

CORRESPONDENCE.

APPROXIMATE VALUES OF ANNUITIES ON THREE
JOINT LIVES.

To the Editor of the Assurance Magazine.

SIR,—A question has presented itself to my mind, which appears to be of considerable interest, viz., whether the approximate value of an annuity on *three* joint lives would be more accurate, if it were obtained by means of the *curtate expectation* of the *two* joint lives, instead of being determined, as at present, by means of the *value of an annuity* on the same joint lives, at the particular rate of interest involved in the calculation.

Considering this to be an important question, I have troubled you with the following very brief investigation of it.

Mr. Milne has taken considerable pains—in note xiii. to art. 529, p. 720, vol. ii., of his valuable work—to investigate one portion of this particular class of approximations, viz., whether it be expedient to combine the age of the younger or of the elder life with the new age resulting from the previous combination of the two other lives; but I am not aware that the subject which I propose to discuss in the present letter has been brought under consideration by any of the various mathematicians who have written on the doctrine of life contingencies.

Simpson's rule, as given by Morgan, for approximating to the value of an annuity on the joint continuance of three lives, A, B, and C, is as follows:—

“Let A be the youngest, and C the oldest, of the three lives; take the value of the joint lives B and C, and find the age of a single life, D, of the same value nearly; then find the value of the joint lives A and D, which will be the answer.”

This rule, in my notation, would be expressed thus:—Find in the tables of the values of annuities $\overline{D} = \overline{BC}^1$; then find the value of $\overline{AD}^1 = \overline{ABC}^1$ for the answer.

Now the question I would propose is this—Whether the new age, D, resulting as above and as above ascertained, is probably more or less accurate than if the age were found from the tables of the curtate expectations of *two* joint lives (see the Carlisle Table, Jones, tab. xl.); as thus:—

$\overline{ED'} = \overline{BC}^1$; and from thence, $\overline{AD'}^1 = \overline{ABC}^1$ for the answer.

The assumption on which these approximations are founded is, that

$$\begin{aligned} \frac{b_1c_1}{bcr} + \frac{b_2c_2}{bc^2r^2} + \frac{b_3c_3}{bc^3r^3} + \frac{b_4c_4}{bc^4r^4} + \&c. = \overline{BC}^1 \\ &= \frac{d_1}{dr} + \frac{d_2}{dr^2} + \frac{d_3}{dr^3} + \frac{d_4}{dr^4} + \&c. = \overline{D}; \end{aligned}$$

which will necessarily give

$$\frac{a_1d_1}{adr} + \frac{a_2d_2}{adr^2} + \frac{a_3d_3}{adr^3} + \frac{a_4d_4}{adr^4} + \&c. = \overline{AD}^1$$

$$= \frac{a_1 b_1 c_1}{abc r} + \frac{a_2 b_2 c_2}{abc r^2} + \frac{a_3 b_3 c_3}{abc r^3} + \frac{a_4 b_4 c_4}{abc r^4} + \&c. = \overline{I_{ABC}}^1$$

but

$$\frac{b_1 c_1}{bc} + \frac{b_2 c_2}{bc} + \frac{b_3 c_3}{bc} + \frac{b_4 c_4}{bc} + \&c. = \overline{E_{BC}}^1$$

may be equal to

$$\frac{d'_1}{d'} + \frac{d'_2}{d'} + \frac{d'_3}{d'} + \frac{d'_4}{d'} + \&c. = \overline{E_{D'}}^1$$

without, however, D' being equal to D ; and our business is to determine which is probably the true age, D' or D . Now it is quite manifest that D' and D must be both older than either B or C ; and the two series

of which $\overline{E_{D'}}^1$ and $\overline{I_{D}}^1$ are respectively the sums, must each consist of fewer terms than either of the two corresponding series, of which $\overline{E_{BC}}^1$ and $\overline{I_{BC}}^1$ are respectively the sums. The series

$$\frac{d'_1}{d'} + \frac{d'_2}{d'} + \frac{d'_3}{d'} + \frac{d'_4}{d'} + \&c.,$$

therefore converges with greater swiftness than the series

$$\frac{b_1 c_1}{bc} + \frac{b_2 c_2}{bc} + \frac{b_3 c_3}{bc} + \frac{b_4 c_4}{bc} + \&c.;$$

and the series

$$\frac{d_1}{dr} + \frac{d_2}{dr^2} + \frac{d_3}{dr^3} + \frac{d_4}{dr^4} + \&c.,$$

converges with greater swiftness than the corresponding series

$$\frac{b_1 c_1}{bc r} + \frac{b_2 c_2}{bc r^2} + \frac{b_3 c_3}{bc r^3} + \frac{b_4 c_4}{bc r^4} + \&c.$$

It is therefore manifest that the sum of the first n terms of

$$\frac{d'_1}{d'} + \frac{d'_2}{d'} + \frac{d'_3}{d'} + \frac{d'_4}{d'} + \&c.,$$

will bear a greater proportion to the full value $\overline{E_{D'}}^1$, or, in other words, more nearly represent the sum of the entire series, than the sum of the first n terms of the series

$$\frac{b_1 c_1}{bc} + \frac{b_2 c_2}{bc} + \frac{b_3 c_3}{bc} + \frac{b_4 c_4}{bc} + \&c.,$$

although the sums of the two series complete may be in fact identical. Hence we deduce that the swifter the convergency of the series, the fewer will be the terms of significant value. Hence, also, any operation which tends to increase the convergency of the series, tends also to diminish the number of terms of significant value, and to throw the weight of the value of the total series into the earlier terms. Therefore the introduction of the element interest, by increasing the denominator of every fraction in the series by $\frac{1}{r}$ and its powers, renders it more rapidly convergent, and tends to throw a still greater proportion of the total value into the early terms of

the series. Hence it would seem that the age D is more likely to be the true age than the age D' ; and, further, the higher the rate of interest, the more likely is the approximation to be correct. And thus the answer to the question proposed, is, that the present mode of approximation by means of the values of annuities is preferable to that by the curtate expectation of the lives.

I am, Sir, your obedient servant,

PETER HARDY.

London Assurance, April 11, 1853.

QUALIFICATION AND ENROLMENT OF ACTUARIES.

To the Editor of the Assurance Magazine.

SIR,—With reference to the “Sketch of a Plan for the Establishment of a Professorship in connection with the Institute,” that appeared in the last number of this *Magazine*, and particularly with reference to the remarks made by Mr. E. J. Farren at the special meeting held on the 19th April last, on the subject of one of the resolutions, moved by that gentleman, to the effect that “no legislation on Life Assurance Associations can be permanently effective that does not exact tests as to the respectability and acquirements of persons allowed to practise as actuaries,” I beg to offer a few remarks.

Mr. Farren has suggested that there should be a register of persons who desire to practise as actuaries; and he proposes that, as a test of general attainments, the candidates for enrolment should be required to have passed the Matriculation Examination of the University of London.

Full particulars of the regulations of that body will be found in the *University Calendar* for the current year. I enclose, however, for your own information, a schedule of the subjects of that examination, as well as of those for the Degree Examination, and an extract from the prospectus of one of the affiliated Colleges of the University; from which you will see that the opportunity of attending the courses required by the University for a degree is, by a recent alteration in the regulations, now afforded to those whose daily engagements might, under the former *régime*, have prevented them from availing themselves of the advantages offered.

Mr. Farren’s scheme stops short, I regret to say, at the Matriculation Examination; after passing which, the candidate becomes only an undergraduate of the University, and is still supposed to be *in statu pupillari*. This examination may be passed by a youth of sixteen. Now I feel sure that upon reflection Mr. Farren will see that this examination is not sufficient, and that he will probably agree with me that the degree of B.A., at least, should be one of the qualifications for registration as an actuary.

Mr. Farren would also require the production of certificates of attendance at the lectures at some recognised Institution.

It is well known that the mere attendance at lectures is no test whatever of the attainments of the student (unaccompanied, I mean, by examination in the subjects of the lectures). Now if the candidate for registration is to be required to produce certificates of attendance at lectures, he may as well at once be required to attend such particular courses as will entitle him to be examined for the degree of B.A.

I think the possession of this degree, in addition to the diploma of the