THE MAGNETIC FIELD STRUCTURES OF A CLASS OF FAST DYNAMOS

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Abstract. We study the magnetic field structures of fast kinematic dynamos for a class of steady chaotic flows with stagnation points. We find that the dynamos are generated by a stretch-fold-shear mechanism, which is effective only when the chaotic flow region is large enough and overlaps significantly with the rotating flux tubes.

Key words: Fast Dynamo – Kinematic Dynamo – Chaotic flow

1. Introduction

Dynamo activity is thought to be responsible for the magnetic fields of the earth, the sun, and other astrophysical bodies. For a conducting fluid with resistivity η, the generation of magnetic fields is governed by the induction equation $\partial \mathbf{B}/\partial t = \nabla\times(\mathbf{v}\times\mathbf{B})-\nabla\times(\eta\nabla\times\mathbf{B})$, where $\mathbf{v}(\mathbf{x},t)$ is the fluid flow. In the kinematic problem one treats $\mathbf{v}$ as given and not influenced by $\mathbf{B}$. For $\nabla\cdot\mathbf{v} = 0$, the induction equation can be written as (assuming uniform resistivity)

$$\partial \mathbf{B}/\partial t + \mathbf{v}\cdot\nabla\mathbf{B} = \mathbf{B}\cdot\nabla\mathbf{v} + \eta\nabla^2\mathbf{B}. \tag{1}$$

Equation (1) is a linear equation for $\mathbf{B}$. If it possesses eigenfunctions that grow exponentially in time, the flow is said to possess a dynamo. If the growth rate remains positive in the limit of infinite conductivity $R_m = 1/\eta \to \infty$ (R_m is the magnetic Reynolds number), the dynamo is said to be *fast*. The existence of a fast dynamo has been found to be associated with chaos in the streamlines (Finn *et al.* 1991), which are solutions of $d\mathbf{x}/dt = \mathbf{v}(\mathbf{x},t)$. For a steady flow $\mathbf{v}(\mathbf{x})$, this means that the streamlines must be chaotic.

In this paper, we study the magnetic field structures of fast kinematic dynamos for a class of steady incompressible flows with stagnation points. The flow structure has been investigated before (Lau & Finn 1992) and the corresponding dynamo solutions have been presented (Lau & Finn 1993).

2. Flow Model and Dynamo Solutions

The three-dimensional steady flow introduced in (Lau & Finn 1992) is

$$v_x = y, \quad v_y = 1/4 - (x-1)^2 - z^2, \quad v_z = B_0(x - x_N). \tag{2}$$

F. Krause et al. (eds.), The Cosmic Dynamo, 231–235.

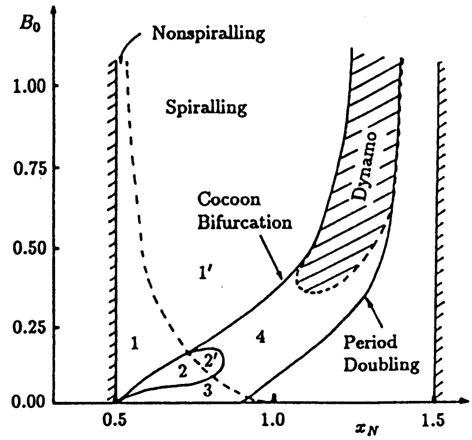

Fig. 1. The parameter space (x_N, B_0) of flow (2), showing the existence of stagnation points within $0.5 < x_N < 1.5$, the dashed curve where the stagnation points become spiralling, and several domains divided by global bifurcations (solid curves). In particular, the streamlines are chaotic in domains 3 and 4. The dynamo window is shaded.

This two-parameter (x_N and B_0) flow is aperiodic in space. It consists basically of a single convective vortex, which may be viewed as a qualitative model for a convection cell in a turbulent environment, such as the solar convection zone. This model was chosen because it exhibits characteristics of the stretch-fold-shear dynamo (Vainshtein & Zel'dovich 1972), and of the models of (Finn *et al.* 1991), but with stagnation points.

The streamline structures of this flow have been studied in (Lau & Finn 1992). For $0.5 < x_N < 1.5$, the flow has two stagnation points, whose stable and unstable manifolds may intermingle, resulting in chaos in the streamlines. Figure 1 [from (Lau & Finn 1992)] is a bifurcation diagram for the flow. Among the domains separated by the solid global bifurcation curves, domains 3 and 4 contain chaotic streamlines and therefore are most interesting. We note also that domain 4 is associated with spiralling stagnation points, while domain 3 is not.

We pick a representative point in Fig. 1 and find the solution of Eq. (1) for flow (2) by a split step finite difference scheme. [The numerical method and convergence study are described in (Lau & Finn 1993).] Figure 2 shows (on a logarithmic scale) $B_{\rm rms} = \langle \mathbf{B}^2 \rangle^{1/2}$ and $|\langle B_z \rangle|$ versus time for a case with $x_N = 1.25$, $B_0 = 0.75$ ($\langle\ \rangle$ being a volume average over 90% of the volume). These quantities indicate an eigenmode $\mathbf{B} = \mathbf{B}_0(\mathbf{x}) \exp(\gamma t - i\omega t)$ with $\gamma \approx 0.06$ and $\omega \approx 1.0$. (The quantities $\ln(\nabla\cdot\mathbf{B})_{\rm rms}$ and $\ln\langle \sum_i |\partial B_i/\partial x_i| \rangle$ are for monitoring the divergence cleaning in the numerical scheme.)

Under our numerical scheme, the growth rate γ is found to obey $\gamma = \gamma_\infty - C/N^\alpha$, where C is a constant and α is a positive integer dependent on the interpolation scheme. With this we obtain the growth rates of fast dynamos ($\eta \to 0$) for grid number $N \to \infty$. In Fig. 1 , the domain with positive growth rates is shaded and

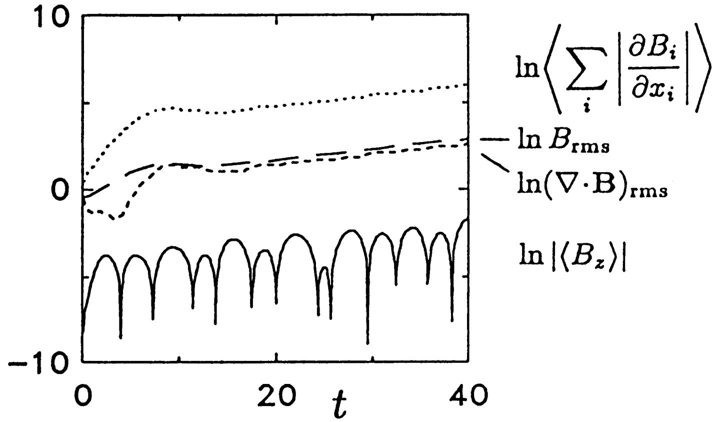

Fig. 2. The quantities $\ln B_{\rm rms}$, $\ln |\langle B_z \rangle|$, $\ln(\nabla \cdot \mathbf{B})_{\rm rms}$, and $\ln\langle \sum_i |\partial B_i/\partial x_i| \rangle$ as functions of time for a dynamo mode with $x_N = 1.25$ and $B_0 = 0.75$. The resistivity $\eta = 10^{-6}$ and the number of grid points in each direction $N = 150$.

labeled dynamo. A noted feature is that the dynamo domain lies within the chaotic domain 4. This leads to the conclusion that chaos is a necessary (but not sufficient) condition for fast dynamos (Lau & Finn 1993).

3. Magnetic Field Structures

To find out what additional conditions are required for a fast dynamo, we need to examine the structure of the dynamo magnetic field. Figure 3 shows the $B_z(x, y, 0)$ contours of a typical dynamo mode for the present flow. It is seen that the two spots of opposite B_z rotate around the unstable manifold γ_A, which is a streamline coming out of one of the stagnation points. These spots correspond to tubes of concentrated magnetic flux. Other flux regions slide along Σ_B, the two-dimensional unstable manifold of one stagnation point, and merge with the rotating flux tubes. The field outside of these areas is practically zero. The rotation frequency of the tubes is the same as the oscillation frequency of the eigenmode (ω), indicating that the dynamo mechanism is closely related to these rotating tubes.

Next in Fig. 4 we trace the magnetic field lines for the above dynamo. The (rotating) dark tubes are roughly parallel to field lines passing through the maximal field regions. By tracing field lines at different times, we obtain the sketch in Fig. 5, which exhibits the stretch-fold-shear feature of (Vainshtein & Zel'dovich 1972). The actual process in our steady flow is of course continuous, yielding an oscillating exponential growth of the field. The twisting is related to the flow rotation. For the present model, the rotation is attributed to the spiralling nature of the stagnation points. In fact, the frequency ω is roughly equal to the imaginary part of the complex eigenvalue of the stagnation points. This is consistent with the absence of dynamos in domain 3 of Fig. 1. [Oscillating dynamo modes also exist in ABC flows, which have stagnation points with real eigenvalues (Lau & Finn 1993). These flows, however,

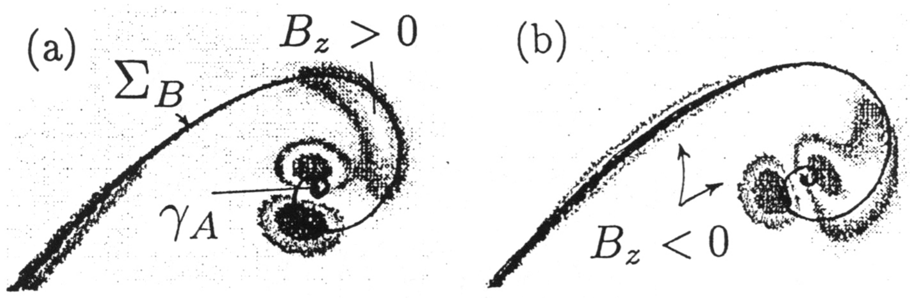

Fig. 3. Two snapshots of the $B_z(x, y, 0)$ contours for a dynamo mode. The contours with white regions correspond to $B_z > 0$ and the others to $B_z < 0$. The spiral curve is the intersection of Σ_B with the $z = 0$ plane. The tip of the spiral is the one-dimensional unstable manifold γ_A.

posses KAM regions.]

In order to have the stretch-fold-shear mechanism act on the field repeatedly, it is necessary for the flow to be chaotic. More specifically, a horseshoe map exists in the chaotic region of the flow (Lau & Finn 1992). This is why dynamos do not exist in domain 1′ of Fig. 1 (no chaotic invariant set in the flow), nor for x_N greater than the period doubling curve (the regular toroidal flow region dominating over the chaotic region). Similarly, the part of domain 4 near 2′ and 3 has very small chaotic regions. These regions barely overlap with the rotating flux tubes, which is very thin because of the large eigenvalues for the chaotic invariant set in this part of domain 4. Thus dynamo action does not occur there.

In summary, the stretch-fold-shear mechanism is effective only when the chaotic flow region is large enough (compared to the regular flow regions) and overlaps significantly with the rotating flux tubes.

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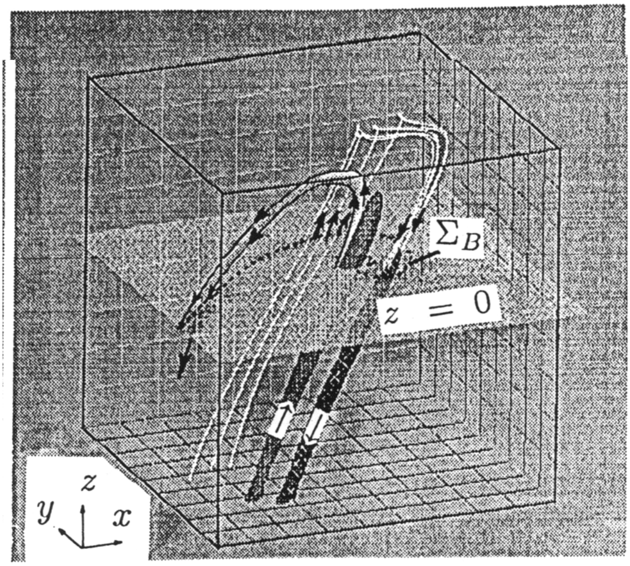

Fig. 4. A three-dimensional view of a dynamo magnetic field. The dotted curve is the intersection of the unstable manifold Σ_B with the cross-sectional plane ($z = 0$). The dark tubes are isosurfaces of $|B_z|$ near its maximal value. The curves with arrows are field lines that pierce through these tubes and connect to other flux regions along Σ_B.

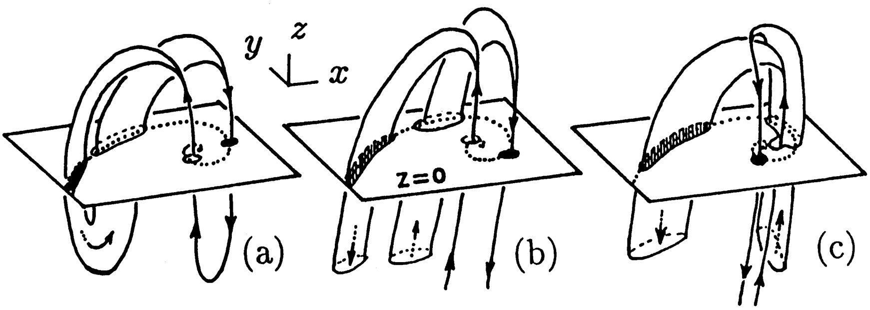

Fig. 5. The evolution of an ideal flux loop. (a) Stretching along the unstable manifolds. Here a line stands for strong magnetic flux and and a band means weak flux. (b) Rotation of the strong flux tubes yields twisting in the loop. Parts of the loop are taken out of the region of interest by continued stretching. (c) A right amount of rotation brings together the two positive B_z regions, which merge into a stronger flux tube. The same happens to the negative B_z regions later. The new flux tube is then stretched and folded to the left side of Σ_B as in (a). Note that (b), corresponding to Fig. 4, occurs just before Fig. 3(a). Also, (c) corresponds to Fig. 3(b).

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