

RESEARCH ARTICLE

Bayesian optimization of underground railway tunnels using a surrogate model

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Abstract

The assessment of soil–structure interaction (SSI) under dynamic loading conditions remains a challenging task due to the complexities of modeling this system and the interplay of SSI effects, which is also characterized by uncertainties across varying loading scenarios. This field of research encompasses a wide range of engineering structures, including underground tunnels. In this study, a surrogate model based on a regression ensemble model has been developed for real-time assessment of underground tunnels under dynamic loads. The surrogate model utilizes synthetic data generated using Latin hypercube sampling, significantly reducing the required dataset size while maintaining accuracy. The synthetic dataset is constructed using an accurate numerical model that integrates the two-and-a-half-dimensional singular boundary method for modeling wave propagation in the soil with the finite element method for structural modeling. This hybrid approach allows for a precise representation of the dynamic interaction between tunnels and the surrounding soil. The validation and optimization algorithms are evaluated for two problems: underground railway tunnels with circular and rectangular cross-sections, both embedded in a homogenous full-space medium. Both geometrical and material characteristics of the underground tunnel are incorporated into the optimization process. The optimization target is to minimize elastic wave propagation in the surrounding soil. The results demonstrate that the proposed optimization framework, which combines the Bayesian optimization algorithm with surrogate models, effectively explores trade-offs among multiple design parameters. This enables the design of underground railway tunnels that achieve an optimal balance between elastic wave propagation performance, material properties, and geometric constraints.

Impact statement

Railway-induced vibrations can disturb residents, interfere with sensitive instruments, and damage historical structures near railway corridors. In Europe, over 7.5 million residents are potentially affected by railway noise and vibrations. An effective approach to attenuate vibration levels at receiving points is to minimize the vibration source, which requires an accurate predictive model for assessing tunnel–soil interaction under dynamic loading conditions. While various models exist, none offer rapid and precise predictions suitable for complex tunnel–soil interaction problems and computationally demanding optimization tasks. This study presents an instant prediction model for optimizing the geometry and materials of underground railway tunnels, enabling industries to design tunnels based on specified constraints, with the objective of minimizing the elastic waves propagated into the surrounding soil.

 This research article was awarded Open Data badge for transparent practices. See the Data Availability Statement for details.

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1. Introduction

Dynamic soil–structure interaction (SSI) is an engineering discipline situated at the intersection of soil science and structural dynamics. It is closely related to fields such as earthquake engineering, geophysics, geomechanics, materials science, and computational mechanics (Kausel, 2006). Over recent decades, scientific and technical communities have shown growing interest in the dynamic assessment of SSI problems. Concurrently, a wide variety of modeling methodologies have been developed to address these complex engineering challenges. For SSI problems that can be assumed to be longitudinally invariant, such as railway tracks (Galvín et al., 2010), tunnels (Galvín et al., 2010; Jin et al., 2018), roads (François et al., 2010), and pipelines (Ozdemir et al., 2013), 2.5D modeling approaches (Lombaert et al., 2015) have proven to be a more efficient alternative to full 3D models (Andersen and Jones, 2006). The wavenumber–frequency domain has been widely adopted in models examining dynamic SSI between the soil and various types of infrastructure, including at-grade railway tracks (François et al., 2010; Galvín et al., 2010), underground tunnel systems (Galvín et al., 2010; Jin et al., 2018), roads (Lombaert et al., 2000), and pipelines (Ozdemir et al., 2013). This paper particularly focuses on underground railway tunnel–soil–structure interaction problems, as shown in Figure 1. It is important to note that the methodology developed in this study takes into account the full-space model of the soil. In the following sections, the existing studies on numerical models for the assessment of underground railway tunnel–soil interaction problems are presented, along with the introduction of surrogate models and optimization algorithms in this context.

1.1. 2.5D numerical models for SSI problems in elastodynamics

The applicability of 2.5D modeling approaches to SSI problems was first introduced through an application of 2.5D finite element method (FEM) to soil–structure systems (Hwang and Lysmer, 1981). Subsequently, a 2.5D approach incorporating finite and infinite elements was developed to model longitudinally invariant unbounded systems subjected to moving loads, with infinite elements employed to represent the unbounded domain (Yang and Hung, 2001). A widely adopted method for addressing SSI problems is the coupled FEM–boundary element method (BEM), where FEM is applied to model the structure and BEM is used to represent the soil medium. In this context, a 2.5D FEM–BEM approach was developed (Sheng et al., 2005). Another notable contribution involved the utilization of Green’s functions for a layered half-space as fundamental solutions, instead of full-space ones, resulting in a significant reduction in the number of boundaries requiring meshing (François et al., 2010). This method has been applied to predict railway-induced vibrations (Galvín et al., 2010). An alternate strategy for deriving

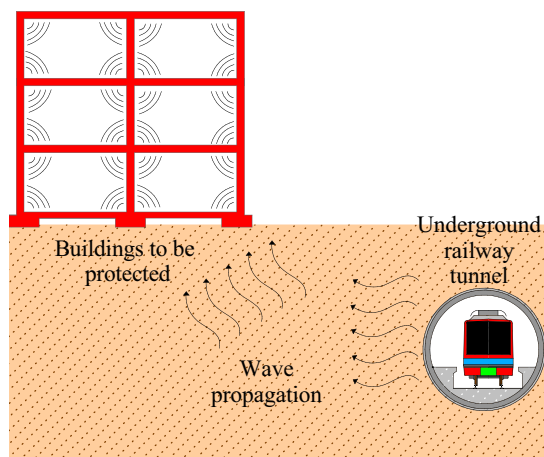


Figure 1. Schematic description of the underground railway tunnel–soil interaction problem.

Green's functions needed for 2.5D BEM in elastodynamics is the thin layer method (De Oliveira Barbosa et al., 2015). Furthermore, a 2.5D FEM–perfectly matched layer (PML) approach was introduced to study vibrations in buildings caused by underground railway traffic, employing FEM to model the structure and PML to represent the unbounded soil domain (Lopes et al., 2014).

Meshless methods, as an alternative to traditional mesh-based approaches, have garnered increasing interest from the research community over the past decades. These methods do not rely on a predefined mesh topology, leading to simpler formulations and computational implementation procedures, while maintaining the robustness and accuracy of mesh-based approaches (Liravi, 2022). In recent years, numerous studies have focused on the development of novel meshless approaches. Among these, the method of fundamental solutions (MFS) and the singular boundary method (SBM) are particularly well known. For instance, a 2.5D FEM–BEM–MFS approach has been proposed, wherein the BEM is first used to calculate SSI forces, followed by the application of the MFS to compute radiated elastic waves (Liravi et al., 2021). Despite the advantages of hybrid approaches such as FEM–MFS (Amado-Mendes et al., 2015; Colaço et al., 2021) and FEM–BEM–MFS, significant challenges remain when dealing with complex boundary shapes, including geometric singularities. To address these challenges, modified meshless strategies have been introduced, such as the FEM–SBM approach (Liravi, 2022). In the most recent developments, a hybrid method combining both the SBM and the MFS, referred to as the hybrid SBM–MFS, has been developed. This method capitalizes on the strengths of both techniques and is particularly effective for addressing problems involving geometric singularities in elastic (Liravi et al., 2024) and acoustic (Fakhraei et al., 2024) media.

Numerous alternative meshless methods have been proposed in the literature. For instance, the regularized meshless method was first introduced by Young et al. (2005) and later enhanced by incorporating the double-layer potential into the desingularization procedure (Liu, 2017). Another technique, known as the boundary-distributed source approach, replaces concentrated sources with circularly distributed ones, resulting in a singularity-free method (Kim, 2013). Additional meshless methods discussed in the literature include the dual surface method (Fakhraei et al., 2023), the boundary knot method (Sun et al., 2020), the localized boundary knot method (Wang et al., 2020; Yue et al., 2021), and the local Petrov–Galerkin method (Dehghan and Salehi, 2014). Although meshless methods are competitive in terms of efficiency for dynamic SSI problems, they still incur substantial computational time. Moreover, due to the inherent complexity of these problems, they limit the feasibility of a systematic investigation of design options.

1.2. Surrogate models

To reduce the lead time from design, that is, by varying the simulation parameters, to analysis results, surrogate models can be employed. This technique reconstructs the solution space by appropriate interpolation paradigms. It is successfully applied in several geotechnical problems, such as tunnel track design (Bui et al., 2023), real-time prediction of settlement (Cao et al., 2016), or vibrations induced by pile driving (Abouelmaty et al., 2024). Typical surrogate models include proper orthogonal decomposition using radial basis functions, Artificial Neural Network (ANN)-based models, and gradient boosting approach.

In the Proper Orthogonal Decomposition (POD) approach, snapshots are constructed by extracting information from the numerical results. This is often encoded in the vector form. Principal component analysis, such as Singular Value Decomposition (SVD) decomposition, is then used to extract the eigenvalues and eigenvectors. For the large problem, only several largest eigenvalues are kept to retain the most information of the input data. The interpolated solution of the original problem can then be reconstructed by employing Radial Basis Functions (Freitag et al., 2015; Bui et al., 2023).

Instead of performing numerical analysis on the snapshots, ANN approach mimics the way the human brain facilitates information by embedding the prediction data in the network weights via progressive training (Ninić and Meschke, 2015; Ninic et al., 2021). For numerical prediction tasks, a feedforward network is usually sufficient. Nevertheless, convolutional neural networks show to be exceptional in prediction involving temporal and historical data. However, in Bui et al. (2023), ANN is shown not to be as accurate as the POD approach for the numerical prediction task for the same computational effort.

Hybrid approaches are also developed to improve the efficiency and reliability of surrogate models. For example, Freitag et al. (2015) and Cao et al. (2016) combine POD and ANN for application in mechanized tunneling. Another branch of machine learning, the so-called Ensemble learning, has also gained widespread use in the geotechnical community (Thai et al., 2021; Xu et al., 2023). This technique relies on simpler learning models and uses boosting methods to connect and improve the robustness of the integral model.

1.3. Optimization algorithms

The surrogate model usually provides sufficiently smoothed output space for the optimization problem. In this regard, local optimization can be used, providing that the second gradient of the output with respect to the input space exists. In this case, the steepest descent approach or Newton Raphson algorithm will provide a quadratic converged solution efficiently. Nevertheless, for real applications, when the input parameters vary greatly and have intricate coupling, the local optimization may resort to a local minima solution (Nocedal and Wright, 2006). This effect becomes more profound if there are additional constraints. As a result, a locally optimized solution may not be the global one. To overcome this, a global optimization algorithm is required (Liravi et al., 2025). This method of searching for an optimized solution typically employs an evolutionary approach where an initial solution is created. The solution is then improved gradually by searching in the appropriate domains. To avoid the local minima, an exhaustive search can be used or a disruptive solution can be fabricated as in the case of genetic algorithms (Petrowski and Ben-Hamida, 2017). In particle swarm optimization, multiple solutions are defined in the swarm of particles, which then be steered toward the global optimum by simulating the social behavior of moving organisms (Ninić et al., 2017). This type of optimization method, together with the genetic algorithm, adopts the natural behavior and is heuristic by default.

Despite advancements in global optimization methods, many rely on iterative evaluations of computationally expensive objective functions, limiting their applicability in real-world engineering problems (Moens and Hanss, 2011). This gap highlights the need for a more efficient approach capable of balancing exploration and exploitation while minimizing computational costs. Bayesian optimization offers a compelling alternative for such scenarios (Shahriari et al., 2016; Diessner et al., 2024). By leveraging surrogate models, Bayesian optimization intelligently selects the next evaluation points based on probabilistic estimates of the objective function. This method not only reduces the number of required evaluations but also adapts to high-dimensional, noisy, and constrained problems (Shahriari et al., 2016), making it particularly well suited for tunnel–soil interaction problems or other scenarios with complex input–output coupling.

1.4. Problem statement and novelty

The optimization of geometrical and material properties in tunnel–soil interaction problems under elastodynamic conditions remains a challenging task due to the high computational cost of conventional meshless or mesh-based approaches. While surrogate models have shown promise in smoothing the output space to enhance optimization performance (Liu et al., 2016), their application to minimizing elastic wave propagation in the surrounding soil, by optimizing specific tunnel design parameters, has not been thoroughly investigated. This gap highlights the need for research into integrating surrogate models with optimization algorithms to address this computationally intensive problem effectively (Khowaja et al., 2019).

In this paper, a 2.5D FEM–SBM numerical method is employed to develop a surrogate model for longitudinally invariant SSI problems. In the 2.5D FEM–SBM approach, the structure is modeled using the 2.5D FEM, while a 2.5D SBM approach is adopted to represent the surrounding soil. The novelty of the present study lies in two key aspects. First, the combination of the versatility of the 2.5D FEM with the accuracy of the 2.5D SBM, along with the use of the Latin hypercube sampling (LHS) technique, ensures that the developed regression ensemble model is both accurate and well suited for optimization tasks. This

makes it a valuable alternative for the rapid assessment of underground railway tunnel–soil interaction problems in elastodynamics, specifically for cases with invariant longitudinal cross-sections. Second, the optimization algorithm takes into account both material and geometrical aspects of the underground railway tunnel structure. The integration of these two key elements makes this study the first to develop a surrogate for the optimization of underground railway tunnels from both material and geometrical perspectives, with the objective of minimizing the elastic waves propagated from the tunnel into the surrounding soil.

The remainder of this paper is structured as follows: [Section 2](#) introduces the governing equations of the problem and the mathematical formulation of the 2D FEM–SBM approach. [Section 3](#) describes the surrogate model used in this study and presents two examples to validate the developed surrogate model. [Section 4](#) focuses on the optimization of underground railway tunnels with circular and rectangular cross-sections. [Section 5](#) evaluates the performance of the optimized cases across the entire frequency range. Finally, the key findings of this study are summarized in [Section 6](#).

2. Numerical formulation

The general description of the 2.5D FEM–SBM approach, which will be replaced by the surrogate model, is illustrated in [Figure 2](#). The structure is modeled using FEM, while the unbounded domain representing the soil is modeled using SBM. The SBM method approximates the solution for the displacement and traction fields in the soil by employing a set of virtual sources placed along the boundary, which satisfy the boundary conditions evaluated at the collocation points, also distributed along the boundary. Further details of the method can be found in [Liravi \(2022\)](#).

2.1. Governing equations

The governing equations of elastodynamics are presented in this section. Consider the motion of a homogeneous, isotropic elastic solid occupying a three-dimensional region Ω , enclosed by the boundary Γ . Assuming the absence of body forces, the governing equation in Cartesian coordinates in the frequency domain for the solid is given by ([Kausel, 2006](#))

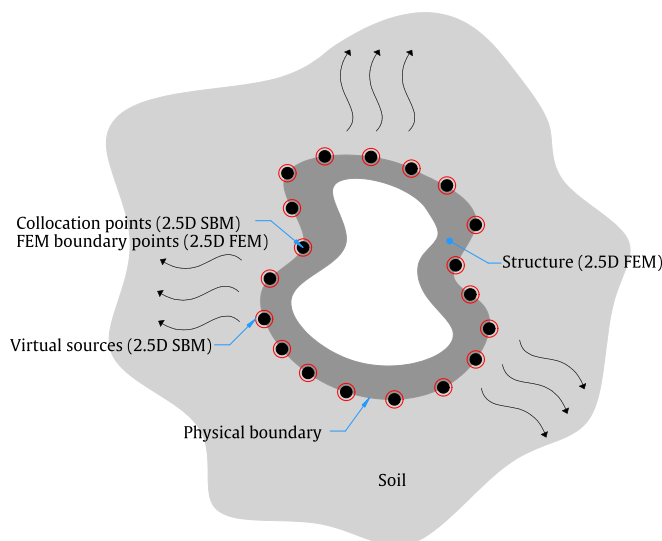


Figure 2. General description of the proposed 2.5D FEM–SBM methodology. Collocation and FEM boundary nodes are represented by black solid points, while virtual forces are indicated by red circles.

$$\mu \nabla^2 \mathbf{U} + (\lambda + \mu) \nabla (\nabla \cdot \mathbf{U}) + \rho \omega^2 \mathbf{U} = 0, \quad \mathbf{x} \in \Omega, \quad (2.1)$$

where $\mathbf{U} = [U_x, U_y, U_z]^T$ denotes the displacement vector at a position $\mathbf{x} = [x, y, z]^T$, $\nabla = \left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right]^T$ represents the gradient operator, and ρ is the mass density. The angular frequency is defined as $\omega = 2\pi f$, where f is the frequency in Hertz. The Lamé constants are given by $\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}$ and $\mu = \frac{E}{2(1+\nu)}$, with E being Young's modulus and ν the Poisson ratio.

For this boundary value problem, Dirichlet or Neumann boundary conditions are typically imposed as follows:

$$\mathbf{U} = \mathbf{U}_b, \quad \mathbf{x} \in \Gamma, \quad (2.2a)$$

$$\mathbf{T} = \boldsymbol{\sigma} \cdot \mathbf{n} = \mathbf{T}_b, \quad \mathbf{x} \in \Gamma, \quad (2.2b)$$

where $\mathbf{n} = [n_x, n_y, n_z]^T$ represents the outward unit normal vector to the boundary, $\mathbf{T}$ is the vector containing the three components of the traction field, $\boldsymbol{\sigma}$ is the stress tensor, and the prescribed values for the displacement and traction boundary conditions are denoted by the vectors $\mathbf{U}_b$ and $\mathbf{T}_b$, respectively. The components of the stress tensor for an isotropic medium are defined as

$$\sigma_{ij} = \lambda \delta_{ij} \nabla \cdot \mathbf{U} + \mu \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right), \quad (2.3)$$

where the indices i and j can take values 1, 2, or 3, representing the coordinates x , y , and z , respectively, and where δ_{ij} is the Kronecker delta.

When the system under study satisfies the condition of longitudinal invariance of its mechanical and geometrical parameters, it is often convenient to formulate the problem in the wavenumber–frequency domain, referred to as the 2.5D domain. The method presented in this paper is specifically designed for such problems. Assuming x as the direction of invariance, the governing equation and boundary conditions are transformed into the wavenumber domain using a Fourier transform of the form:

$$\bar{F}(k_x, y, z, \omega) = \int_{-\infty}^{+\infty} F(x, y, z, \omega) e^{ik_x x} dx, \quad (2.4)$$

where k_x is the wavenumber associated with the longitudinal direction x , F represents displacements U or tractions T in the spatial–frequency domain, and $\bar{F}$ denotes the corresponding quantities in the wavenumber–frequency domain. The detailed derivation is provided in Tadeu et al. (2001). For the sake of brevity, it is not included in the present paper.

2.2. 2.5D FEM–SBM methodology

Using radial basis function interpolation, the displacement and traction of the soil are approximated by the following linear combinations of fundamental solutions corresponding to N distinct source points:

$$\bar{\mathbf{U}}(\mathbf{y}) = \sum_{n=1}^N \bar{\mathbf{H}}(\mathbf{y}, \mathbf{x}^n) \bar{S}_n, \quad (2.5)$$

$$\bar{\mathbf{T}}(\mathbf{y}) = \sum_{n=1}^N \bar{\mathbf{H}}^T(\mathbf{y}, \mathbf{x}^n) \bar{S}_n, \quad (2.6)$$

Here, $\bar{\mathbf{U}}(\mathbf{y})$ and $\bar{\mathbf{T}}(\mathbf{y})$ represent the soil displacements and tractions, respectively, at an arbitrary field point $\mathbf{y}$. The vector $\bar{S}_n$ denotes the unknown strengths of the n th virtual source located at $\mathbf{x}^n$, while $\bar{\mathbf{H}}(\mathbf{y}, \mathbf{x}^n)$ and $\bar{\mathbf{H}}^T(\mathbf{y}, \mathbf{x}^n)$ are the soil's dynamic Green's functions for displacement and traction, respectively.

The SBM method assumes that these equations can be reformulated as follows (Liravi, 2022):

$$\bar{\mathbf{U}}(\mathbf{y}^m) = \sum_{n=1, n \neq m}^N \bar{\mathbf{H}}(\mathbf{y}^m, \mathbf{x}^n) \bar{\mathbf{S}}_n + \bar{\mathbf{H}}_{mm} \bar{\mathbf{S}}_m, \quad (2.7)$$

$$\bar{\mathbf{T}}(\mathbf{y}^m) = \sum_{n=1, n \neq m}^N \bar{\mathbf{H}}^t(\mathbf{y}^m, \mathbf{x}^n) \bar{\mathbf{S}}_n + \bar{\mathbf{H}}_{mm}^t \bar{\mathbf{S}}_m, \quad (2.8)$$

where $\mathbf{y}^m$ refers to the position of the m th collocation point, and $\bar{\mathbf{H}}_{mm}$ $\bar{\mathbf{H}}_{mm}^t$ are known as origin intensity factors (OIFs) in the SBM literature. The term “OIF” for the Neumann boundary conditions is defined in Eq. (2.8):

$$\bar{\mathbf{H}}_{mm}^t = \frac{1}{L_m} \left[\mathbf{I} + \mathbf{A}_m - \sum_{n=1, n \neq m}^N L_n \mathbf{H}^t(\mathbf{y}^m, \mathbf{y}^n) \right], \quad (2.9)$$

where $\mathbf{I}$ is the identity matrix and L_i is the half-length of the curve between the source points $\mathbf{x}^{i-1}$ and $\mathbf{x}^{i+1}$. In the case of the OIF associated with the Dirichlet boundary condition, the OIF corresponding to the m th collocation point can be directly computed as the average value of the fundamental solution over the physical boundary, which represents the small portion of the boundary containing the singular point. Detailed equations on the 2.5D FEM and the coupling equations between soil and structure can be found in Liravi et al. (2021, 2022).

3. Validation of the surrogate model

Surrogate models, serving as efficient predictive tools, bridge the gap between computationally intensive numerical simulations and demanding multi-objective optimization processes. In this study, the density and Young’s modulus of the tunnel, along with the tunnel lining thickness, concrete slab height, and the radius or side length of the tunnel, are included in the learning database to account for both geometrical and material properties of the tunnel within the surrogate model. A schematic representation of the geometrical parameters associated with both tunnel types is provided in Figure 3.

The range for the variation of each parameter in the parametric study is provided in Table 1. It is important to note that the selected parameter ranges fall within the realistic geometrical and mechanical properties of underground railway tunnels (Gupta et al., 2009; Ruiz et al., 2019; He et al., 2020). It is

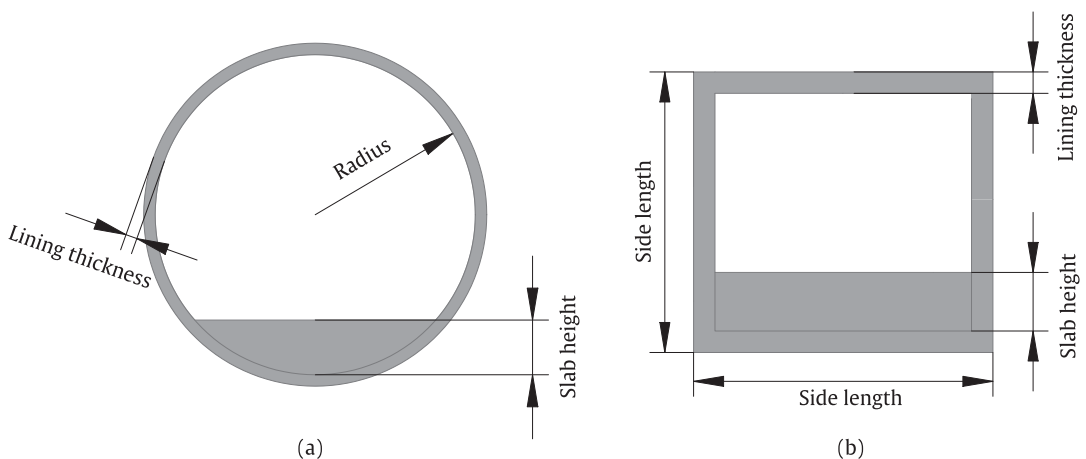


Figure 3. Schematic representation of underground railway tunnel with circular (a) and rectangular (b) cross-sections.

Table 1. Ranges of variation of each parameter for the surrogate model

Variable	Description
Tunnel lining's density (kg/m ³)	2,000–3,000
Tunnel lining's Young's modulus (MPa)	25,000–35,000
Tunnel lining's thickness (m)	0.2–0.5
Concrete slab height (m)	0.5–1
Radius of the circular tunnel (m)	3–6
Side length of the rectangular tunnel (m)	4–7

crucial to represent these parameters in the learning database in a balanced and systematic manner to prevent misleading results, such as overestimations or underestimations. To achieve this balance, the LHS technique is employed to generate input parameters. The primary aim of utilizing LHS is to reduce the number of simulations required while ensuring that the results remain reasonable and reliable (Iordanis et al., 2025). In this investigation, 300 samples are utilized for training, as increasing the sample size beyond this point did not result in a significant improvement in the model's accuracy. It should be noted that 80% of the samples are used for training, while the remaining 20% are allocated to validation. Preprocessing the data ensures that all variables are given equal importance during the training process, as discrepancies in the distributions of input parameters can lead to biases in the model, favoring features with larger values or variances. Furthermore, preprocessing enhances the speed and efficiency of the learning process. In this study, input parameters have been preprocessed using normalization, whereby the data are scaled to fall within a range of 0–1 using the following equation:

$$X = \frac{X - X_{\min}}{X_{\max} - X_{\min}}, \quad (3.1)$$

where $X_{\min}$ and $X_{\max}$ represent the minimum and maximum values of the input parameters, respectively.

In this study, a random forest (RF) model is employed to develop the surrogate method based on the dataset generated using the 2.5D FEM–SBM approach (Liravi, 2022, 2023) for predicting displacement and traction fields. RF is a regression machine learning model, primarily composed of an ensemble of randomized decision trees, denoted as $\left\{f\left(x; \Theta_t^{(i)}, R_n\right)\right\}_{1 \leq i \leq T}$. The sequence $\left\{\Theta_t^{(i)}\right\}_{1 \leq i \leq T}$ encapsulates the random variables Θ that govern the probabilistic mechanism underlying the construction of each tree. For a finite number of trees, T , the RF estimate can be expressed as follows:

$$f_T(X; \Theta_{1_t}, \Theta_{2_t}, \dots, \Theta_{T_t}, R_n) := \frac{1}{T} \sum_{i=1}^T f(X; \Theta_i, R_n). \quad (3.2)$$

The RF model uses a least-squares boosting algorithm with 300 boosting iterations and a learning rate of 0.05. Additionally, the minimum number of samples required in a leaf node of the tree is set to 10, and the maximum number of splits allowed in a tree is set to 30. The least-squares algorithm selects several bootstrap samples, $\left(R_n^{\Theta_{1_t}}, \dots, R_n^{\Theta_{T_t}}\right)$, and applies the previous tree-based decision algorithm to these samples to construct a collection of T prediction trees. It should be noted that the 2.5D FEM–SBM approach has been thoroughly validated in a previous study, which ensures the reliability of the synthetic datasets (Liravi, 2022). The framework for using RF regression for prediction is illustrated in Figure 4.

The developed surrogate model is validated through two examples: a circular and a rectangular railway tunnel, both embedded in a homogeneous full space. Two sets of evaluation points, designated as point A and point B, are defined in the ground, as shown in Figures 5a and 9a. The geometrical and material properties of the tunnel structure are randomly selected from the validation datasets. The target outputs of the surrogate model are the displacement and traction values at the evaluation points, while the

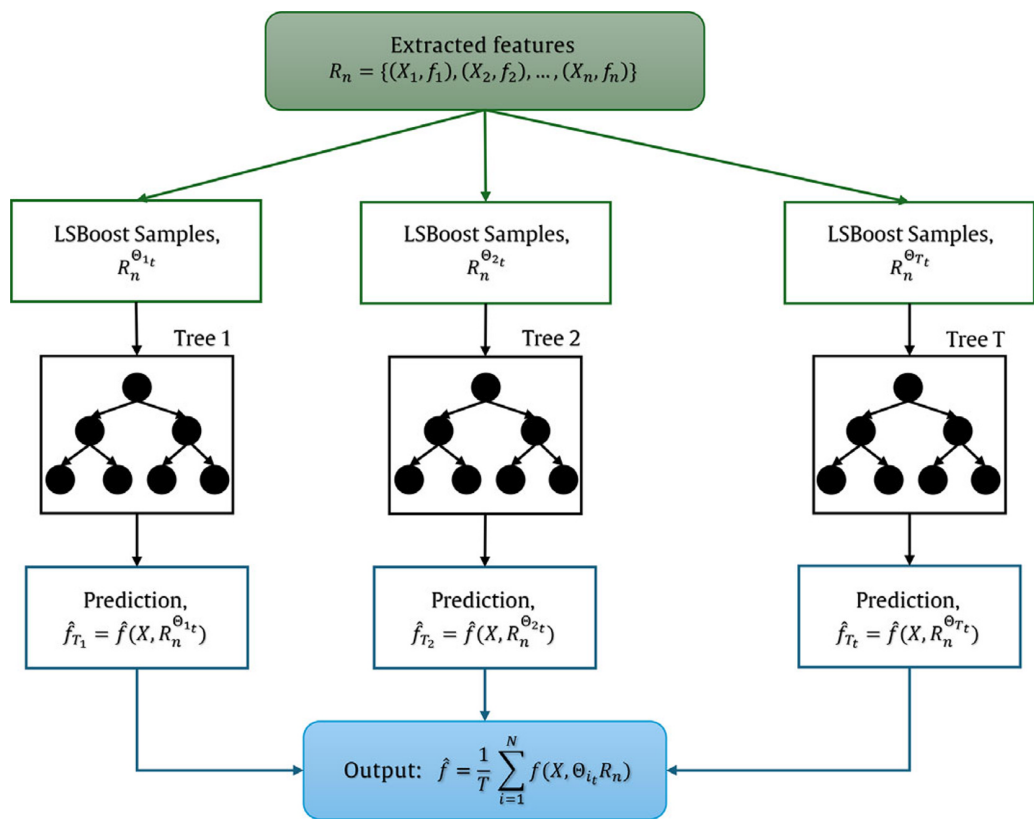


Figure 4. Illustration of Random forest regression construction.

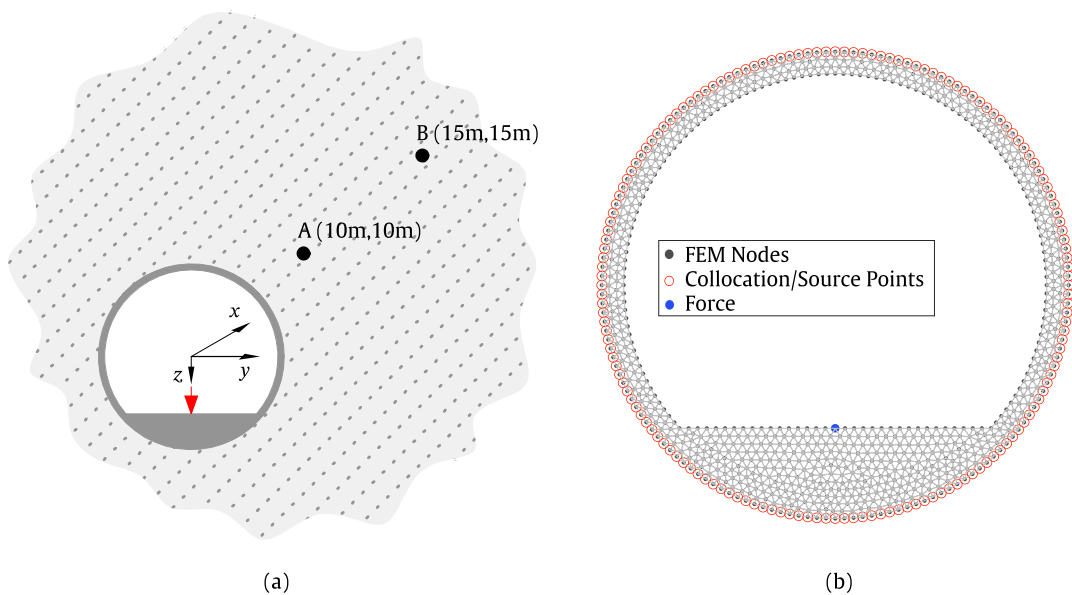


Figure 5. Schematic view of the example (a) and the FEM mesh for the circular tunnel (b). The FEM nodes, the collocation/source nodes, and the position of the applied forces used in the case study are also included.

receptances and traction transfer function (TTF) values are subsequently calculated using Eq. (2.4). The results obtained by the surrogate model are compared to the 2.5D FEM–SBM approach. The comparison is made up to 80 Hz, as railway-induced ground-borne vibrations within the frequency range of 1–80 Hz can be perceived by the human body (He et al., 2022). In all simulations, the number of collocation or boundary points along the tunnel boundary is chosen to ensure a minimum of 10 nodes per shear wavelength, based on the maximum frequency of interest, thereby ensuring accurate results (Liravi, 2022). In all calculations conducted in this study, receptance and TTF have been computed numerically with a total of 257 sampling points, consisting of $k_x = 0$ and a logarithmically spaced vector of wavenumbers ranging from 10^{-3} to 10^2 rad/m. Due to the symmetries of the displacements and tractions in the wavenumber–frequency domain caused by a vertical load, the x components of the receptances and TTF for this direction are null at $x = 0$ (Liravi, 2022). The results of the receptances and the TTFs are presented in decibels (dB), with references of 10^{-12} m/N and 1 (N/m²)/N, respectively.

The accuracy of the surrogate model is evaluated by calculating the relative root-mean-square error (rRMSE) with respect to a reference solution. The relative error is computed using the following expression:

$$\text{rRMSE} = \sqrt{\frac{\frac{1}{N_E} \sum_{j=1}^{N_E} |U^j - U_r^j|^2}{\sum_{j=1}^{N_E} |U_r^j|^2}}, \tag{3.3}$$

where j is the index representing the evaluation points and N_E is the total number of evaluation points considered in the calculation. Here, U^j and U_r^j denote the receptances at the j th evaluation point, calculated using the assessed method and the reference method, respectively. It should be noted that the rRMSE plots are only presented for receptance values, as these will later be utilized in the optimization algorithm in the application example.

3.1. Underground railway tunnel with circular cross-section

Circular cross-section tunnels are commonly used in deep tunneling due to their uniform structural force distribution, effective load-bearing capacity, and ease of shield construction. In this section, the performance of the developed surrogate model is assessed in the context of a circular tunnel embedded in a homogeneous full space. The mechanical and geometrical parameters of the tunnel and soil are presented in Table 2. The FEM mesh associated with this particular example is shown in Figure 5b.

As illustrated in Figure 6, there is good agreement between the receptances obtained by the developed model and those from the reference, 2.5D FEM–SBM approach, both in the y - and z -directions and at two evaluation points, with differences of up to 1 dB. However, slightly larger discrepancies are observed at point A responses in the y -direction, with differences reaching up to 2 dB. This good agreement is also evident in the TTF responses as indicated in Figure 7. The comparison against the 2.5D FEM–SBM results indicates that the surrogate model is an accurate tool for predicting both receptances and TTFs.

The rRMSE of the receptance values is plotted in Figure 8. As shown in this figure, the rRMSE values are generally low, with an average error of approximately 3% within the considered frequency range, reaching as low as 0.2% at lower frequencies. However, at some specific frequencies, slightly higher

Table 2. Mechanical parameters of the circular tunnel and the soil

Type	Radius (m)	Slab height (m)	Thickness (m)	E (MPa)	ρ (kg/m ³)	ν	Damping
Tunnel	3.1	0.9	0.3	30,589	2,754	0.2	0.001
Soil	—	—	—	108	1,800	0.3	0.05

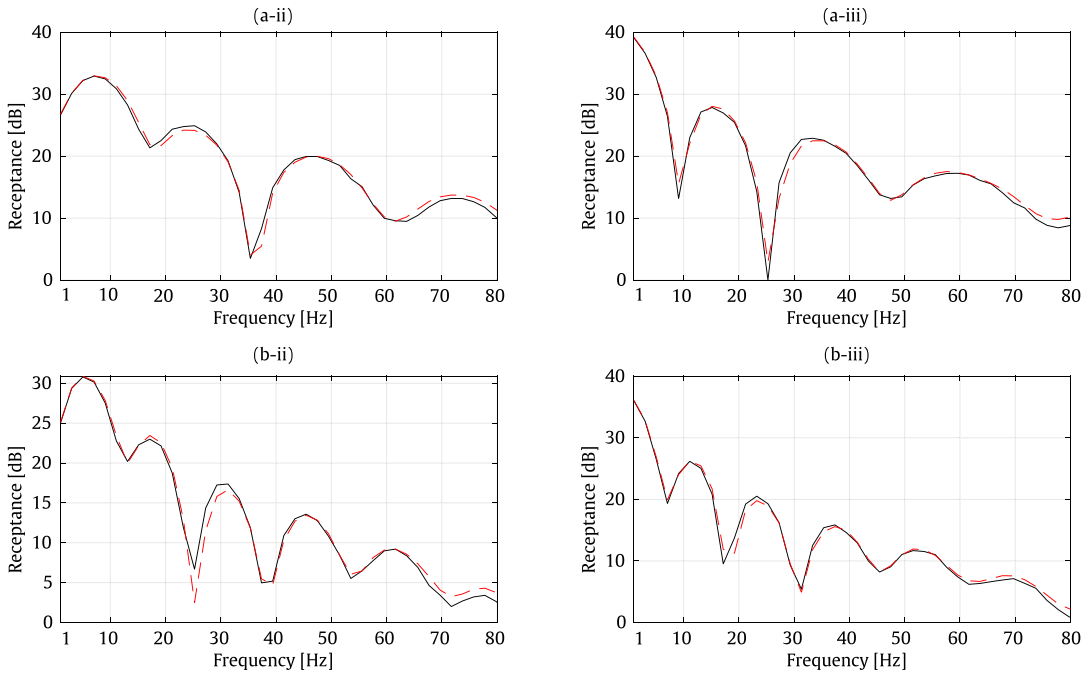


Figure 6. Receptances at points A (a) and B (b) for the y- (ii) and z- (iii) directions. Methods: 2.5D FEM–SBM (dashed red line) and surrogate model (solid black line).

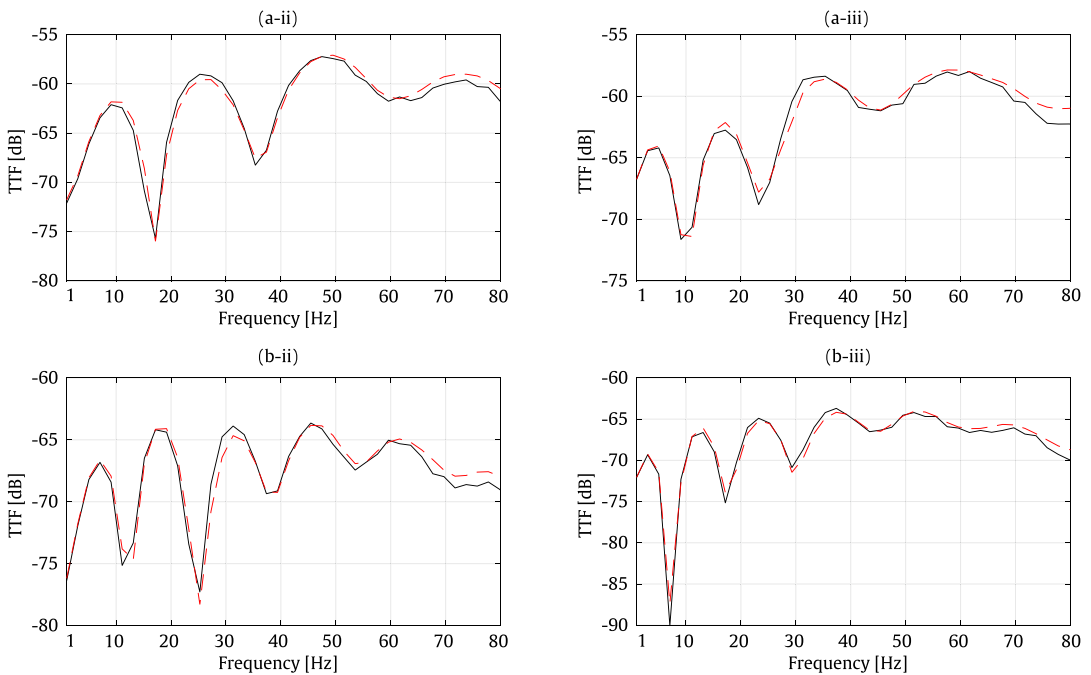


Figure 7. Traction transfer functions at points A (a) and B (b) for the y- (ii) and z- (iii) directions. Methods: 2.5D FEM–SBM (dashed red line) and surrogate model (solid black line).

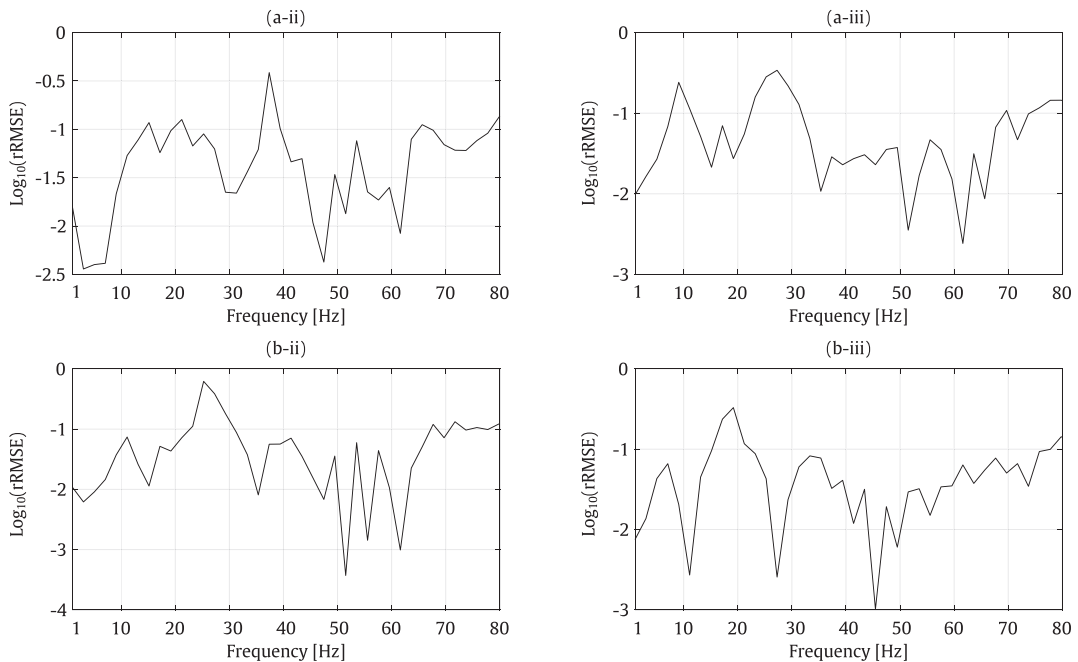


Figure 8. *rRMSE of Receptances at points A (a) and B (b) for the y- (ii) and z- (iii) directions for the underground tunnel with a circular cross-section.*

rRMSE values are observed, particularly at the positions of the resonant/anti-resonant frequencies. Nevertheless, as mentioned earlier, the average maximum error, when compared to the 2.5D FEM–SBM approach used as the reference solution, is approximately 2 dB.

3.2. Underground railway tunnel with rectangular cross-section

In this section, the accuracy of the developed model is compared to the reference method, in the framework of a rectangular underground tunnel embedded in a homogenous full space. The FEM mesh used for the numerical calculations of this example and the evaluation points in the soil are presented in Figure 9. The mechanical and geometrical properties of the tunnel and soil are presented in Table 3.

As demonstrated in Figure 10, the receptances obtained by the developed surrogate model are in good agreement when compared to the reference. A maximum of 3 dB differences can be observed, particularly at the location of the trough in the receptance trend. This good accuracy is also observed in the TTF responses, where, as shown in Figure 11, errors of up to 2 dB can be seen at specific locations on the trend. Overall, the proposed method demonstrates accurate results for the case of a railway tunnel with a rectangular cross-section.

For this example, the rRMSE plot of the receptance values is presented in Figure 12. As indicated, the rRMSE values are generally low, with an average error of approximately 3.5% within the considered frequency range, reaching as low as 1%, particularly at lower frequencies. However, similar to the previous example, at some specific frequencies, slightly higher rRMSE values are observed. As mentioned earlier, the maximum error does not exceed 2 dB.

4. Optimization

This study employs a Bayesian optimization method to optimize underground railway tunnels from specific geometrical and material perspectives. As previously mentioned, the optimization is conducted within the logical ranges of the considered parameters, as indicated in Table 1. The optimization process is

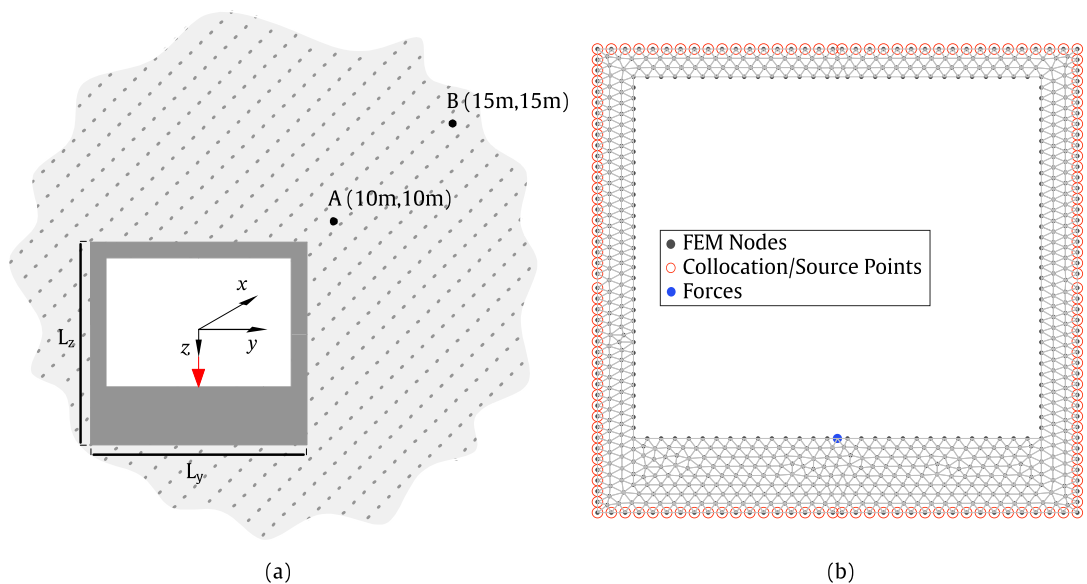


Figure 9. Schematic view of the example (a) and the FEM mesh for rectangular tunnel (b). The FEM nodes, the collocation/source nodes, and the position of the applied forces used in the case study are also included.

Table 3. Mechanical parameters of the rectangular tunnel and the soil

Type	Side length (L_y/L_z) (m)	Slab height (m)	Thickness (m)	E (MPa)	ρ (kg/m ³)	ν	Damping
Tunnel	5.1/5	0.57	0.22	29,475	2,675	0.2	0.001
Soil	—	—	—	108	1,800	0.3	0.05

illustrated through two case studies involving underground railway tunnels with circular and rectangular cross-sections. It should be noted that the optimization process is conducted at a single frequency. In this study, the frequencies of 20 and 80 Hz are considered, representing low and high frequencies, respectively. In this section, the objective of the optimization is to minimize the vertical receptance values at the target point (point A, as illustrated in earlier examples) while simultaneously optimizing the considered geometrical and material parameters of the underground tunnel. Specifically, the five parameters used for training the surrogate model, including Young's modulus, density, and thickness of the tunnel lining, as well as the concrete slab height and radius/side length of the tunnel, are the parameters to be optimized. Optimization iterations continue until the results converge and show no further changes. To ensure the accuracy of the optimization, each example has been run five times, and the optimal values presented are the average of these five repetitions. The optimization problem for an embedded structure in an elastic medium is defined as a classical optimization problem of the form:

$$\begin{aligned}
 &\text{minimize} && \Phi(\mathbf{x}) = \text{Receptance/TTF}, \\
 &\text{optimize} && \mathbf{g}(\mathbf{x}) = [L_y(\mathbf{x}) \& L_z(\mathbf{x})/r(\mathbf{x}), h_s(\mathbf{x}), t(\mathbf{x}), E(\mathbf{x}), \rho(\mathbf{x})], \\
 &\text{subject to} && \mathbf{x} \in \mathcal{X}, \\
 &&& g_i(\mathbf{x}) \leq 0, \quad i = 1, \dots, m, \\
 &&& h_j(\mathbf{x}) = 0, \quad j = 1, \dots, p,
 \end{aligned} \tag{4.1}$$

where Φ represents the objective function for the optimization algorithm (in this study, receptances or the TTF), L_y and L_z are the side lengths of the rectangular tunnel, r denotes the radius of the circular tunnel, h_s ,

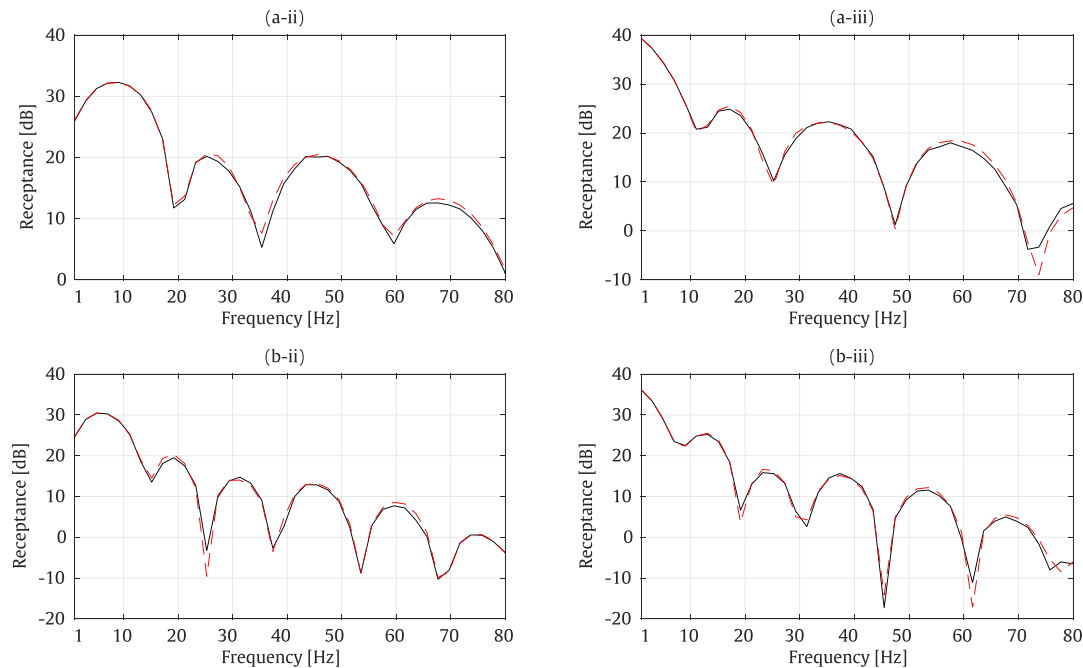


Figure 10. Receptances at points A (a) and B (b) for the y- (ii) and z- (iii) directions. Methods: 2.5D FEM–SBM (dashed red line) and surrogate model (solid black line).

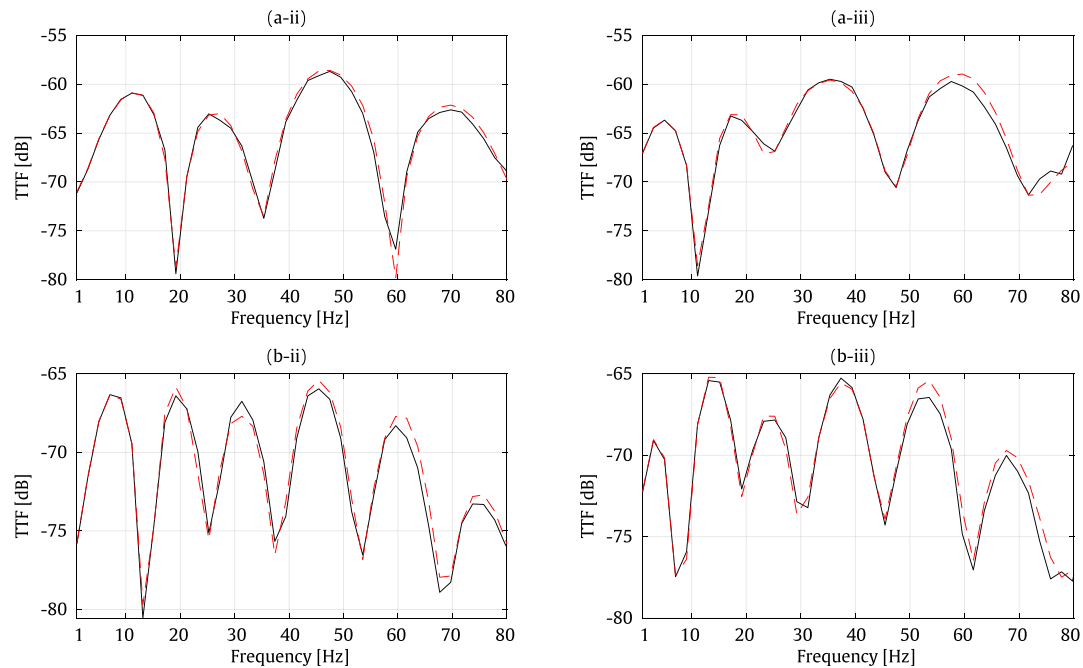


Figure 11. Traction transfer functions at points A (a) and B (b) for the y- (ii) and z- (iii) directions. Methods: 2.5D FEM–SBM (dashed red line) and surrogate model (solid black line).

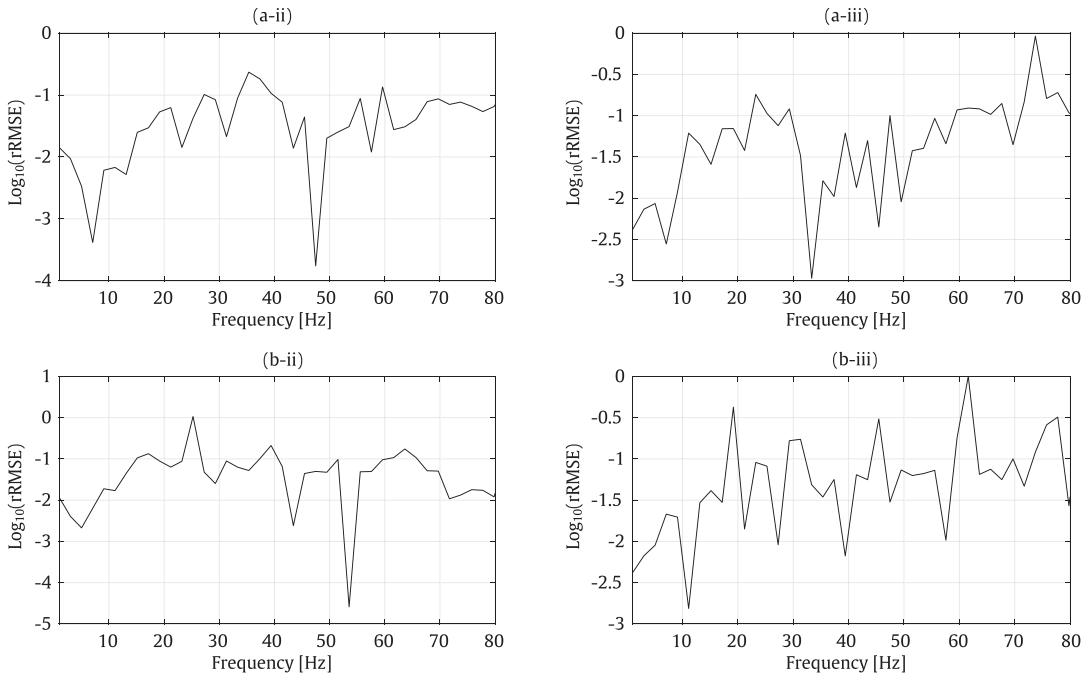


Figure 12. *rRMSE of Receptances at points A (a) and B (b) for the y- (ii) and z- (iii) directions for the underground tunnel with a rectangular cross-section.*

represents the slab height, t denotes the thickness of the tunnel lining, and $\mathbf{x}$ is the vector of design parameters. Meanwhile, $g_i(\mathbf{x})$ and $h_j(\mathbf{x})$ correspond to the inequality and equality constraints, respectively. These constraints are established based on the imposed limits on the design parameters and the elastic wave equation, as defined earlier in Eq. (2.1).

4.1. Underground railway tunnel with circular cross-section

The first optimization example considers an underground railway tunnel with a circular cross-section embedded in a homogeneous full-space medium. The optimization is visualized in Figures 13 and 14 for the frequencies of 20 and 80 Hz, respectively. As shown, the geometrical parameters, including the slab height and the thickness of the tunnel, are presented in one figure, while the material parameters,

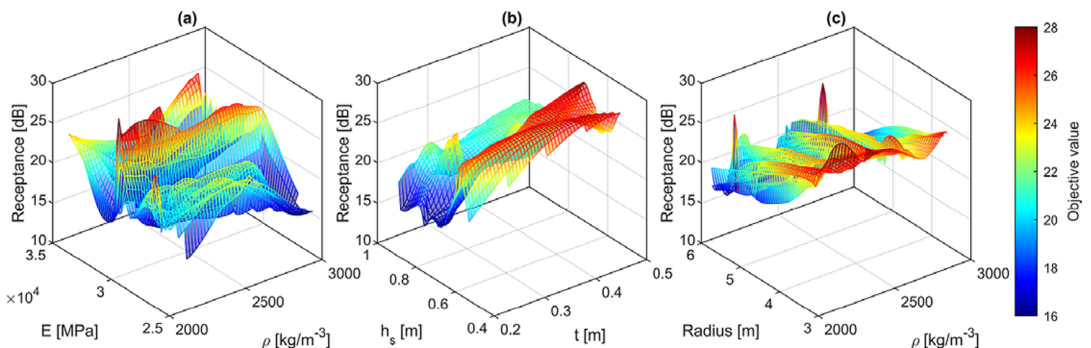


Figure 13. *Optimization plot for a railway tunnel with a circular cross-section at a frequency of 20 Hz, illustrating the variations in performance across different considered parameters.*

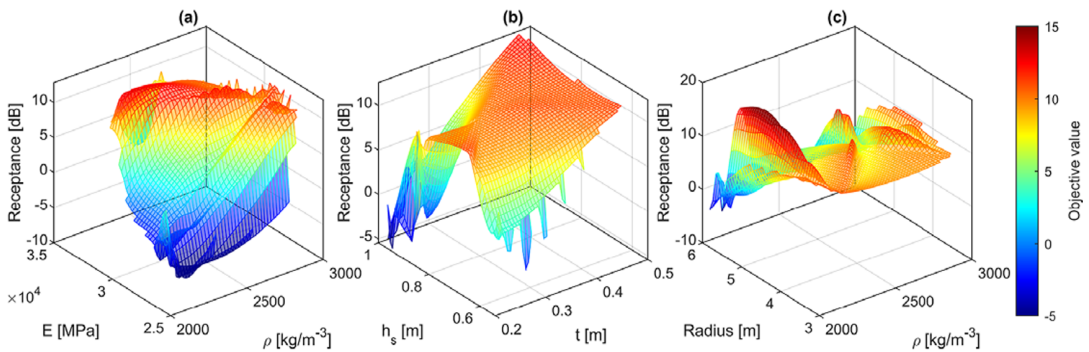


Figure 14. Optimization plot for a railway tunnel with a circular cross-section at a frequency of 80 Hz, illustrating the variations in performance across different considered parameters.

Table 4. Results of the optimization algorithm for the case of underground tunnel with circular cross-section

Point	Frequency (Hz)	Radius (r) (m)	Lining’s thickness (t) (m)	Slab height (h _s) (m)	E (MPa)	ρ (kg/m ³)	Receptance (dB)
A	20	5.9	0.21	0.6	31,808	2,990	18.4
	80	5.9	0.23	0.95	25,889	2,112	−3.1

such as Young’s modulus and density, are depicted together. In both figures, the minimum objective function is observed when the tunnel thickness is set between 0.2 and 0.3 m, and the slab height is greater than 0.6 m. At a frequency of 20 Hz, the minimum objective function is observed for material densities greater than 2,800 kg/m³ and Young’s modulus between 28,000 and 33,000 MPa. For a frequency of 80 Hz, these ranges shift to 2,000–2,200 kg/m³ for material density and 23,000–27,000 MPa for Young’s modulus, respectively.

Additionally, Figures 13c and 14c show the optimization iterations between a geometrical design parameter and a mechanical property, in particular, the radius of the tunnel is plotted against its density for the frequencies of 20 and 80 Hz, respectively. At both frequencies, the minimum receptance value is observed when the largest tunnel radius is selected. The lining thickness in both cases is optimized to its minimum value. The optimal values of the parameters under consideration are provided in Table 4.

When comparing the vertical receptances at point A for the optimized case against a random validation example presented in Figure 6b(iii), an attenuation of approximately 8 dB in vertical receptance can be observed at both considered frequencies.

4.2. Underground railway tunnel with rectangular cross-section

The second optimization case involves an underground railway tunnel with a rectangular cross-section, similar to the previous example, embedded in a homogeneous full-space medium. In this case, as illustrated in Figures 15 and 16, the visualization of the optimization process for the side lengths of the tunnel is presented together, while the thickness of the tunnel and the slab height are displayed in a separate plot. Additionally, the material properties of the tunnel, specifically the density and Young’s modulus, are presented together. The optimal values reveal that, at a frequency of 20 Hz, the side lengths of the tunnel should be set to 6.9 and 6.5 m, with the minimum thickness within the considered range selected to minimize the vertical receptance at point A. Similarly, at a frequency of 80 Hz, L_y and L_z should

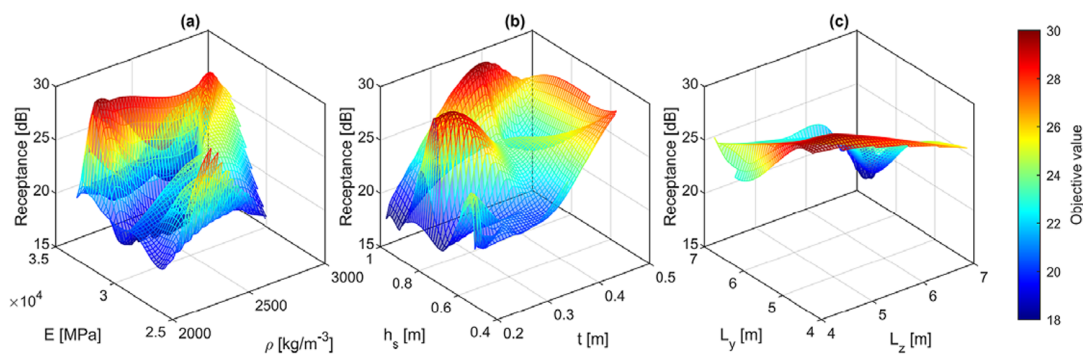


Figure 15. Optimization plot for a railway tunnel with a rectangular cross-section at a frequency of 20 Hz, illustrating the variations in performance across different considered parameters.

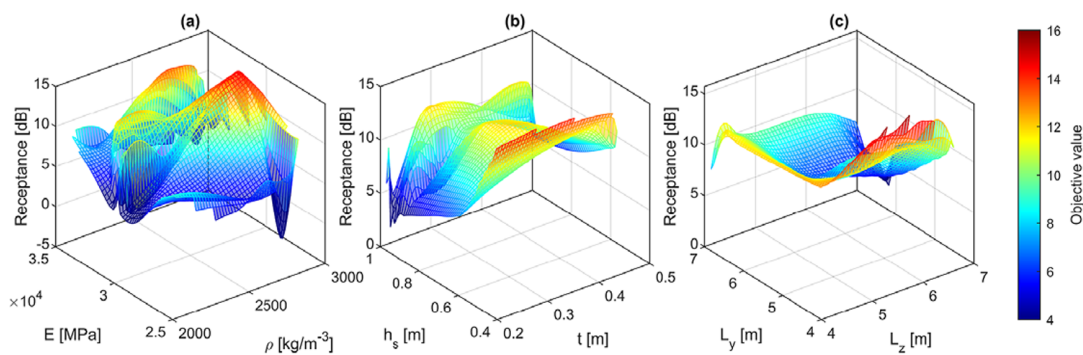


Figure 16. Optimization plot for a railway tunnel with a rectangular cross-section at a frequency of 80 Hz, illustrating the variations in performance across different considered parameters.

Table 5. Results of the optimization algorithm for the case of underground tunnel with rectangular cross-section

Point	Frequency (Hz)	L_y/L_z (m)	Lining's thickness (t) (m)	Slab height (h_s) (m)	E (MPa)	ρ (kg/m^3)	Receptance (dB)
A	20	6.9/6.5	0.2	0.92	30,955	2,553	16.7
	80	6.9/6.4	0.2	0.95	30,598	2,219	0.2

be set to 6.9 and 6.4 m, respectively, again with the minimum thickness within the considered range achieving the same goal. Furthermore, the optimal concrete slab height is approximately 0.9 m at both frequencies, while the density at 80 Hz is approximately 300 kg/m^3 lower than that at 20 Hz. The optimal values of the considered parameters across all optimization cases associated with this example are summarized in Table 5.

It is noteworthy that, when comparing the vertical receptance values presented in Figure 10b(iii) for a random validation case of an underground tunnel with a rectangular cross-section, an approximate reduction of 6 and 5 dB can be achieved by optimizing the tunnel from both geometrical and material perspectives, at the frequencies of 20 and 80 Hz, respectively.

5. Assessment of the performance of the optimized cases

To further highlight the differences between the optimized cases and the random validation case presented in Figures 6 and 10 across the entire frequency range, bar plots are provided in Figures 17 and 18 for the circular and rectangular tunnel cases, respectively. These plots show the receptance differences between the optimized cases and the validation case at 10-Hz intervals. The comparison considers both optimized cases at 20 and 80 Hz. In these plots, negative values indicate the extent of improvement achieved in the optimized case relative to the non-optimized case, while positive values represent a deterioration in the optimization outcomes.

For the case of an underground tunnel with a circular cross-section, a significant improvement is observed across the entire frequency range. Similarly, for the underground tunnel with a rectangular cross-section, the bar plots clearly demonstrate that the optimized cases result in improvements in vertical receptance throughout the frequency range. However, minor negative impacts of up to 2 dB are observed around 30–40 Hz.

Since the optimization process results in different design parameters for the tunnel at different frequencies, as shown in Tables 4 and 5, the design parameters can be selected based on the required target frequency, considering the problem under study. As demonstrated in this section, optimization at a specific frequency results in an improvement in the system's performance over a wider range of frequencies.

Overall, the results indicate that the proposed optimization approach is highly effective in enhancing elastic wave propagation performance at the entire frequency range. In summary, this optimization method offers a promising tool for improving the design of buried structures, making it suitable for practical applications where mitigating elastic wave propagation is a key concern.

6. Conclusion

This study presents a surrogate model for the optimization of underground railway tunnels with respect to specific geometrical and material aspects, developed in the wavenumber–frequency domain. The

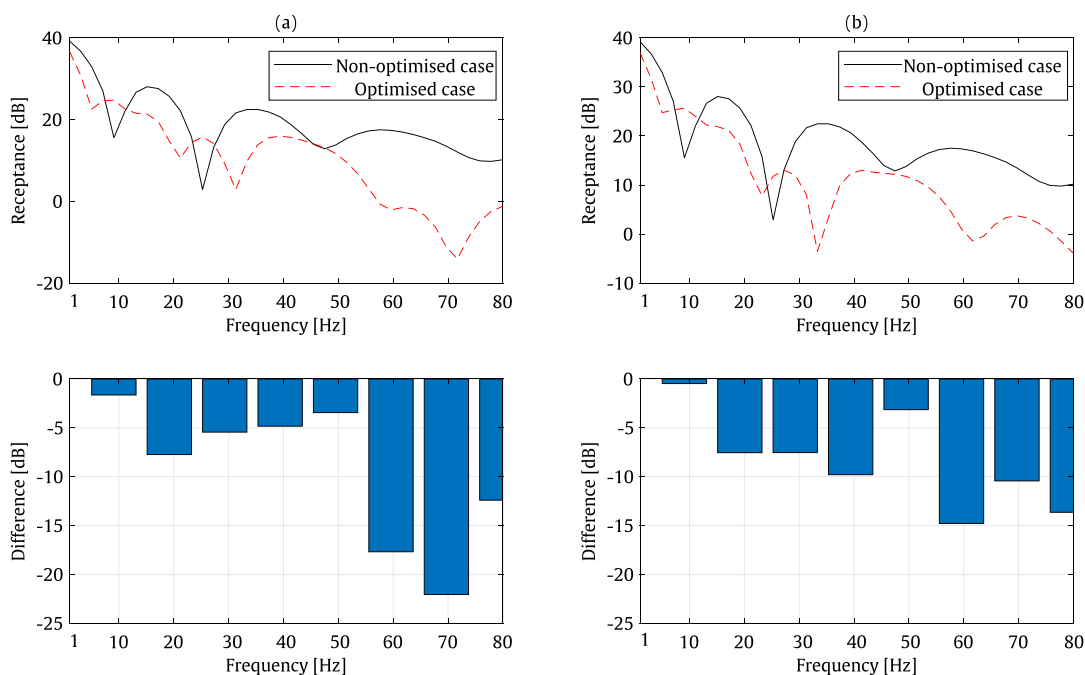


Figure 17. Comparison of vertical receptances computed at evaluation point A between the non-optimized case and optimized cases at 20 Hz (a) and 80 Hz (b), along with bar plots showing the differences between the cases in decibels for an underground tunnel with a circular cross-section.

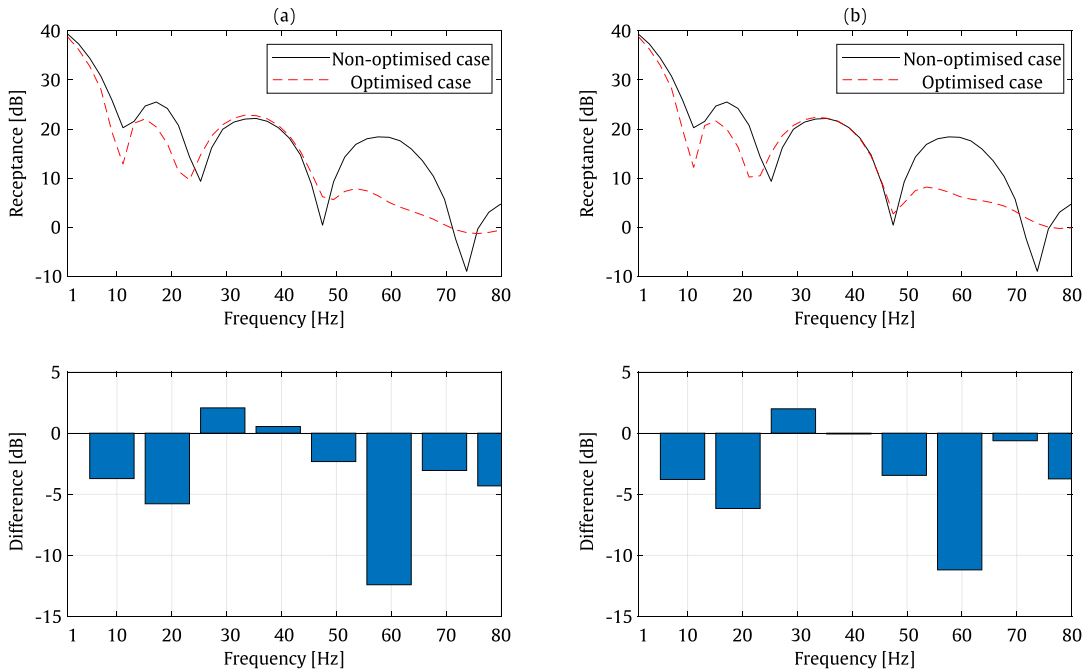


Figure 18. Comparison of vertical receptances computed at evaluation point A between the non-optimized case and optimized cases at 20 Hz (a) and 80 Hz (b), along with bar plots showing the differences between the cases in decibels for an underground tunnel with a rectangular cross-section.

validation and optimization processes are conducted in the context of underground railway tunnels with circular and rectangular cross-sections. The primary objective of the optimization algorithm is to minimize elastic wave propagation in the surrounding soil. The key findings of this research are as follows:

- The 2.5D FEM–SBM approach is an accurate, efficient, and robust numerical method for generating datasets for training the surrogate model. Furthermore, it effectively handles geometries that are more complex than circular tunnels, such as railway tunnels with rectangular cross-sections.
- The developed surrogate model demonstrates a high degree of accuracy in responses obtained at evaluation points when compared to the reference method, with an average rRMSE of receptances of approximately 3% and 3.5% observed within the considered frequency range for the examples of underground railway tunnels with circular and rectangular cross-sections, respectively. This precision is also evident in the TTF responses when compared to the reference method.
- The optimization case of an underground railway tunnel with a circular cross-section indicates that by optimizing specific geometrical and material perspectives of the underground tunnel structure, the vertical receptances at an arbitrary evaluation point in the soil, approximately near the tunnel, can be significantly mitigated. For the specific example presented in this study, an average reduction of 10 dB is observed in the target frequencies.
- The optimization example of an underground railway tunnel with a rectangular cross-section also provides promising results. For the specific case presented in this study, an average attenuation of 6 dB in vertical receptances can be achieved by optimizing the tunnel shape and material properties at target frequencies.

As a limitation, the optimization algorithm must be carried out based on the target frequency, although it may also demonstrate improvements at other frequencies, as evidenced by the results presented in this

paper. Furthermore, the model could be improved and extended in future work to incorporate both frequency and wavenumber as inputs for the surrogate model. This enhancement would lead to a significantly faster training process and reduced storage requirements. Overall, the optimization algorithm, combined with the surrogate model, proves to be a suitable prediction and optimization toolbox for addressing tunnel–soil interaction problems in elastodynamics. However, it should be noted that the current study has been implemented and developed exclusively for a homogeneous full-space soil medium. To achieve greater realism, the model should also consider a horizontally layered half-space medium.

Data availability statement. Replication data will be found in GitHub of Meta-NOVIB project (<https://github.com/HassanLiravi/Meta-NOVIB>) and on Zenodo (<https://doi.org/10.5281/zenodo.15491461>) Liravi (2025).

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Author contribution. Conceptualization: H.L., J.N.; Data curation: H.L.; Data visualization: H.L.; Methodology: H.L., J.N.; Writing—original draft: H.L., H.G.B.; Writing—review and editing: J.N., A.C., S.K. All authors approved the final submitted draft.

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Competing interests. The authors declare no competing interests.

Ethical standard. The research meets all ethical guidelines, including adherence to the legal requirements of the study country.

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