

# Calculation of fusion rates at extremely low energies in laser plasmas

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**Abstract.** At temperatures and densities that are typical of plasmas produced by lasers pulses interacting with solid targets, at power intensities  $I > 10^{12} \text{ W/cm}^2$ , the classical Debye screening factor in nuclear reactions becomes comparable with the one of the solar core. Preliminary calculations about the total number of fusion reactions have been performed following an hydrodynamical approach for the description of the plasma dynamics. This approach is propaedeutic for future measurements of D-D fusion reaction rates.

**Keywords.** plasmas, nuclear reactions, hydrodynamics

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## 1. Introduction

At power intensities  $I \geq 10^{12} \text{ W/cm}^2$  laser generated plasmas supersonically expand in vacuum with ion temperatures of several hundreds of eV, and densities around  $10^{20} - 10^{21} \text{ cm}^{-3}$ . Their importance in nuclear astrophysics is constantly growing, because some unexplored energy domains persist in calculation of fusion reaction rates, especially at low energy when clouds of cold electrons may play a role as electron screening, boosting the number of fusion events. By following a methodology largely employed in plasma physics, it is possible to rescale plasma parameters (e.g. temperature and density) in order to make more similar to the real world our laboratory conditions. In our laser generated plasmas, in fact, the classical Debye screening factor (it depends somehow by the temperature over density ratio) becomes comparable with the one of the solar core. Preliminary calculations about the total number of fusion reactions are reported in Mascali *et al.*, 2010, assuming a parametrization of the initial plasma temperature and density and following the evolution of such parameters by means of the 'one fluid' hydrodynamical model of Anisimov *et al.*, (1993). In this work we present an updated version of the numerical code, which easily evaluates the 3D fusion rate, and finally the number of expected fusions per laser shot, including the electron screening. The model predictions about the dependence of the plasma temperature and density on time, on the initial laser energy (fluence) and on type of ablated material are in good agreement with the experimental data collected in our laboratory during the last year.

## 2. Theory and models

Among the various treatments existing in literature about fusion reactions (Rolphs and Rodney, 1998), we used the Gamow theory, which includes the tunnel effect of the nuclear Coulomb barrier:

$$\langle \sigma v \rangle = \left( \frac{8}{\pi \mu} \right)^{1/2} \frac{1}{kT^{3/2}} \int_0^\infty S(E) \exp \left[ -\frac{E}{kT} - \frac{b}{E^{1/2}} \right] dE \tag{2.1}$$

where  $E = \frac{1}{2} m_r v^2$  is the energy of the particles pair with reduced mass  $m_r$  and relative velocity  $v$ , assuming a Maxwell Boltzmann distribution with temperature  $T$ .  $\mu = \frac{m_r}{m_p}$  and  $\sigma(E)$  can be obtained by the Gamow theory with an energy value  $E_G = b^2 = (2\mu)\pi^2 e^4 \left( \frac{Z_1 Z_2}{\hbar} \right)^2$ ; here  $Z_1$  and  $Z_2$  are the atomic numbers of the pair and  $S(E)$  is the astrophysical factor. For non resonant reactions  $S(E)$  varies smoothly with energy and it can be developed in Mc-Laurin series, stopped at the quadratic term. For D-D reactions the series' coefficients are:  $S(0) = 0.05 \text{ MeV} \cdot b$ ,  $S(0)' = 0.0183 b$ ,  $S(0)'' = 4.24 \text{ MeV}^{-1} b$  (NACRE website). Therefore, assuming that the plasma is made of identical particles, the total number of fusions will be:

$$f_{TOT} = \frac{1}{2} N \langle \sigma v \rangle \int_0^\infty \rho(t) dt \tag{2.2}$$

where  $N$  is the total number of particles in the plasma and  $\rho(t)$  is the plasma density. Note that  $f_{TOT}$  depends on the density and the temperature variation in space and time (in laser produced plasmas they both strongly vary, spatially and temporally), other than on their absolute values. Hence eq. 2.2 must be discretized over small 3D cells in which density and temperature are evaluated step by step from the hydrodynamical code. Time intervals of 0.5 ns have been chosen, integrating over  $300 \times 300 \times 300$  cells, each one having a volume of  $10 \times 10 \times 10 \mu m^3$ .  $\rho$  and  $T$  have been determined by the equations:

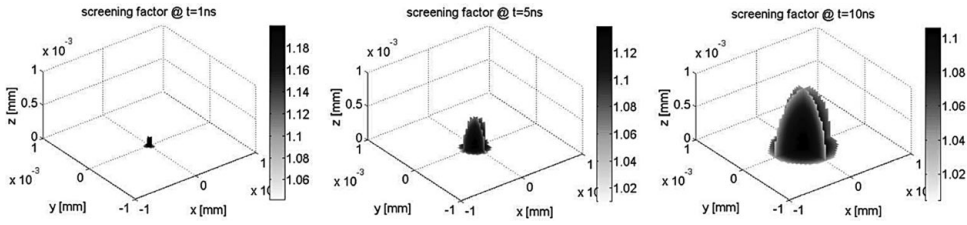
$$\rho(x, y, z, t) = \frac{M}{I_1 X Y Z} \left[ 1 - \frac{x^2}{X^2} - \frac{y^2}{Y^2} - \frac{z^2}{Z^2} \right]^{\frac{1}{1-\gamma}} \tag{2.3}$$

$$T(x, y, z, t) = \beta \frac{\gamma - 1}{2\gamma} \frac{m_p}{k_B} \left[ \frac{X_0 Y_0 Z_0}{X Y Z} \right]^{\gamma-1} \left[ 1 - \frac{x^2}{X^2} - \frac{y^2}{Y^2} - \frac{z^2}{Z^2} \right]^{\frac{1}{1-\gamma}} \tag{2.4}$$

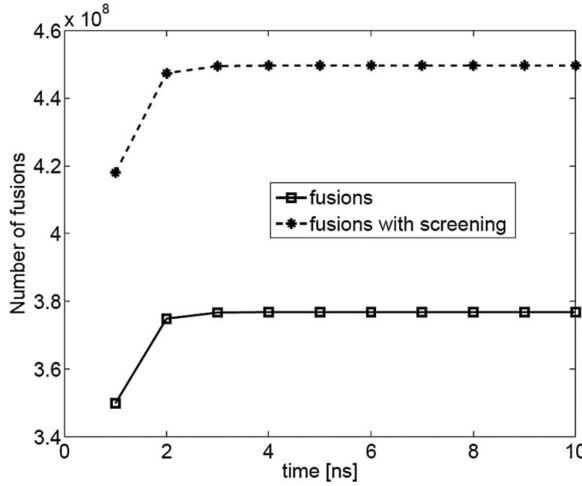
where  $I_1, \beta, \gamma, m_p, k_B$  are parameters connected with the adiabatic expansion of the plume, and  $X, Y, Z$  are the coordinates of the plasma front at a given time  $t$ , which can be calculated by the following set of second order differential equations:

$$\xi \frac{d^2 \xi}{d\tau^2} = \eta \frac{d^2 \eta}{d\tau^2} = \left( \frac{\sigma}{\xi^2 \eta} \right)^{\gamma-1} \tag{2.5}$$

where  $\xi(t) = X(t)/R_0$ ,  $\eta = Z(t)/R_0$ ,  $\sigma = Z(t)/R_0$ ,  $\tau = t/t_0$  denote the spatial and temporal dimensionless coordinates, and  $Z_0, R_0$  are the plasma dimensions at the end of the laser pulse. The model assumes a self-similar, isoentropic expansion of the plasma plume, whose shape is half-ellipsoidal at  $t = 0$  (for more details see Anisimov *et al.*, (1993)). In our calculations we assumed  $T_i \neq T_e$ , with initial temperatures  $T_{i0} \sim 450 \text{ eV}$  and  $T_{e0} \sim 45 \text{ eV}$ . This assumption is in agreement with empirical data; at power intensities comparable with the simulated ones, far from the target (i.e. in the free flight region) it is  $T_i \leq 10^2 \text{ eV}$  (these numbers are consistent with our initial condition if one considers the adiabatic cooling, which goes like  $T \propto \rho^{\gamma-1}$ );  $T_e$  may be determined by optical emission spectroscopy or electrostatic probes, and it is usually a factor 5 to 10 lower (Torrise *et al.*, 2010). This discrepancy is mostly due to the plasma acceleration



**Figure 1.** 3D calculation of the screening at different times after the laser shot.



**Figure 2.** Cumulated number of fusions after 24h of operations in laser repetition rate (10 Hz) mode versus the time after each laser shot.

mechanism, partially driven by self-generated electric fields. The resulting velocity of the moving plume is too small to give any appreciable contribution to the electrons’ kinetic energy (because of their small mass), but large enough to give a contribution to the ion energy content, that is even larger of the electrons thermal energy (collisional temperature equalization should give  $T_e \sim T_i$ ). The ordered, forward peaked energy of the ions is then transformed by the ion-ion collisions in thermal motion, finally resulting in an effective ion temperature  $T_i \gg T_e$ .

The initial plasma density (that resulted to be around  $10^{21} \text{ cm}^{-3}$ , assuming the validity of the quasi-neutrality condition  $\rho_e \simeq \rho_i$ ) was self-calculated by the code, assuming a laser pulse of duration  $\tau_L = 6 \text{ ns}$  and spot diameter of  $200 \mu\text{m}$  impinging on a virtual  $\text{CD}_2$  thick target. Once known the temporal variation of  $\rho$  and  $T$ , the electron screening was calculated according to the well-known Debye-Huckel formula (Salpeter, 1954):

$$f_{scr} = \exp\left(\frac{Z_1 Z_2 e^2}{k T_e \lambda_D}\right) \tag{2.6}$$

where  $\lambda_D$  is the Debye length ( $\lambda_D \sim 743 \sqrt{\frac{T_e [\text{eV}]}{n_e [\text{cm}^{-3}]}} [\text{cm}]$ ) and  $T_e$  the electron temperature.

### 3. Fusion rates calculations

Data coming from simulations are reassumed in figures 1 and 2. Figure 1 shows that the screening factor  $f_{scr}$  smoothly varies either in time and along the spatial coordinates; this occurs because the ratio  $\rho/T$  remains almost unvaried, although  $\rho$  and  $T$  rapidly

drop during the plasma expansion. The above mentioned discrepancy between electron and ion temperatures confers to laser generated plasmas at medium-low power intensity the unique property to have high enough ion temperature to favor a consistent number of fusion events, but low enough electron temperature to ensure a non negligible screening factor. About the absolute numbers, simulation results reported in figure 2 reveal that if the laser is operated in the repetition rate mode (10 Hz), after 24 hours the accumulated number of fusion is increased of a factor 1.2 by the electron screening. Note that almost the totality of the fusion events takes place in the first 2-3 ns after each laser pulse. The calculated screening factor is very similar to the stellar ones (Rolphs and Rodney, 1998).

The collected data put in evidence that an experiment with a deuterated target, and  $4\pi$  detectors for fusion products like neutrons, is possible and physically meaningful; the strong temporal concentration of the significative events will help to adequately trigger the acquisition system, improving the experimental precision. The forecasted experiment will give the opportunity to calculate the electron screening in a stellar-like environment, to test the validity of the classical Debye-Huckel theory and finally to evaluate the astrophysical factor at energy domains of the order of hundreds of eV, never explored up to now by nuclear astrophysics.

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