

Quite nice is Chapter IV; Bohr's theorem on bounded integrals of a.p functions and its extension to linear systems given by Bochner are included here.

Chapter VI on Banach space-valued a.p functions presents the approximation theorem following the reviewer [Ann. Ecole Normale Supérieure, 1962].

A quite complete list of references ends this short, but probably useful, monograph.

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Introduction of the methods of real analysis, by Maurice Sion. Holt, Rinehart and Winston Inc., New York, 1968. x + 134 pages. Canad. \$9.85.

In a mere 130 pages the author presents the basic ideas of the topological concepts of real analysis and measure theory. In fact, the book is divided into two parts. Part I concerns the topological concepts and Part II is entirely devoted to measure theory.

The first part starts with a chapter on set theory which distinguishes itself because of brevity. Chapter two deals with spaces of functions such as the classical sequence spaces, spaces of continuous function. In this short chapter the author only aims at the pertinent definitions. The remaining two chapters of the first part are devoted to the elements of point set topology. It includes subjects such as completeness, compactness, connectedness and the Baire category theory which are all essential in analysis.

Part II, which is devoted to measure theory, starts with a discussion of measures on abstract spaces. It includes the Jordan decomposition theorem and the theory of Carathéodory's outer measures. The Lebesgue-Stieltjes measure on the line are discussed and Lebesgue measure in \mathbb{R}^n . A chapter is devoted to the theory of integration. It discusses the basic limit theorems and the Fubini theorem. The book concludes with a chapter on the Riesz representation theorem.

In view of the many subjects which are covered in this book, the reviewer feels that it is a welcome addition to the existing literature in real analysis.

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A Seminar on graph theory, edited by F. Harary with L. Beineke. Holt, Rinehart and Winston Inc., New York, 1967. v + 116 pages.

The book contains in 111 pages the fourteen lectures of a seminar on graph theory held at University College, London, in 1962/63. The lecturers were: F. Harary (Lecture 1 - 6); L. Beineke (7); P. Erdős (8 - 9); P. Erdős and P. Kelly (10); J. W. Moon (11); C. St. J.A. Nash-Williams (12); R. Rado (13); C. A. B. Smith (14).

The lectures 7 - 14 are prefaced by quite remarkable "Steckbriefe" of the lecturers given by the editor F. Harary. The volume being dedicated to Pólya, Lectures 4, 5, 6 and 11 are concerned with his famous enumeration theory. In 4 one finds a proof, and in 5, a general pattern of applying Pólya's enumeration theorem; in 6, the counting series of graphs and digraphs are determined. Lecture 11 presents nine proofs of Cayley's theorem that the

number of trees with n labeled vertices is n^{n-2} . In 1, some basic concepts and problems near at hand are introduced. In 2, topological aspects are treated, especially connectivity and embeddability in orientable surfaces of given genus. The fundamental theorems of Menger and Kuratowsky are quoted. Theorem 12 of this lecture states that a planar graph can be drawn in the plane such that the edges are represented by line segments. This was independently found by several mathematicians. The first publication of this result known to the reviewer is: K. Wagner [Bemerkungen zum Vierfarbenproblem, Jber. DMV 46 (1936) 26-32].

Lecture 3 is about Ulam's conjecture that graphs U, V with vertices u_i and v_i respectively ($1 \leq i \leq n$) are isomorphic if $U - u_i$ and $V - v_i$ are always isomorphic ($1 \leq i \leq n$). The main aim of 7 is the determination of the thickness $t(m, n)$ of the complete bipartite graph $K_{m, n}$, where $t(m, n)$ is the least integer k such that $K_{m, n}$ is the union of k planar subgraphs. It is shown that $t(m, n) = \left\{ \frac{mn}{2(n+m-2)} \right\}$ with possibly some exceptions. The graph $G_r = K_{2r+1, 4r^2-2r+1}$ is critical, i.e. G_r has thickness $r+1$ and every proper subgraph of G_r has thickness at most r .

In 8, an account of extremal problems is given, of a topic which was initiated by P. Turán [Eine Extremalaufgabe aus der Graphentheorie, Mat. Fiz. Lapok 48 (1941) 436-452]. The results are of the following type: if the number m of edges of a graph with n vertices is sufficiently large, then G contains a graph of prescribed type. Example: If G has n vertices and $(2m-1)n-2m^2+m+1$ edges with $n > 24m$ then G contains m disjoint cycles.

In 9, applications of probabilistic methods in graph theory are discussed. Sometimes one is not able to construct graphs with certain properties, although the existence of such graphs can be shown by probabilistic reasoning: one shows that the number of (in most cases labeled) graphs in a suitable finite set and having not the prescribed properties is smaller than the number of the (labeled) graphs in the set.

In 10, the following problem is solved. A graph G with maximum degree d given, which is the least integer r such that G is the induced subgraph of some regular graph of degree d with r vertices? From the result follows that this least integer depends only on the degrees of G .

In 12, a short proof is given for J.B. Kruskal's result that for any sequence T_1, T_2, \dots of finite trees there are indices $m < n$ such that T_n contains as subgraph a subdivision of T_m . Nash-Williams extended this result later to infinite trees. [On Well-quasi-ordering Infinite Trees, Proc. Camb. Phil. Soc 61 (1965) 697-720].

Let C be a class of graphs. Then $G^* \in C$ is universal in the weak (strong) sense when every $G \in C$ is isomorphic to some (induced) subgraph of G^* . In 13, two results are proved - the first being due to N.G. de Bruijn, namely that in the class of locally finite graphs no universal graph can be found. The second result is that in the class of all denumerable graphs, a universal graph in the strong sense exists.

In 14, games G of two players A, B with physical positions P, Q, \dots and moves $(P, A) \rightarrow (Q, B), (P', B) \rightarrow (Q', A), \dots$ are considered. The corresponding bicoloured digraph $C(G)$ has vertices $(P, A), (Q, B), \dots$ being coloured A, B respectively. A central concept of 14 is that of remoteness which is an integral measure for optimal play: if A (B) can force winning A (B) tries to win as soon as possible, and if A (B) cannot prevent losing A (B) tries to postpone the defeat as long as possible. More generally selective, conjunctive and disjunctive compounds are considered, i.e. sets of games of two players A, B where at each step A (B) moves in some, in all, in exactly one of the component games respectively. For selective and conjunctive compounds the remoteness is defined and studied. For the study of disjunctive compounds the concept of Sprague-Grundy function is developed. For example, the Sprague-Grundy function f of the game G , the graph of which contains no cycles, is defined inductively for the vertices of $C(G)$: $f(v)$ is the least non-negative integer different from $f(v')$ for all vertices v' in which ends an edge from v .

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Progress in mathematics. Edited by R.V. Gamkrelidze. Translated from the Russian. Plenum Press, New York, 1968.

Vol. 1 : Mathematical analysis, ix + 246 pages. U.S. \$15.00.

Vol. 2 : Mathematical analysis, viii + 161 pages. U.S. \$15.00.

These volumes which are translations of Itogi Nauki-Seriya Matematika give a survey of recent work on mathematical analysis. The first volume contains three articles and the second contains two, each supplemented with an impressive list of references. Concepts and Notations are defined carefully to make these surveys readable by mathematicians moderately familiar with the general field. Important results are given in the form of theorems while for others, which are too big or involved, a brief indication is given. The text and bibliography are both sufficiently detailed and systematic to serve as reliable guides in the topics discussed. The five articles in the two volumes are as follows.

Operational Calculus, by V.A. Ditkin and A.P. Prudnikov (74 pages and 486 references) gives a brief survey of Mikusinski's operational calculus and related topics including operational calculus on a finite interval, and on the whole axis, operational calculus in several variables and operational calculus using Bessel operators. Fantappi 's analytic functionals, numerical methods and some applications of operational calculus are mentioned.

Spaces of Analytic Functions by V.P. Khavin (93 pages and 330 references) gives a general description of investigations made in recent years related to the application of the theory of topological normed linear spaces to problems concerning functions of one and more complex variables. It includes, among other topics, the relation between boundary values of analytic functions and generalized functions (distributions), Hardy's H^p -spaces, interpolation problems and axiomatic theories of functions, and spaces of vector-valued functions.

Operational Differential Equations, by V.V. Nemytskii, M.M. Vainberg, and R.S. Gusarova (78 pages and 330 references) gives a review of recent work on the theory of differential equations in linear spaces. The linear and non-linear differential equations with bounded operators are discussed in the first part. The abstract Cauchy problem for linear equations with unbounded operators is