New Cosmological Data and the Values of the Fundamental Parameters
IAU Symposium, Vol. 201, 2005
Anthony Lasenby and Althea Wilkinson, eds.

The "still point" cosmology

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Recent results on supernovae as standard candles (Riess et al. 1998; Perlmutter et al. 1999) and on CMB anisotropy (Lineweaver 1998) indicate that $\Omega_{\rm M} \approx 0.3$–$0.4$, $\Omega_{\rm V} \approx 0.6$–$0.7$, $\Omega_{\rm M} + \Omega_{\rm V} \approx 1$. By definition, $\Omega_{\rm M} = \rho_{\rm M}/\rho_{cr}$, $\Omega_{\rm V} = \rho_{\rm V}/\rho_{cr}$, where $\rho_{\rm M}$ is the matter density, $\rho_{\rm V}$ is the vacuum density; the critical density $\rho_{cr} = 3H^2/8\pi G$; H is the Hubble parameter, G is the gravitational constant. In the standard Friedmann–Lemaître cosmologies, these results seriously constrain the non-dimensional cosmological constant (as defined below): $\lambda \gg 1$, meaning that the Universe expands forever. If a scalar field is present, the future evolution may be different.

Consider the equations for the evolution of the scale factor of a closed universe with a massive non-interacting scalar field (e. g., Dolgov, Zel'dovich, & Sazhin 1988). We add the terms accounting for the matter (pressureless "dust") and the classical cosmological constant, and write the equations in non-dimensional form:

$$\left(\frac{da}{d\tau}\right)^2 = \frac{(\lambda + 8\pi\widetilde{\rho}_{sc})}{3}a^2 + \frac{2}{3a} - 1,$$
$$\frac{d^2\widetilde{\varphi}}{d\tau^2} + \frac{3}{a}\frac{da}{d\tau}\frac{d\widetilde{\varphi}}{d\tau} = -\frac{\partial\widetilde{V}}{\partial\widetilde{\varphi}}. \qquad (1)$$

The unit system $\hbar = c = 1$ is adopted. The non-dimensional time variable $\tau = t/R_E$, where $R_E = \Lambda_E^{-1/2}$, Λ_E is Einstein's value of the cosmological constant Λ: $\Lambda_E = (4\pi G\rho_{\rm M}R^3)^{-2}$, where R is the scale factor; $\lambda = \Lambda/\Lambda_E$, $a = R/R_E$. The quantity $\widetilde{\rho}_{sc} = \frac{1}{2}\left(\frac{d\widetilde{\varphi}}{d\tau}\right)^2 + \widetilde{V}(\widetilde{\varphi})$ is the non-dimensional density of the scalar field; $\widetilde{V}(\widetilde{\varphi}) = C\widetilde{\varphi}^2$ is the potential (in usual dimensional notation $V(\varphi) = \frac{1}{2}m^2\varphi^2$, m is the mass of the scalar field), $C = \frac{2}{\pi^2}\frac{M_{\rm M}^2 m^2}{m_P^4}$, where $M_{\rm M} = 2\pi^2\rho_{\rm M}R^3$ is the total mass of the matter in the Universe; $\widetilde{\varphi} = \varphi/m_P$, $m_P = G^{-1/2}$ is the Planck mass. Note that one does not have any physical constraints on the value of C from above, since its definition involves the unknown big cofactor $M_{\rm M}$.

Numerically integrating Eqs. (1) and using the definitions of $\Omega_{\rm M}$ and $\Omega_{\rm V}$ (where the non-dimensional vacuum density is $\widetilde{\rho}_{\rm V} = \frac{\lambda}{8\pi} + \widetilde{\rho}_{sc}$), we calculate evolutionary routes on the $\Omega_{\rm M}$–$\Omega_{\rm V}$ plane. In Figs. 1*a*, *b*, two collections of evolutionary routes are shown. It is set that $C = 10^6$, and initial conditions are $a_0 = 10^{-3}$ (for smaller a_0 the pictures are practically the same), $\dot{\widetilde{\varphi}}_0 = 0$; $\widetilde{\varphi}_0$ and λ are varied. Here and below overdots represent derivatives by τ.

The evolution starts near the point (1, 0), goes approximately along the line $\Omega_{\rm M} + \Omega_{\rm V} = 1$, but of a sudden retreats, meanders, and then follows a new

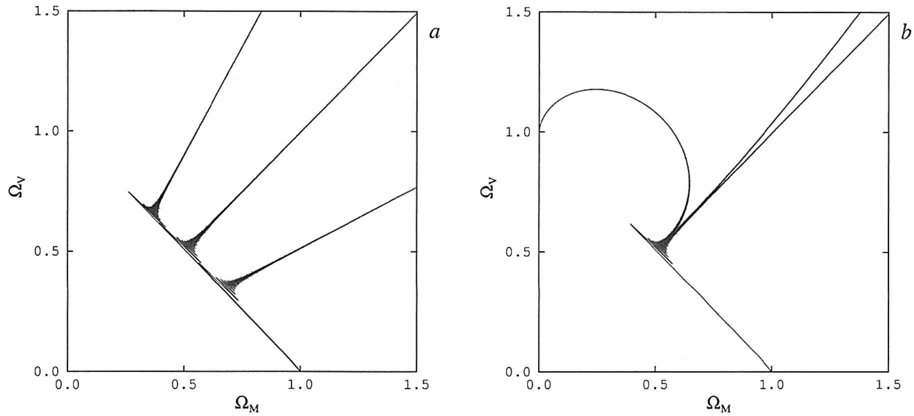

Figure 1. Evolutionary routes. *a:* $\lambda = 0$ and $\tilde{\varphi}_0 = 0.2,\ 0.252,\ 0.3$ (from right to left); *b:* $\tilde{\varphi}_0 = 0.252$ and $\lambda = 0,\ 0.251,\ 5$.

smooth path. The turn of evolution is explained by dying out of the vacuum density. If C is large enough, the meanders near the turning point form a bundle (a "clot"), where the trajectory stays relatively long. However, the Universe cannot be deep in the clot, since then $a \propto \tau^{2/3}$, contrary to observations. The scenario with a clot takes place for $C \gtrsim 10^3$ and the values of $|\lambda|$, $|\dot{\tilde{\varphi}}_0|$, a_0 limited from above. The value of $\tilde{\varphi}_0$ controls location of the clot (Fig. 1*a*). Empirically, the coordinates of the first turning point (where $\dot{\Omega}_M = 0$ for the first time) are $\Omega^*_M \approx \exp(-15\varphi_0^2)$, $\Omega^*_V \approx 1 - \Omega^*_M$; the accuracy in Ω^*_M is better than 0.01 if $C \geq 10^7$, $\lambda = 0$, $\dot{\tilde{\varphi}}_0 = 0$ and a_0 is small enough (e. g. $a_0 < 0.001$ if $C = 10^7$). If $\tilde{\varphi}_0 \approx 0.25$–$0.3$, the currently observed values of Ω_M and Ω_V may correspond to the turning point or its neighbourhood.

Fig. 1*b* illustrates the role of λ. A route with $\lambda = 0.251\ldots$ leads to a static state $a = \text{const}$ at infinite future. In the standard Friedmann–Lemaître cosmologies, such kind of evolution is described by the A1 model. Among the standard closed models, the A1 universe has the maximum conformal life-time, and on this reason it is most preferable from the viewpoint of a variant of the strong anthropic principle (Shevchenko 1993).

So, if the scalar field is present, modern observations do not impose constraints on λ and the Universe may still recollapse, or expand forever, or follow the separatrix route. What is more, the "turning point" scenario may explain why the Universe is not close to the standard (0, 1) attractor.

References

Dolgov, A. D., Zel'dovich, Ya. B. & Sazhin, M. V. 1988, Cosmology of the Early Universe (Moscow Univ.: Moscow) (in Russian)

Lineweaver, C. H. 1998, ApJ, 505, L69

Perlmutter, V. S. et al. 1999, ApJ, 517, 565

Riess, A. G. et al. 1998, AJ, 116, 1009

Shevchenko, I. I. 1993, Astrophys. and Space Sci., 202, 45

https://doi.org/10.1017/S0074180900216926 Published online by Cambridge University Press