MATHEMATICAL STATUS OF THE FISSION THEORY

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ABSTRACT. A review is given of the current mathematical and theoretical status of the possibility of the formation of binary stars by fission. It is emphasized that none of the competing theories is completely deductive: each begins with a preconception of how fission might occur, and considers mathematical problems that bear on the viability of that preconception. The classical fission theory turned out to lack internal consistency. The present author's version of the theory, revised from the classical theory in that viscosity plays a negligible rather than a dominant role, appears to be internally consistent and compatible with the idea that close binaries with mass ratios near one may form in this way. The version of the theory described elsewhere in this volume by Durisen, which fails to produce binaries, is based on a preconception of the route to fission different from the classical one. Its failure to produce binaries therefore reflects only on that route, and not on other possible routes to fission.

1. Introduction

It has become common to use the term 'fragmentation' to refer to the breakup of a rapidly collapsing gas cloud and the consequent formation of binary or multiple systems; articles in this volume by Boss and Miyama are related to fragmentation. It occurs in the earliest phase of star formation, that of dynamical collapse, and is characterized by a single timescale (the dynamical timescale). Its mathematical and numerical treatment is via a standard initial-value problem.

By contrast, the word 'fission' refers to the formation of a binary star from a single, rotating star during a phase of slow, or secular contraction. The present article and that by Durisen are devoted to this subject. Two timescales (the dynamical and the contraction timescales) characterize the pre-main-sequence phase when fission is contemplated. A standard initial-value problem is impractical because of the widely separated timescales (but Lucy [1977] has attempted such a treatment anyway). A quasistatic approximation, like that used in the theory of the evolution of spherical stars, is the logical approach, but there are serious mathematical obstacles to applying it in the circumstances relevant to fission. It has so far been implemented only in the context of idealized models of uniform density, as described below in section 2.

The traditional, quasistatic approach toward fission envisages the following sequence of events (or preconception). Initially, the figure is axisymmetric, or nearly so. In consequence of a slow loss of energy through radiation, the axisymmetric figure

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becomes progressively more flattened, resulting in a progressively larger fraction of its energy in the form of kinetic energy of rotation. When the ratio of kinetic to gravitational energy reaches a critical value, the axisymmetric figure becomes unstable to a symmetry -breaking perturbation carrying the circular cross-section into an elliptical one. Further evolution takes place quasistatically along a nonaxisymmetric (or bar-shaped) family on which the equatorial cross-section lengthens progressively in one direction. This continues until this nonaxisymmetric family itself becomes unstable. The nature of this secondary instability and of the family bifurcating at this point is crucial for the theory. One imagines that instability is initiated by a perturbation tending to concentrate matter toward the ends of the bar, resulting in a dumbbell shape. This process continues as evolution proceeds until the matter connecting the two ends of the dumbbell has thinned to the vanishing point, and one is left with a pair of binary stars orbiting each other.

There is nothing in this preconception of binary-star formation that would preclude the use of realistic models in checking the various assertions. However, the mathematical difficulties remain formidable, and, as a result, the only models in which attempts at verification have been made are ellipsoidal figures of uniform density and perturbations therefrom. The work described elsewhere in this volume by Durisen uses realistically stratified models, but in a preconception different from that described above and (in my opinion) not compelling; we discuss this further in section **3** below.

2. Ellipsoidal Figures

We turn now to a brief description of the classical fission theory and of its revised form, wherein one attempts to verify the preconception of fission described above in the ellipsoidal context. A fuller description of these topics is given by Lebovitz (1987).

2.1 THE CLASSICAL VERSION

Here the assumption is made that viscosity is important. This has the effect of enforcing a uniform rotation and influences the stability properties of the figures as well. The axisymmetric family consists of Maclaurin spheroids, which become unstable when the ratio of kinetic to gravitational energy reaches a certain critical value. The family of Jacobi ellipsoids bifurcates from this critical point, and becomes the stable, bar-shaped family along which further evolution takes place. The Jacobi family in turn becomes unstable when the bar reaches a critical elongation. Up to this point of instability along the Jacobi family, all the assertions of the preconception above are verified (in fact, they were originally abstracted from this model). Beyond this point, however, the facts fail to agree with the preconception. The perturbation initiating the instability of the critical Jacobi figure does not shape it into a dumbbell. More seriously, the family of figures bifurcating from this critical point is itself unstable: there are no stable, quasistatic figures just beyond the critical Jacobi figure. Hence quasistatic evolution ceases at this point and motion must take place (at least for a while) on the dynamical timescale. The precise behavior of the figures at this point has never been fully resolved.

2.2 THE REVISED VERSION

Here viscosity is neglected, on the ground that this assumption is nearer to the truth than the opposite assumption of the classical version. A wider variety of ellipsoidal configurations is now possible (the Riemann ellipsoids, in which there are fluid motions beyond those of rigid-body rotation). The initial evolution again takes place along the Maclaurin family (approximately). Again an instability along this family signals a shift in the evolution onto a family of ellipsoids with three unequal axes. This family itself subsequently becomes unstable. While details differ, there is a rather complete analogy with the classical version up to this point. However, there is an important difference beyond this point. On the revised version, the perturbation initiating the instability carries the ellipsoid into a dumbell shape. Moreover, the family of figures bifurcating at the critical point is stable beyond the critical point, allowing quasistatic evolution to continue.

3. Conclusions

The principal conclusion is that, by removing the artificial assumption of a dominating viscosity from the theory, one finds a pattern of behavior supporting the preconception of fission as described above. There remain a number of mathematical details to confirm before this picture is mathematically solid (Lebovitz [1987]), but it appears at this writing that the impasse that affected the classical version of the fission theory is overcome on the revised version.

The theory suffers from an obvious defect: the models are uniformly dense whereas stars are centrally condensed. One may wonder whether the qualitative conclusions drawn from these idealized models would persist in the context of realistically stratified models.

The work described elsewere in this volume by Durisen employs realistically stratified models and extensive numerical computation, but also suffers from defects, in the nature of its preconception of fission. One of the elements of this preconception is that fission should be the immediate and direct outcome of the instability of the axisymmetric figure. Since the initiating perturbation carries the figure into a bar (not a dumbbell), it is difficult to understand the motivation for this idea. In fact the computations do not support it. One can construe this as confirming that the preconceived route to fission is not possible; one need not construe it to mean, as some have, that fission is not possible.

References

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