BOOK REVIEWS

CASSON, A. J. and BLEILER, S. A., *Automorphisms of surfaces after Nielsen and Thurston* (London Mathematical Society Student Texts 9, Cambridge University Press, Cambridge 1988) 104 pp., cloth 0 521 34203 1, £20; paper 0 521 34985 0, £7.50.

Thurston's preprint "On the geometry and dynamics of diffeomorphisms of surfaces", circulated from Princeton around 1978, contains his results classifying the elements of the mapping class group of a compact surface. His theorem states (for genus ≥ 2) that a surface diffeomorphism which is not homotopic to a periodic diffeomorphism is homotopic either to one which fixes a partition on the surface, or to one which "fixes" a pair of transverse measured foliations; the first is called reducible, the second pseudo-Anosov. In retrospect and with the clarity provided by Thurston's conceptual ideas, it became apparent that Nielsen's work in the early 1930s had already covered a considerable part of this classification. Thurston's preprint was eventually published (*Bull. Amer. Math. Soc.* **19** (1988), 417–431), but much of its contents were earlier reworked and developed in a book, *Travaux de Thurston sur les surfaces*, by A. Fathi, F. Laudenbach, V. Poenaru, *et al.* (*Astérisque* **66–67** (1979)).

The short book under review is a jointly revised version of the lecture notes of the second author on a course given by the first. It sets out to introduce the beginning graduate student to the beauties of the interplay between hyperbolic geometry and low-dimensional topology through the specific goal of giving a virtually self-contained account of the classification theorem. The language and context of the approach is through hyperbolic structures on surfaces and homeomorphisms of these surfaces. The structure of most interest is that of a geodesic lamination on the surface which is a closed subset that is a disjoint union of simple geodesics. An orientation-preserving homeomorphism between closed hyperbolic surfaces lifts to a homeomorphism of the hyperbolic plane that extends uniquely to the circle at infinity. This crucial result shows that such a homeomorphism induces a homeomorphism of the spaces of geodesic laminations, topologized using the Hausdorff metric. The nature of geodesic laminations is investigated in detail. Examples of surface automorphisms which are non-periodic and irreducible are spelled out (without using train tracks), and the stable and unstable laminations associated with these automorphisms are explained. In the final chapter, the connection with the transverse measured foliations of Thurston is made.

The style is that of a set of lecture notes and as such makes demands on the reader. But there is a helpful picture on every other page (albeit drawn with the unsteady hand of an aged computer-printer), the style is unhurried, and the book's aims are unquestionably achieved. This volume will provide a welcome stimulus to the graduate student entering this area and a readable account of this result for those straying in from elsewhere.

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