

# MEASUREMENTS AND ANALYSES OF VELOCITY PROFILES AND FRAZIL ICE-CRYSTAL RISE VELOCITIES DURING PERIODS OF FRAZIL-ICE FORMATION IN RIVERS

by

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## ABSTRACT

The vertical concentration distribution of frazil-ice crystals in a stream during the formation and growth of frazil ice was discussed in a preliminary way by Gosink and Osterkamp (1981). This paper extends and completes the analysis of buoyant rise velocities of frazil-ice crystals and applies the results to an interpretation of measured velocity profiles in rivers during frazil-ice events. Additional experimental data are also presented. Two time scales are defined: the buoyant time scale  $T_B$ , which represents the time required for a frazil crystal to rise, buoyantly, from the river bottom to the water surface, and the diffusive time scale  $T_D$ , which represents the time required for a frazil crystal to be transported by turbulence through the depth. It is shown that the ratio of the time scales  $T_B/T_D$  defines the nature of the layering processes; in particular, if  $T_B/T_D < 1$ , then buoyant forces will lift a frazil crystal faster than turbulent diffusion can redistribute it and the flow will be layered. Conversely, if  $T_B/T_D > 1$ , turbulent mixing will proceed faster than buoyant lifting and the flow will be well-mixed. This ratio, for frazil particles of diameter 2 mm or more, corresponds to rule-of-thumb velocity criteria developed in Norway and Canada to distinguish layered frazil-ice/water flow from well-mixed flow.

The development of this theory depends in large part upon the determination of  $T_B$ , which depends upon the rise velocity of frazil-ice crystals. A force balance model was developed for the rise velocity of a frazil crystal. Field observations during frazil-ice formation in Goldstream Creek and in the Chatanika River north of Fairbanks are reported, including a series of measurements of the rise velocities of frazil-ice crystals. Typical particle size of frazil ice was about 2 mm with a rise velocity of about  $10.0 \text{ mm s}^{-1}$ . The agreement of measured rise velocities with the theoretical model is good considering uncertainties in the drag coefficient and in the determination of frazil crystal sizes under field conditions.

Velocity profiles in the Chatanika River and in Goldstream Creek during frazil formation suggest that the time-scale ratio may serve as a transition criterion between layered frazil-ice/water flow and well-mixed flow. This ratio was calculated with the rise

velocity of frazil-ice crystals arbitrarily chosen to be  $0.01 \text{ m s}^{-1}$ .

## INTRODUCTION

Observations of ice formation in northern rivers during the freeze-up period show that three ice-flow regimes are common: sheet ice, floating frazil ice and a well-mixed flow (frazil-ice crystals and water). In the last case, if the velocity of the river is sufficiently high, open water conditions may persist throughout the winter.

Designers of hydraulic structures such as canals, water transportation facilities, hydroelectric power structures, water-intake structures, etc., may require information on which flow regime to expect under a given set of flow conditions and on the transition from an ice cover to open water flow. Thus, there is substantial interest in developing criteria which can be used to predict the conditions that lead to each of these three flow regimes and the presence or absence of an ice cover.

It could be argued that the criteria for the transition from floating frazil ice to a well-mixed flow should be the same as the criteria for the formation of an ice cover. The reason for this is that if the frazil crystals remain mixed with the flow then they cannot accumulate on the surface to form an ice cover. However, it is clear that other hydrological aspects of rivers enter into criteria on the presence or absence of an ice cover. These include the sinuosity, channel and slope variability, bed roughness and also the mechanics of frazil crystal, floc, pan and floe interactions (Osterkamp and Gosink in press) and changes of these parameters during the growth of an ice cover.

For many locations, rule-of-thumb flow velocities have been determined, which define the transitions from well-mixed flow to floating frazil ice and the formation of a coherent ice cover. For example, in Norway (where rivers are often wide, shallow, with steep slopes and underlain with rocks or boulders), the water surface is usually covered with moving frazil slush if the water velocity is  $>0.6$  and  $<1.2 \text{ m s}^{-1}$ , and open if the water velocity is  $>1.2 \text{ m s}^{-1}$  (Carstens 1971). On the Saint Lawrence Waterway in Canada layered frazil-ice flow may be expected at river velocities  $>0.8$  and  $<1.0 \text{ m s}^{-1}$ ,

and at velocities  $>1.0 \text{ m s}^{-1}$  the river surface usually remains free of ice (Starosolszky 1971). However, these empirical formulations are known to be imprecise as the critical velocity varies with water depth, bottom roughness and meteorological conditions. In particular, Bengtsson (1982) reports ice-free conditions on the Råne river in Sweden where flow velocity was  $<0.6 \text{ m s}^{-1}$  and flow depth was  $<0.5 \text{ m}$ . Carstens (1971) claims that the transition river velocity is reduced for flow depths  $>5.0 \text{ m}$ . Studies in Hokkaido, Japan (Hirayama 1982), suggest that river slope is another important parameter for the maintenance of ice-free conditions. Matoušek (1982) prescribes a formula for that velocity "at which all clusters of ice crystals still float on the surface". Unfortunately, no derivation for the formula nor comparison with data is given.

If frazil-ice crystals rise to the water surface faster than they are removed by turbulent mixing, a two-layer flow will result. Gosink and Osterkamp (1981) have proposed a criterion for layered vs well-mixed flow which depends upon competing time scales for buoyancy and vertical turbulent diffusion. The buoyancy time scale  $T_B$  is the time required for a frazil-ice crystal to rise the depth of the stream, and is given by  $T_B = h/V$  where  $h$  is the stream depth and  $V$  is the rise velocity of a frazil-ice crystal. The time scale for turbulent diffusion  $T_D$  is determined from the friction velocity of the stream according to standard vertical mixing theory for open channel flow. The ratio of these competing time scales  $T_B/T_D$  defines the nature of the layering process; if  $T_B/T_D < 1$ , buoyant forces will lift a particle of frazil ice faster than turbulent diffusion can redistribute it and the flow will be layered, and, if  $T_B/T_D > 1$ , turbulent mixing will proceed faster than buoyant lifting and the flow will be well-mixed.

This paper presents results of field and laboratory measurements of the rise velocity of frazil-ice crystals and of the velocity profiles in turbulent streams during periods of frazil-ice production. Data on rise velocities are compared with a simple theoretical model. The velocity profiles were analyzed to determine the suitability of a time-scale ratio as a criterion for the transition from layered to well-mixed flow.

RISE VELOCITIES

Experimental

The field experiments to measure rise velocities of frazil-ice crystals were performed during freeze-up in October 1981 in the Chatanika River north of Fairbanks. A transparent, graduated cylinder about 0.45 m long and 0.08 m in diameter was used to scoop water laden with frazil ice from the river. The cylinder was immediately set upright on a table and the motion of the crystals of frazil ice was observed. Velocities were measured by timing the displacement of the frazil particles past the graduations on the cylinder. Ice-crystal diameters were estimated by comparison with these graduations. Generally individual particles could be observed over displacement distances of at least 0.1 m. For a rise velocity of  $10 \text{ mm s}^{-1}$ , this suggests an observation period of 10 s.

Each measured rise velocity is shown as a function of ice-crystal diameter in Figure 1. The inaccuracy involved in the estimate of diameter may account for some of the scatter in the data. However, residual turbulent eddying could also, on occasion, be observed in the cylinder. Furthermore, in some instances a smaller disc of frazil ice was observed to rise faster than a larger disc, contrary to expectations. In several cases, a smaller disc of frazil ice could be seen accelerating upward in the wake of a larger disc, probably because of pressure drag from the larger disc. The proximity of the cylinder wall, which was eventually lined with needle-ice crystals,

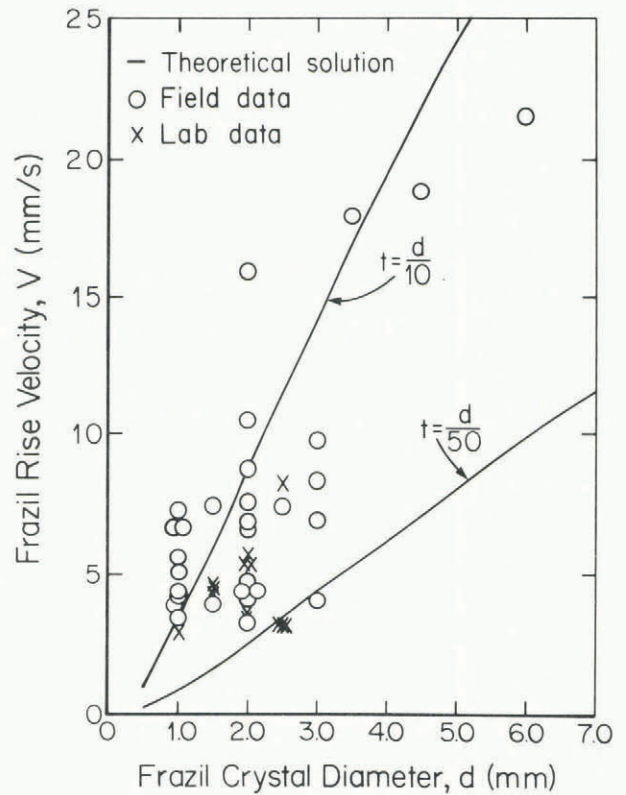


Fig.1. Measured and predicted frazil crystal rise velocity vs diameter.

could also affect the rise velocity of the frazil crystals.

Theoretical

Rise velocities of the frazil-ice crystals were predicted by a simple force balance model. The forces acting on the frazil-ice crystal are the upward net Archimedes force and the retarding drag force. The force balance is

$$\rho_i \pi (d^2/4) t a = (\rho_w - \rho_i) \pi (d^2/4) t g - (\rho_w C_D V^2/2) (\pi d^2/4), \quad (1)$$

where  $\rho_i$  is the density of ice ( $920 \text{ kg m}^{-3}$ ),  $\rho_w$  is the water density ( $1000 \text{ kg m}^{-3}$ ),  $d$  is the diameter of the frazil-ice crystal,  $t$  is the thickness of the frazil-ice,  $a$  is the acceleration of the crystal,  $g$  is the gravitational acceleration,  $V$  is the rise velocity, and  $C_D$  is the drag coefficient. Steady-state motion can be shown to be established within a very small distance equivalent to a few frazil disc thicknesses. The steady-state velocity of a frazil disc is given by

$$V = \sqrt{2 g' t / C_D}, \quad (2)$$

where  $g' = g(\rho_w - \rho_i) / \rho_w$  is the reduced gravitational acceleration.

Since the observed frazil ice diameters and rise velocities varied over a range of 1 to 6 mm and 3 to 22  $\text{mm s}^{-1}$ , respectively, the corresponding Reynolds numbers for these experiments varied between 1 and 75. Here Reynolds number is defined as  $Re = Vd/\nu$  and the water viscosity  $\nu$  is assumed constant ( $1.8 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$ ). According to Schlichting (1968)  $C_D$  ranges between 24 and 1 for these Reynolds numbers. Furthermore, both Stokes's (1851) and Oseen's (1910) approximations for  $C_D$  are inappropriate for  $Re > 5$ . Therefore, an empirical relation for the standard  $Re$  vs  $C_D$  curve (e.g. see Schlichting 1968 or Willmarth

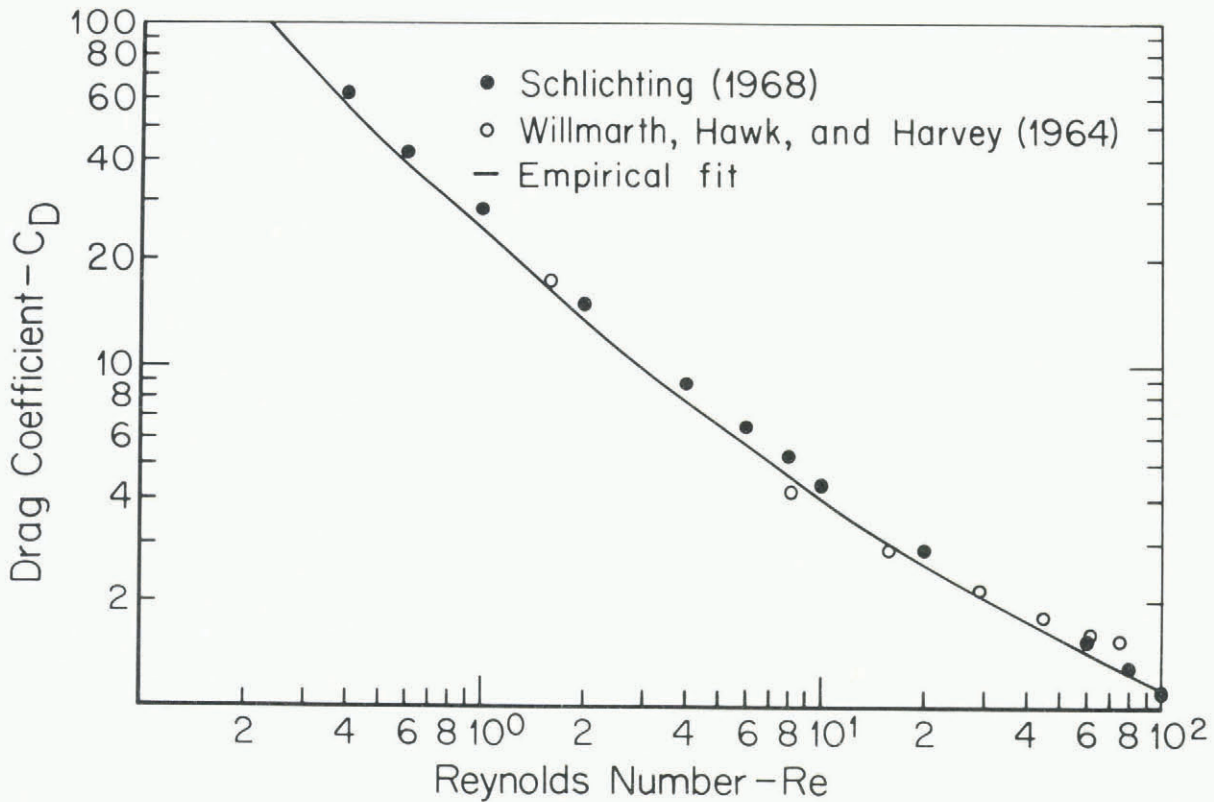


Fig.2. Empirical fit of drag coefficient vs Reynolds numbers of frazil-ice crystals.

and others 1964) was determined to give a best fit over the observed range of Reynolds numbers. This empirical relation is

$$\log C_D = 1.386 - 0.892 \log Re + 0.111 (\log Re)^2 \quad (3)$$

and is depicted in Figure 2 together with the data of Schlichting (1968) and Willmarth and others (1964) for discs. There appears to be some recent disagreement as to the "correct" values of  $C_D$  for Reynolds numbers above 100 (e.g. see Stringham and others 1969, Boillat and Graf 1981), with the more recent measurements indicating slightly higher values of  $C_D$ . However, within the range  $1 < Re < 100$ , the "standard"  $Re$  vs  $C_D$  curve is well-accepted.

Equations (2) and (3) were solved for the thickness  $t$  of the frazil-ice crystal in terms of the crystal diameter, i.e.  $t = d/n$ . Two sets of solutions of Equations (2) and (3) are shown in Figure 1 for the cases  $n = 10$  and  $n = 50$ . These two cases form an envelope for the data on the measured velocity of frazil ice vs diameter of frazil-ice crystals. From the model, a characteristic thickness for the frazil crystals in these experiments appears to be about  $t = d/20$ . Arakawa (1954) measured a rise velocity of  $2 \text{ mm s}^{-1}$  for frazil crystals with  $d = 1 \text{ mm}$ . This velocity is predicted by the present model when a diameter-thickness ratio of 18 is assumed.

As the frazil-ice crystal rises through the super-cooled water, the crystal grows due to the removal of the heat of fusion by convection. Since the rise velocity depends upon the crystal size, it is important to assess the effect of crystal growth rate upon the rise velocities measured in these experiments.

The semi-empirical model of Fernandez and Barduhn (1967) can be used to estimate the frazil crystal growth rate,

$$v = a W^{1/2} \Delta T^n, \quad (4)$$

where  $v$  is the growth rate in  $\text{cm s}^{-1}$ ,  $a$  and  $n$  are experimentally determined constants,  $\Delta T$  is the supercooling, and  $W$  is the shear velocity between the frazil crystal and the flow.

Fernandez and Barduhn's (1967) measurements of  $n$  in Equation (4) indicate that  $n$  is generally in the range  $1.5 < n < 2.0$ , contrasting with earlier studies in quiescent water (Hason 1952) where it was found that  $n \approx 1.0$ . It therefore appears that convective heat transfer, defined by Equation (4), is the maximum heat transfer mode when a shear velocity exists between frazil crystals and the flow. This suggests that whenever a shear velocity exists, frazil crystal growth by convective heat transfer will dominate other forms of growth.

Since  $v$  is controlled by the convective transfer of heat from the stagnation line of the frazil disc to the flowing water,  $W$  should represent the velocity difference between the water and the frazil crystal. Tsang (1982[a]) suggests that  $W$  should be a function of turbulent intensity. However, this assumption contradicts models which successfully predict pollutant and aerosol deposition from turbulent flow on horizontal surfaces (Csanady 1973). In these models, particles are thought to move with the fluctuating turbulent eddies; the motion of the particle then consists of a slow vertical velocity ("free fall velocity") superimposed upon the turbulent velocity of the surrounding fluid. Accordingly, the buoyant velocity  $V$ , or the "free fall velocity", as given by Equation (2), is the maximum shear velocity  $W$  between the frazil crystal and flowing water. Therefore, until the effects of turbulent intensity on growth rate are well established, we will assume that the maximum growth rate is dependent upon  $V$  and that  $V = W$ , and calculate the growth rate using  $V$ .

For relatively large values of supercooling ( $\Delta T = 40 \text{ mK}$ ) and maximum measured velocity ( $V = 20 \text{ mm s}^{-1}$ ), Equation (4) predicts  $v < 5.0 \times 10^{-3} \text{ mm s}^{-1}$ . Since the observation period

was about 10 s, the increase in  $d$  of a 5 mm frazil crystal is 1% which corresponds to a change in  $V$  of 0.5%. Therefore it should not be necessary to include the change in  $d$  in Equation (2) for these rise-velocity experiments.

In deep rivers, the frazil crystal growth rate may be large enough to significantly increase  $V$  and thus decrease the buoyancy time scale ( $T_B = h/V$ ). For example a 1 mm frazil crystal initially rising at about 5 mm s<sup>-1</sup> in water with  $\Delta T = 50$  mK would require only 100 s to double in size. According to Equation (2) this would result in a 40% increase in  $V$  and a corresponding decrease of 30% in  $T_B$ . However, if  $\Delta T = 10$  mK,  $v$  is an order of magnitude less, and the crystal would require 1 000 s to double in size.

For the field experiments described below,  $T_B$  was always <50 s and  $\Delta T < 40$  mK so that it was not necessary to consider growth rate in these field studies.

TURBULENT DIFFUSION TIME SCALE

Vertical mixing in streams is a rapid process dependent upon the intensity of the turbulence. According to Taylor's (1921) diffusion model for turbulence (see also Monin and Yaglom 1965, Fischer and others 1979) the standard deviation of the spreading width of a disturbance is

$$\sigma = (2D_T\tau)^{1/2}, \tag{5}$$

where  $D_T$  is the turbulent diffusion coefficient and  $\tau$  is time. Integrating the standard effective viscosity distribution for a logarithmic velocity profile through the depth to obtain an average vertical diffusion coefficient gives

$$D_T = 0.067 hu^*, \tag{6}$$

where  $u^*$  is the friction velocity. Fischer and others (1979) show that the cross-stream distribution of a neutral tracer released at a wall is everywhere within about 3% of its mean value when the standard deviation of the tracer is equal to the cross-stream distance. Applying this principle to the vertical distribution of frazil crystals, we assume that the frazil crystals are completely mixed when the standard deviation of the spreading width is equal to the river depth. Then the time scale for complete vertical mixing through the depth is found by setting  $\sigma = h$ , which implies that

$$T_D = h/0.134 u^*. \tag{7}$$

COMPARISONS OF BUOYANT AND MIXING TIME SCALES  
Time scale ratio

Since stream friction velocities are often in the range of 0.05 to 0.10 m s<sup>-1</sup>, the time scale for complete vertical mixing is of the same order of magnitude as the buoyancy time scale. The ratio

$$T_B/T_D = 0.134 u^*/V \tag{8}$$

is independent of flow depth. Furthermore,  $u^*$  can be related to the mean river velocity  $U$  by the Chezy coefficient  $C$  yielding

$$T_B/T_D = 0.42 U/CV. \tag{9}$$

The time-scale ratio in the form of Equation (8) suggests that river slope is an important parameter in the determination of well-mixed vs layered flow. This corresponds to the observations in Hokkaido (Hirayama 1982) that surface accumulations of frazil ice would not occur for steeply sloping rivers. That is,  $T_B/T_D$  increases as river slope increases, and for  $T_B/T_D > 1$  the time-scale ratio predicts well-mixed flow. The ratio in the form of Equation (9) empha-

sizes the importance of the river roughness or  $C$ . As roughness increases,  $C$  decreases, and the time-scale ratio increases.

A measure of the time-scale ratio can be found if we assume a specific value for rise velocity  $V$ . The ratio gives a critical condition for frazil crystals of a particular diameter  $d$  with velocity  $V$ . All smaller crystals will have a smaller  $V$ , and therefore a larger value of  $T_B/T_D$ , and subsequently will remain well-mixed. Frazil crystals larger than  $d$  will form a stratified flow. Since observations of diameters of frazil-ice crystals in rivers are generally in the range of 0.1 to 5 mm (Osterkamp 1978, Osterkamp and Gosink in press), the appropriate choice of rise velocity to use in the time-scale analysis is probably close to 0.01 m s<sup>-1</sup>. This value is somewhat arbitrary and more research regarding the size and velocity distributions of frazil crystals is required to define a characteristic velocity more precisely.

With this value of  $V$ , the time-scale ratio from Equation (9) may be written

$$T_B/T_D = 42 U/C, \tag{10}$$

where  $U$  is in m s<sup>-1</sup> and  $C$  is in m<sup>1/2</sup>s<sup>-1</sup>. Tsang (1982[b]) reports Chezy coefficients in the Beauharnois Canal along the Saint Lawrence Waterway less than 42 m<sup>1/2</sup> s<sup>-1</sup> and other estimates range between 30 and 40 m<sup>1/2</sup> s<sup>-1</sup>. For this canal, assuming  $C = 38$  m<sup>1/2</sup> s<sup>-1</sup>,

$$T_B/T_D = 1.1 U,$$

and for well-mixed flow, the criterion  $T_B/T_D > 1$  implies  $U > 0.91$  m s<sup>-1</sup>, which is close to the rule-of-thumb value for the transition from layered to well-mixed frazil flow of 0.8 m s<sup>-1</sup> accepted for the Saint Lawrence Waterway.

Similarly for Norwegian rivers, where  $C$  may be chosen as 25 m<sup>1/2</sup> s<sup>-1</sup>, which is appropriate for steep, shallow and rough rivers, the time-scale ratio gives  $U > 0.60$  m s<sup>-1</sup>, in agreement with the accepted rule-of-thumb value of 0.60 m s<sup>-1</sup>.

Matoušek (1982) has devised a formula for the critical velocity to maintain fully-mixed flow which also defines critical velocity in terms of  $C$ ,

$$U > 0.064 \sqrt{C(0.7C + 6)} \text{ m s}^{-1}. \tag{11}$$

The Matoušek formula yields high values of critical velocity, or, conversely, requires extremely low values of  $C$  to match observed critical velocities. For example,  $C = 4.3$  m<sup>1/2</sup> s<sup>-1</sup> is required to define the critical velocity of 0.60 m s<sup>-1</sup> corresponding to Norwegian rivers. In contrast, the present formula predicts the observed critical velocities with a more reasonable value of  $C$ .

Present data

Buoyant and turbulent diffusive time scales were calculated for two field sites during the initial frazil ice formation. The friction velocity  $u^*$  was calculated from the water slope  $S$  and hydraulic radius  $R$ , ( $u^* = \sqrt{gSR}$ ). The calculated values of friction velocity, buoyant time scale, turbulent diffusive time scale and time-scale ratio are listed in Table I.

TABLE I. CALCULATION OF BUOYANT AND TURBULENT TIME SCALES

	Chatanika River	Goldstream Creek
Surface slope	0.00235	0.0012
Mean width	19.00 m	8.00 m
Mean depth	0.345 m	0.40 m
Hydraulic radius	0.3329 m	0.3636 m
$u^* = \sqrt{gSR}$	87.5 mm s <sup>-1</sup>	66.7 mm s <sup>-1</sup>
$T_B = h/V$	34.5 s	40.0 s
$T_D = h/0.134 u^*$	29.4 s	44.8 s
$T_B/T_D$	1.2	0.9

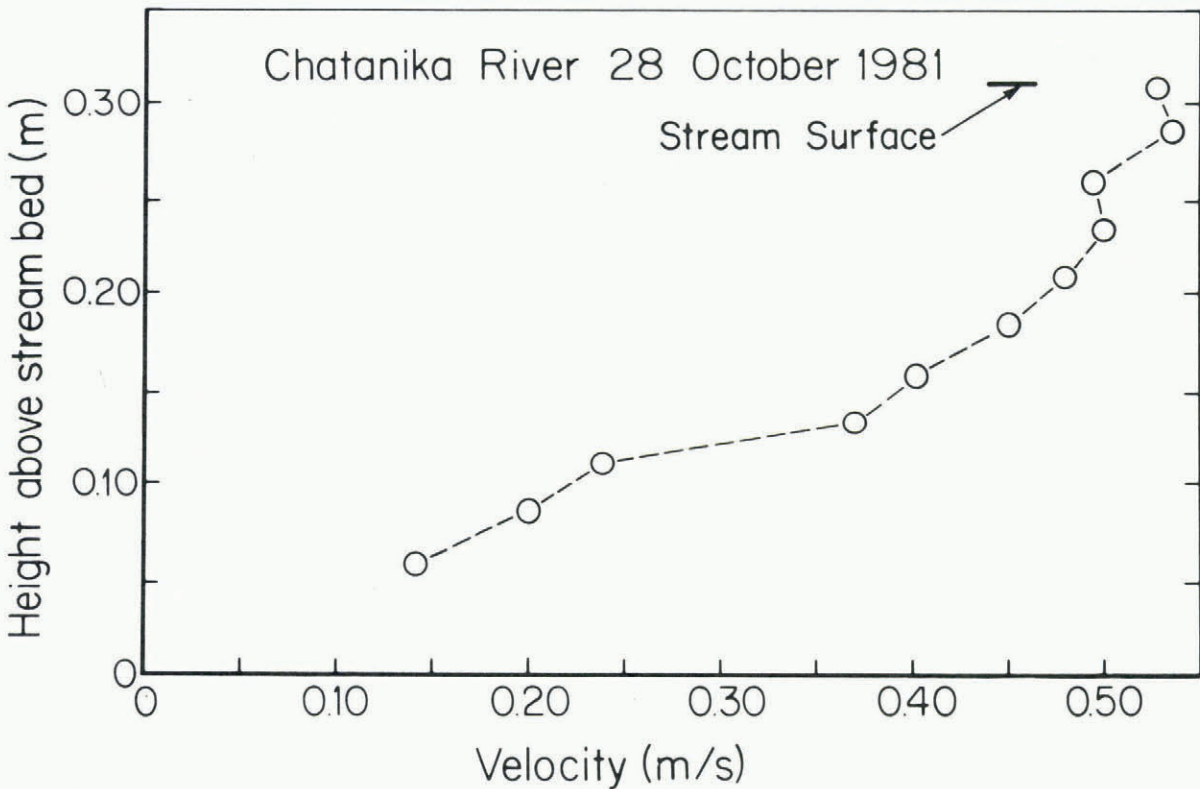


Fig.3. Measured velocity profiles in the Chatanika River, 28 October 1981.

Measured velocity profiles during the frazil formation and growth stage at the Chatanika River and the Goldstream Creek sites are depicted in Figures 3 and 4 respectively. Curve A in Figure 4 is the measured velocity profile before frazil-ice production. The profile is typical of open channel flow, and fits a logarithmic curve with correlation coefficient of

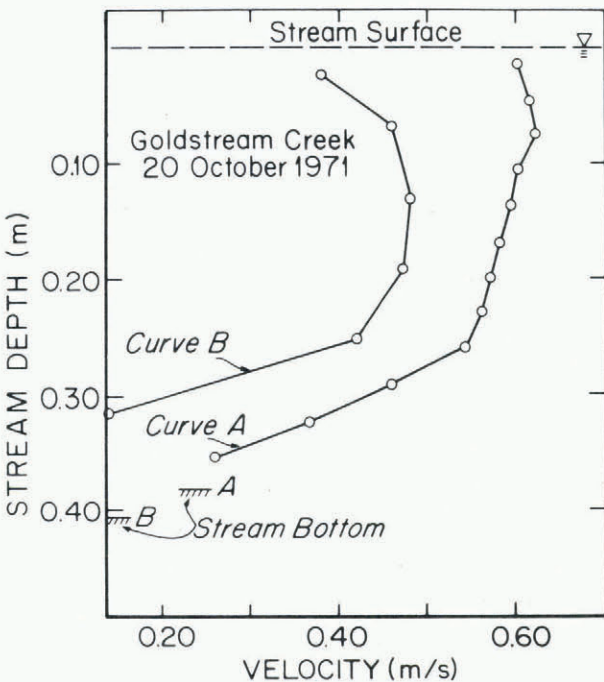


Fig.4. Measured velocity profile in Goldstream Creek, 20 October 1971.

0.96. The decelerated surface layer is relatively thin and represents a 3% velocity decrease from maximum. Curve B was measured during the period of frazil-ice production. During this time the entire profile underwent a decrease in velocity, and the decelerated surface layer thickened to more than 25% of the depth. The surface velocity decrease was 20% of maximum velocity. Layering of the stream is consistent with an interpretation of a developing stratified frazil-ice layer as predicted by the  $T_B/T_D$  criterion. This ratio is 0.9 in Goldstream Creek due to relatively slow mixing. In contrast,  $T_B/T_D = 1.2$  in the Chatanika River which is indicative of more complete mixing. The corresponding velocity profile in Figure 3 could be fitted to a logarithmic curve with a correlation coefficient of 0.96. Therefore, in these two instances the use of the time-scale ratio to determine well-mixed versus layered flow appears to be satisfactory. It should be noted that during the freeze-up, the slope of the river surface will change due to the growth of border and anchor ice, the formation of ice jams and subsequently changing backwater curves. Then, as  $S$  decreases, the local value of  $T_B/T_D$  also decreases until a slush-like frazil layer is formed. In contrast, in rapids or where  $S$  remains high, the local value of  $T_B/T_D$  will also remain high and implies that the flow will be well mixed.

SUMMARY

Rise velocities of frazil-ice crystals were measured in field and laboratory experiments. The rise velocities ranged from 3 to 22  $\text{mm s}^{-1}$  for frazil-ice crystals with diameters of 1 to 6 mm. A force balance model was derived which shows that rise velocities depend on the thickness of the frazil crystals. While the scatter in the experimental data was large, the model predicts rise velocities of the right order. For example, the rise velocities for frazil crystals 2 mm in diameter ranged from 3 to 16  $\text{mm s}^{-1}$  while 6  $\text{mm s}^{-1}$  was predicted by the model. A frazil-ice

crystal diameter to thickness ratio of about 20 was obtained by comparing the model predictions to the experimental data.

A criterion for distinguishing between layered flow, where frazil accumulates on the stream surface, and well-mixed flow, where frazil becomes vertically mixed in the stream, was developed by comparing the buoyancy time scale  $T_B$  with the vertical diffusive mixing scale  $T_D$ . This comparison suggests that when  $T_B/T_D < 1$  a layered flow will develop and when  $T_B/T_D > 1$  a well-mixed flow will develop. The time-scale ratio  $T_B/T_D$  was related to the mean stream velocity  $U$  through the Chezy coefficient  $C$ . For the Saint Lawrence Waterway, the time-scale analysis for the transition to layered flow predicts  $U \approx 0.91 \text{ m s}^{-1}$  compared to the rule-of-thumb value of  $U \approx 0.8 \text{ m s}^{-1}$ . For Norwegian rivers the predicted  $U \approx 0.60 \text{ m s}^{-1}$  is in agreement with the rule-of-thumb value.

Velocity profiles measured in Goldstream Creek showed a developing layered-flow structure during the formation of frazil ice. The value for  $T_B/T_D \approx 0.9$  which predicts layered flow is in agreement with the observations. A velocity profile measured in the Chatanika River during a period of frazil-ice formation suggested a well-mixed flow in agreement with the calculated  $T_B/T_D \approx 1.2$ . These ratios were calculated with the rise velocity of frazil-ice crystals set somewhat arbitrarily at  $0.01 \text{ m s}^{-1}$ . Further research is necessary to define this velocity more precisely.

Due to experimental difficulties, the reported measurements of the frazil-ice crystal rise velocities must be considered preliminary, the velocity profiles measured during periods of frazil-ice formation fragmentary and the rule-of-thumb criteria, for the transition from layered to well-mixed flow, crude. Nevertheless, the proposed theoretical criterion for this transition ( $T_B/T_D > 1$ ) agrees with the present experimental evidence. This suggests that more systematic and detailed comparisons are warranted.

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