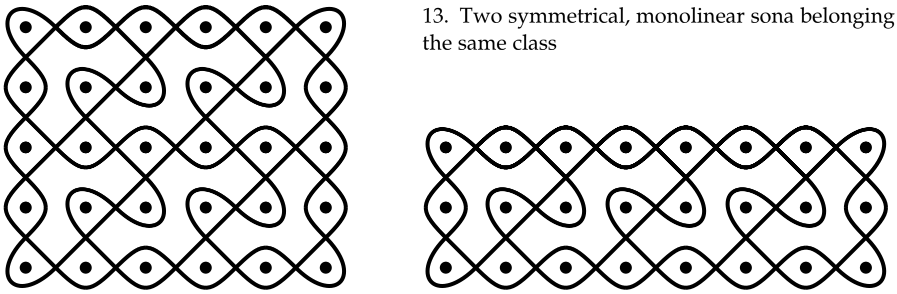


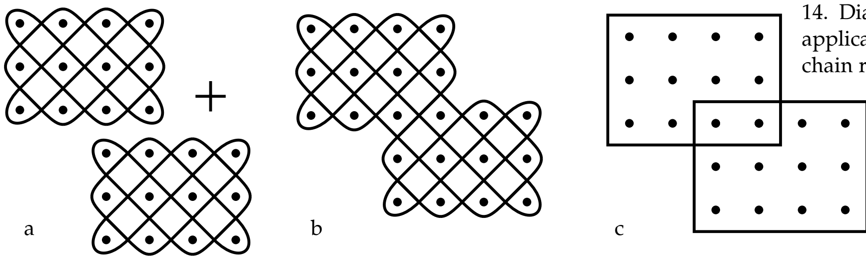
Figure 14 displays an example of the use of a chain rule. It indicates how the appearance of the monolinear drawing in fig. 14c may be explained on the basis of the monolinearity of the two patterns in fig. 14a, and of the way they have been chained together (see fig. 14b).

Several sona reported in the literature clearly do not conform to the cultural values. Sometimes the symmetry or monolinearity was broken in order to give the drawing a specific meaning. More often we seem to be dealing with mistakes or errors. Some drawing experts may have committed mistakes, perhaps because when they were contacted when they were already old. We may be dealing with errors in the transmission of the sona knowledge from one generation to the next, or with mistakes on the part of the reporter, who had little time to make his copies, as the drawings were normally wiped out immediately after telling the story. The wiping out may have been a way to protect the knowledge. Hence some mistakes in the reporting of sona may be an expression of cultural resistance: drawing experts may have consciously made mistakes to deceive the reporter – the 'white' man, associated with slave trade, colonial administration and Christianity.

Once some elements of the sona tradition were reconstructed (Gerdes, 1993/4, 1995b, 1997a; see also Ascher, 1991, Kubik, 1988), it became possible to try to explore



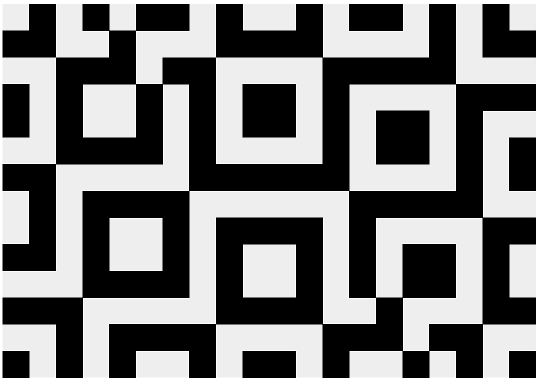
13. Two symmetrical, monolinear sona belonging to the same class



14. Diagram of the application of a chain rule

its geometric potential, both in mathematics education (see Gerdes, 1990b, 1997b, 2002a) and in mathematical theory building. My personal research experience has taught me that the sona are very fertile as a field for further mathematical exploration. I was led successively to the discovery and analysis of mirror curves, Lunda-designs (see the example in fig. 15) and Lunda-symmetry,[7] Liki-designs (Gerdes, 2002b) and of various new types of algebraic structures like periodic cycle matrices, and cylinder, helix and chessboard matrices (Gerdes, 2002b, c, d, e). Robert Lange at Brandeis University (Massachusetts, USA) have created sona-tiles. Franco Favilli and his students at the University of Pisa (Italy) are developing sona-software. Probably, as more researchers enter the field, many more attractive results will be quickly obtained.

The educational and scientific potential of sona further demonstrates the imaginative force and creativity of the Cokwe drawing experts and the profoundness of the mathematical knowledge they had started to build up. This is in sharp contrast to what an ethnographer in the colonial period wrote about the mathematical knowledge of the Cokwe. He described some arithmetic, some time reckoning and some geometrical vocabulary (line, curve, point, etc.), but suggested that the Cokwe did not know mathematics (Santos, 1960). The same author, however, published an interesting study on sona (Santos, 1961). Apparently he did not see any relationship between sona and mathematics. His assessment reflects the horizon of his ethnographic training and his school mathematics education in the colonial metropolis.



15. Example of a Lunda-design

This example may serve to raise the important question of who defines some activity, some idea or some theory as mathematical. Who defines 'what is mathematical thinking', what can be said about his/her socio-cultural background? Speaking at professional gatherings of mathematicians I have seen that the mathematical aspects and potential of the sona are quickly being appreciated and absorbed by the international mathematical community.

Intercultural intelligibility of mathematical thinking

Geometric ideas are developed in diverse cultural contexts, sometimes in different ways, sometimes in parallel ways. The reasoning may differ, but often shows strong, maybe surprising, similarities, reflecting the specific constitution of human beings and the similarities in the contexts in which they are living.

Having grown up in several cultural contexts, a researcher (or any human being even) may have developed some feeling for mathematical ideas. Just like any musician (or any human being even) may develop a certain understanding of and feeling for musical expressions, and any linguist (or any human being even) may develop a certain understanding and recognition of language phenomena. In this sense, mathematical thinking is as pan-human as using a language or involvement in music (playing, listening). This 'feeling for' is the result of a culture-specific mathematical enculturation process (cf. Bishop, 1988).

From an ethnomathematical perspective, mathematics becomes the product of all cultures, being the school mathematical experience of an academic researcher with only one form of mathematical experience. Mathematics is not the product of a particular culture sphere, 'western', but a common human experience. In the process of studying mathematical ideas in diverse cultural contexts, the understanding of what mathematics is, or better of what constitutes mathematical activity, may be deepened. Mathematical thinking is only interculturally intelligible.

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Socio-pedagogical value of ethnomathematics

In *Educate or Perish* Joseph Ki-Zerbo stresses that Africa needs new educational systems, properly rooted in both society and environment, and therefore apt to generate the self-confidence from which imagination springs (1990: 104). I would like to conclude the paper with a short remark on the socio-pedagogical value of ethnomathematics. Each culture has its own starting-points that will allow learning to develop more easily, just like learning anything else is best done in one's mother tongue, even if the final overall mathematical thinking may have common traits among all human beings.

Teacher education plays a crucial role in this process. Reflecting on our mathematics teacher education praxis in Post-Independence Mozambique, some interconnected dimensions of the development of an awareness among mathematics teachers of the social and cultural bases of mathematics and mathematics education were suggested in Gerdes (1998b). Briefly they are:

(a) awareness of mathematics as a universal activity;
(b) awareness of the multilinear development of mathematics;
(c) awareness of mathematics and mathematics education as socio-cultural processes, and;
(d) awareness of the mathematical potential of the pupils.

It is particularly important for teachers to learn never to underestimate the capacities, know-how and wisdom of their pupils and the pupils' communities. People may be doing mathematics, may be engaged in thinking that involves mathematical thought processes, without themselves calling their activity 'mathematical'; they even may say that they do not know mathematics, or that they are not able to do mathematics. Teachers becoming aware of these phenomena and the reasons for them will not accept feelings of 'fear for mathematics' and 'I lack mathematical ability' by their pupils or pupils' parents as natural, normal or insurmountable. On the contrary, the teacher will try to place these feelings in their social, cultural and historical context, being motivated to challenge them through the teacher's activity and attitude in valuing the cultural background of the pupils.

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Acknowledgements

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Notes

1. For an introduction to ethnomathematics, see D'Ambrosio (1985, 1990), Ascher (1991), and Gerdes (1992b, 1993, 1995a). The reader compiled by Powell and Frankenstein (1997) presents an overview of ethnomathematical studies in their relationship to education.
2. Paulin Hountondji criticizes the expression 'indigenous knowledge' and proposes the concept of 'endogenous knowledge' (Hountondji, 1994).
3. For a brief introduction see Gerdes (1998a, chapter 1) or Gerdes (1999, 140–3).
4. For further examples of female activities, art and geometrical thought in Southern Africa, see Gerdes (1996, 1998a).
5. Cf. my study of the awakening of geometrical thought in early culture (Gerdes, 1990a, 1992a, 2003d), Luquet's paper on the psychological-cultural origin of mathematical concepts (Luquet, 1929) and Eglash's book on fractal structures in African cultures (Eglash, 1999).
6. On the basis of an analysis of sona reported by missionaries, colonial administrators and ethnographers, it is possible to reconstruct mathematical elements in the sona tradition (Gerdes, 1993/4, 1995b, 1997a; see also Ascher, 1991; Kubik, 1988).
7. For an introduction, see Gerdes (1999, chapter 40).

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