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On the Existence of a New Class of Contact Metric Manifolds

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Abstract. A new class of 3-dimensional contact metric manifolds is found. Moreover it is proved that there are no such manifolds in dimensions greater than 3.

1 Introduction

Let M be a Riemannian manifold. The tangent sphere bundle T_1M admits a contact metric structure (ϕ, ξ, η, g) and so T_1M together with this structure is a contact metric manifold [1]. If M is of constant sectional curvature, then the curvature tensor R of $T_1M(\phi, \xi, \eta, g)$ satisfies the condition

$$(*) \qquad R(x, y)\xi = \kappa[\eta(y)x - \eta(x)y] + \mu[\eta(y)hx - \eta(x)hy]$$

for any $x, y \in \mathfrak{X}(T_1M)$, where $2h$ is the Lie derivative of ϕ with respect to ξ and κ, μ are constant. Moreover, the converse is also true [3]. This class of contact metric manifolds is especially interesting, because apart from its other characteristics, it contains the well known Sasakian manifolds. In [5], [6], [7] are studied contact metric manifolds satisfying (*) but with κ, μ smooth functions not necessarily constant. In these papers it is proved that, with an extra assumption, the functions κ, μ must be constant. On the other hand, up to now, we didn't know any example of a contact metric manifold satisfying (*) and with κ, μ non-constant smooth functions. The following question comes up naturally. Do there exist contact metric manifolds satisfying (*) with κ, μ non-constant smooth functions, independent of the choice of vector fields x, y? In this paper we give a negative answer to the above question for dimensions > 3. For dimension 3 we give an affirmative answer, through the construction of examples.

2 Preliminaries

A differentiable $(2m + 1)$-dimensional manifold M^{2m+1} is called a contact manifold if it carries a global differential 1-form η such that $\eta \wedge (d\eta)^m \neq 0$ everywhere on M^{2m+1}. It is known that a contact manifold admits an almost contact metric structure (ϕ, ξ, η, g), *i.e.*, a global vector field ξ, which will be called the characteristic vector field, a (1,1) tensor field

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ϕ and a Riemannian metric g such that

$$\phi^2 = -\operatorname{Id} + \eta \otimes \xi, \quad \eta(\xi) = 1, \tag{2.1}$$

$$g(\phi x, \phi y) = g(x, y) - \eta(x)\eta(y), \tag{2.2}$$

for all vector fields x, y on M^{2m+1}. Moreover, (ϕ, ξ, η, g) can be chosen such that $d\eta(x, y) = g(x, \phi y)$ and thus the structure is called a contact metric structure and the manifold M^{2m+1} a contact metric manifold. Equations (2.1) and (2.2) imply

$$\phi\xi = 0, \quad \eta \circ \phi = 0, \quad d\eta(\xi, x) = 0. \tag{2.3}$$

Denoting by $\mathcal{L}$ and R, Lie differentiation and the curvature tensor respectively, the operators l and h are defined by

$$lx = R(x, \xi)\xi, \quad hx = \frac{1}{2}(\mathcal{L}_\xi \phi)x. \tag{2.4}$$

The (1,1) tensors h and l are self-adjoint and satisfy

$$h\xi = 0, \quad l\xi = 0, \quad h\phi + \phi h = 0. \tag{2.5}$$

If ∇ is the Riemannian connection of g, equations (2.1)–(2.5) imply

$$\nabla_x \xi = -\phi x - \phi h x, \tag{2.6}$$

$$\phi l \phi - l = 2(\phi^2 + h^2), \tag{2.7}$$

$$\nabla_\xi \phi = 0, \tag{2.8}$$

$$\nabla_\xi h = \phi - \phi l - \phi h^2. \tag{2.9}$$

A contact structure on M^{2m+1} gives rise to an almost complex structure on the product $M^{2m+1} \times R$. If this structure is integrable, then the contact metric manifold is said to be Sasakian. Equivalently, a contact metric manifold is Sasakian if and only if

$$R(x, y)\xi = \eta(y)x - \eta(x)y. \tag{2.10}$$

For more details concerning contact manifolds the reader is referred to [1].

3 Main Results

Let $M^{2m+1}(\phi, \xi, \eta, g)$ be a contact metric manifold. We suppose that

$$R(x, y)\xi = \kappa[\eta(y)x - \eta(x)y] + \mu[\eta(y)hx - \eta(x)hy], \tag{3.1}$$

for some smooth functions κ and μ on M independent of the choice of vector fields x and y. We call such a manifold M, a *generalized* (κ, μ)*-contact metric manifold*. In the special case $\kappa, \mu =$ constant, the manifold will be called simply a (κ, μ)-contact metric manifold.

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The 3-dimensional case, $(m = 1)$

Now, we are going to construct examples of 3-dimensional generalized (κ, μ)-contact metric manifolds, which are not (κ, μ)-contact metric manifolds.

Example 1 We consider the 3-dimensional manifold $M = \{(x_1, x_2, x_3) \in R^3 \mid x_3 \neq 0\}$, where (x_1, x_2, x_3) are the standard coordinates in R^3. The vector fields

$$e_1 = \frac{\partial}{\partial x_1}, \quad e_2 = -2x_2x_3\frac{\partial}{\partial x_1} + \frac{2x_1}{x_3^3}\frac{\partial}{\partial x_2} - \frac{1}{x_3^2}\frac{\partial}{\partial x_3}, \quad e_3 = \frac{1}{x_3}\frac{\partial}{\partial x_2}$$

are linearly independent at each point of M. Let g be the Riemannian metric defined by $g(e_i, e_j) = \delta_{ij}$, $i, j = 1, 2, 3$. Let ∇ be the Riemannian connection and R the curvature tensor of g. We easily get

$$[e_1, e_2] = \frac{2}{x_3^2}e_3, \quad [e_2, e_3] = 2e_1 + \frac{1}{x_3^3}e_3, \quad [e_3, e_1] = 0.$$

Let η be the 1-form defined by $\eta(z) = g(z, e_1)$ for any $z \in \mathfrak{X}(\mathcal{M})$. Because $\eta \wedge d\eta \neq 0$ everywhere on M, η is a contact form. Let ϕ be the (1,1)-tensor field, defined by $\phi e_1 = 0$, $\phi e_2 = e_3$, $\phi e_3 = -e_2$. Using the linearity of ϕ, $d\eta$ and g we find $\eta(e_1) = 1$, $\phi^2 z = -z+\eta(z)e_1$, $d\eta(z, w) = g(z, \phi w)$ and $g(\phi z, \phi w) = g(z, w)-\eta(z)\eta(w)$ for any $z, w \in \mathfrak{X}(\mathcal{M})$. Hence (ϕ, e_1, η, g) defines a contact metric structure on M and so M together with this structure is a contact metric manifold.

Putting $\xi = e_1$, $x = e_2$, $\phi x = e_3$ and using the well known formula

$$2g(\nabla_y z, w) = yg(z, w) + zg(w, y) - wg(y, z) - g(y, [z, w]) - g(z, [y, w]) + g(w, [y, z])$$

we calculate

$$\nabla_x\xi = -\left(1 + \frac{1}{x_3^2}\right)\phi x, \quad \nabla_{\phi x}\xi = \left(1 - \frac{1}{x_3^2}\right)x,$$
$$\nabla_\xi x = \left(-1 + \frac{1}{x_3^2}\right)\phi x, \quad \nabla_\xi\phi x = \left(1 - \frac{1}{x_3^2}\right)x,$$
$$\nabla_x x = 0, \quad \nabla_x\phi x = \left(1 + \frac{1}{x_3^2}\right)\xi, \quad \nabla_{\phi x}x = \left(-1 + \frac{1}{x_3^2}\right)\xi - \frac{1}{x_3^3}\phi x, \quad \nabla_{\phi x}\phi x = \frac{1}{x_3^3}x.$$

Therefore for the tensor field h we get $h\xi = 0$, $hx = \lambda x$, $h\phi x = -\lambda\phi x$, where $\lambda = \frac{1}{x_3^2}$. Now, putting $\mu = 2(1 - \frac{1}{x_3^2})$ and $\kappa = \frac{x_3^4-1}{x_3^4}$ we finally get

$$R(x, \xi)\xi = \kappa\big(\eta(\xi)x - \eta(x)\xi\big) + \mu\big(\eta(\xi)hx - \eta(x)h\xi\big)$$
$$R(\phi x, \xi)\xi = \kappa\big(\eta(\xi)\phi x - \eta(\phi x)\xi\big) + \mu\big(\eta(\xi)h\phi x - \eta(\phi x)h\xi\big)$$
$$R(x, \phi x)\xi = \kappa\big(\eta(\phi x)x - \eta(x)\phi x\big) + \mu\big(\eta(\phi x)hx - \eta(x)h\phi x\big).$$

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These relations yield the following, by a straightforward calculation,

$$R(z,w)\xi = \kappa\big(\eta(w)z - \eta(z)w\big) + \mu\big(\eta(w)hz - \eta(z)hw\big),$$

where κ and μ are non-constant smooth functions. Hence M is a generalized (κ,μ)-contact metric manifold.

Example 2 We consider the 3-dimensional manifold $M = \{(x_1,x_2,x_3) \in R^3 \mid x_3 \neq 0\}$ and the vector fields

$$e_1 = \frac{\partial}{\partial x_1}, \quad e_2 = \frac{1}{x_3^2}\frac{\partial}{\partial x_2}, \quad e_3 = 2x_2x_3^2\frac{\partial}{\partial x_1} + \frac{2x_1}{x_3^6}\frac{\partial}{\partial x_2} + \frac{1}{x_3^6}\frac{\partial}{\partial x_3}.$$

We define ξ, g, η, ϕ by $\xi = e_1$, $g(e_i,e_j) = \delta_{ij}$, $(i,j = 1,2,3)$ and $\phi e_1 = 0$, $\phi e_2 = e_3$, $\phi e_3 = -e_2$. Working as in the previous example we finally get that $M(\phi,\xi,\eta,g)$ is a generalized (κ,μ)-contact metric manifold with $\kappa = 1 - \frac{1}{x_3^8}$, $\mu = 2(1 + \frac{1}{x_3^4})$.

Let us give some more examples. Starting with the examples given previously we will now construct new 3-dimensional generalized (κ,μ)-contact metric manifolds for any positive real number.

Let $M(\phi,\xi,\eta,g)$ be a 3-dimensional generalized (κ,μ)-contact metric manifold. By a D_a-homothetic deformation [8] we mean a change of structure tensors of the form $\bar{\eta} = a\eta$, $\bar{\xi} = \frac{1}{a}\xi$, $\bar{\phi} = \phi$, $\bar{g} = ag + a(a-1)\eta\otimes\eta$, where a is a positive constant. It is well known that $M(\bar{\phi},\bar{\xi},\bar{\eta},\bar{g})$ is also a contact metric manifold. Moreover the curvature tensor R and the tensor h transform in the following manner [3], $\bar{h} = \frac{1}{a}h$ and

$$\begin{aligned} a\bar{R}(x,y)\bar{\xi} &= R(x,y)\xi + (a-1)^2\big(\eta(y)x - \eta(x)y\big) \\ &\quad - (a-1)\{(\nabla_x\phi)y - (\nabla_y\phi)x + \eta(x)(y+hy) - \eta(y)(x+hx)\}, \end{aligned}$$

for any $x, y \in \mathcal{X}(\mathcal{M})$.

Additionally it is well known [9, pp. 446–447], that any 3-dimensional contact metric manifold satisfies $(\nabla_x\phi)y = g(x+hx,y)\xi - \eta(y)(x+hx)$. Using the above relations we finally obtain

$$\bar{R}(x,y)\bar{\xi} = \frac{\kappa + a^2 - 1}{a^2}\big(\bar{\eta}(y)x - \bar{\eta}(x)y\big) + \frac{\mu + 2(a-1)}{a}\big(\bar{\eta}(y)\bar{h}x - \bar{\eta}(x)\bar{h}y\big)$$

for any $x, y \in \mathcal{X}(\mathcal{M})$. So we have proved the following Theorem.

Theorem 3.1 *For any positive number, there exists a 3-dimensional generalized (κ,μ)-contact metric manifold.*

The case $m > 1$ Let $M^{2m+1}(\phi,\xi,\eta,g)$ be a generalized (κ,μ)-contact metric manifold and $B = \{p \in M \mid \kappa(p) = 1\}$. The set $N = M - B$ is an open subset of M and thus $N^{2m+1}(\phi,\xi,\eta,g)$ is a contact metric manifold, which satisfies the equation (3.1) with $\kappa \neq 1$ everywhere.

Lemma 3.2 *The following relations are valid on* $N^{2m+1}(\phi, \xi, \eta, g)$

$$l\phi - \phi l = 2\mu h\phi, \tag{3.2}$$

$$h^2 = (\kappa - 1)\phi^2, \quad \kappa < 1 \tag{3.3}$$

$$R(\xi, x)y = \kappa[g(x, y)\xi - \eta(y)x] + \mu[g(hx, y)\xi - \eta(y)hx], \tag{3.4}$$

$$\begin{aligned}(\nabla_x h)y - (\nabla_y h)x = (1-\kappa)[2g(x, \phi y)\xi + \eta(x)\phi y - \eta(y)\phi x] \\ + (1-\mu)[\eta(x)\phi hy - \eta(y)\phi hx],\end{aligned} \tag{3.5}$$

$$\xi\kappa = 0. \tag{3.6}$$

for any $x, y \in \mathfrak{X}(N)$.

Proof The proof of (3.2)–(3.5) is similar to that of Lemma 3.1 of [3] and hence we omit it. To prove (3.6), we operate (3.2) by ϕ and use (2.7) and (3.3) we get $l = -\kappa\phi^2 + \mu h$ and so through (2.8) we find

$$\nabla_\xi l = -(\xi\kappa)\phi^2 + (\xi\mu)h + \mu(\nabla_\xi h). \tag{3.7}$$

Moreover from (2.9), (3.3) and $l = -\kappa\phi^2 + \mu h$ we obtain

$$\nabla_\xi h = \mu h\phi. \tag{3.8}$$

The use of (3.8) in (3.7) shows

$$\nabla_\xi l = -(\xi\kappa)\phi^2 + (\xi\mu)h + \mu^2 h\phi. \tag{3.9}$$

Differentiating (2.7) along ξ and using (3.8) we get $\phi(\nabla_\xi l)\phi - \nabla_\xi l = 0$. This together with (3.9) complete the proof of the Lemma.

Lemma 3.3 *For any vector fields* x, y *on a* $(2m+1)$*-dimensional generalized* (κ, μ)*-contact metric manifold the following differantial equation is valid*

$$(y\kappa)\phi^2 x - (x\kappa)\phi^2 y + (x\mu)hy - (y\mu)hx + (\xi\mu)[\eta(y)hx - \eta(x)hy] = 0. \tag{3.10}$$

Proof Differentiating (3.1) along an arbitrary vector field z and using (2.6) we find

$$\begin{aligned}\nabla_z R(x, y)\xi = {} & (z\kappa)[\eta(y)x - \eta(x)y] + (z\mu)[\eta(y)hx + \eta(x)hy] \\ & + \kappa\big[\big(\eta(\nabla_z y) - g(y, \phi z) - g(y, \phi hz)\big)x + \eta(y)\nabla_z x \\ & \quad - \big(\eta(\nabla_z x) - g(x, \phi z) - g(x, \phi hz)\big)y + \eta(x)\nabla_z y\big] \\ & + \mu\big[\big(\eta(\nabla_z y) - g(y, \phi z) - g(y, \phi hz)\big)hx + \eta(y)\nabla_z hx \\ & \quad - \big(\eta(\nabla_z x) - g(x, \phi z) - g(x, \phi hz)\big)hy + \eta(x)\nabla_z hy\big].\end{aligned}$$

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The use of the last relation, (3.1) and (2.6) in Bianchi second identity yield to the following relation, by a direct calculation,

$$\begin{aligned}\bigoplus_{\{x,y,z\}} \{(z\kappa)[\eta(y)x - \eta(x)y] + (z\mu)[\eta(y)hx + \eta(x)hy]\\
+ \kappa\big[\big(\eta(\nabla_z y) - g(y,\phi z) - g(y,\phi hz)\big)x + \eta(y)\nabla_z x\\
- \big(\eta(\nabla_z x) - g(x,\phi z) - g(x,\phi hz)\big)y + \eta(x)\nabla_z y\big]\\
+ \mu\big[\big(\eta(\nabla_z y) - g(y,\phi z) - g(y,\phi hz)\big)hx + \eta(y)\nabla_z hx\\
- \big(\eta(\nabla_z x) - g(x,\phi z) - g(x,\phi hz)\big)hy + \eta(x)\nabla_z hy\big]\\
- \kappa[\eta(y)\nabla_z x - \eta(\nabla_z x)y] - \mu[\eta(y)h\nabla_z x - \eta(\nabla_z x)hy]\\
- \kappa[\eta(\nabla_x z)y - \eta(y)\nabla_x z] - \mu[\eta(\nabla_x z)hy - \eta(y)h\nabla_x z]\\
+ R(x,y)\phi z + R(x,y)\phi hz\} = 0,\end{aligned}$$

where $\bigoplus_{\{x,y,z\}}$ denotes the cyclic sum of x, y, z. Putting ξ instead of z in the last relation and using (3.4) and (3.6) we obtain

$$\begin{aligned}-(y\kappa)x + (x\kappa)y + [(\xi\mu)\eta(y) - (y\mu)]hx + [-(\xi\mu)\eta(x) + (x\mu)]hy\\
+ \eta(y)(\nabla_\xi h)x - \mu\eta(x)(\nabla_\xi h)y + \mu(\nabla_x h)y - \mu(\nabla_y h)x\\
+ \big[-(x\kappa)\eta(y) + (y\kappa)\eta(x) + \kappa\big(g(y,\phi hx) - g(x,\phi hy)\big)\\
+ \mu\big(g(hx,\phi hy) - g(hy,\phi hx) - g(hy,\phi x) + g(hx,\phi y)\big)\big]\xi\\
- \mu\eta(x)h\nabla_y\xi - \mu\eta(y)h\nabla_x\xi = 0.\end{aligned}$$

Substituting (2.1), (2.5) and (3.5) in the last relation we finally get (3.10) and it completes the proof of the Lemma.

Lemma 3.4 *For any $P \in N$ there exist an open neighbourhood U of P and orthonormal local vector fields x_i, ϕx_i, ξ, $i = 1,\dots,m$, defined on U, such as*

$$hx_i = \lambda x_i, \quad h\phi x_i = -\lambda\phi x_i, \quad h\xi = 0, \quad i = 1,\dots,m, \tag{3.11}$$

where $\lambda = \sqrt{1-\kappa}$.

Proof Using (3.3), we see that, at any point of N the tensor h has three eigenvalues; 0 with multiplicity 1, $\sqrt{1-\kappa}$ with multiplicity m and $-\sqrt{1-\kappa}$ with multiplicity m. The function $\lambda = \sqrt{1-\kappa}$ is smooth on N. Let $y_1,\dots,y_m,y_{m+1},\dots,y_{2m},y_{2m+1}$ be a basis of T_PN, such that $hy_i = \lambda y_i$, $i = 1,\dots,m$, $hy_j = -\lambda y_j$, $j = m+1,\dots,2m$, $y_{2m+1} = \xi$. We extend y_k's to vector fields on N and define the vector fields $w_i = (h+\lambda I)y_i - \lambda\eta(y_i)\xi$, $i = 1,\dots,m$, $w_j = (h-\lambda I)y_j + \lambda\eta(y_j)\xi$, $j = m+1,\dots,2m$ and ξ. At P we have $w_i = 2\lambda y_i$, $i = 1,\dots,m$, and $w_j = -2\lambda y_j$, $j = m+1,\dots,2m$. Thus $w_1,\dots,w_m,w_{m+1},\dots,w_{2m},\xi$

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are linearly independent at P and hence in a neighbourhood U of P. At each point of U we have

$$hw_i = h\big((h+\lambda I)y_i - \lambda\eta(y_i)\xi\big) = \lambda w_i, \quad i = 1,\dots,m,$$
$$hw_j = h\big((h-\lambda I)y_j + \lambda\eta(y_j)\xi\big) = -\lambda w_j, \quad j = m+1,\dots,2m,$$
$$h\xi = 0.$$

The vector fields ξ, $x_i = \frac{w_i}{|w_i|}$ and ϕx_i, $i = 1,\dots,m$, satisfy (3.11) and so the proof is completed.

From now on, we will call the vector fields of Lemma 3.4 a local *h-basis*. We suppose that $\{x_i, \phi x_i, \xi\}$, $i = 1,\dots,m$, is a local h-basis on N. Substituting $x = x_i$, $y = \phi x_i$ in (3.10) we get

$$\lambda x_i\mu = x_i\kappa, \quad -\lambda\phi x_i\mu = \phi x_i\kappa, \quad i = 1,\dots,m. \tag{3.12}$$

Since $m > 1$, replacing x, y by x_i, x_j $(i \neq j)$ respectively, equation (3.10) gives

$$-\lambda x_i\mu = x_i\kappa, \quad i = 1,\dots,m. \tag{3.13}$$

Finally, substituting $x = \phi x_i$, $y = \phi x_j$, $(i \neq j)$, in (3.10) we have

$$\lambda\phi x_i\mu = \phi x_i\kappa, \quad i = 1,\dots,m. \tag{3.14}$$

By virtue of (3.6), (3.12), (3.13) and (3.14) we obtain

$$x_i\kappa = \phi x_i\kappa = \xi\kappa = x_i\mu = \phi x_i\mu = 0, \quad i = 1,\dots,m. \tag{3.15}$$

Considering the 1-form $d\mu$ and using (3.15) we have $d\mu = (\xi\mu)\eta$, and so

$$0 = d^2\mu = d(\xi\mu)\wedge\eta + (\xi\mu)d\eta. \tag{3.16}$$

Using (3.15) and (3.16) we obtain $d(\xi\mu) = \xi(\xi\mu)\eta$ and so $\xi\mu = 0$. This together with (3.15) show that the functions κ and μ are constant on N. Therefore by the continuity of κ, μ we conclude that the functions κ, μ are constant on M. If $\kappa \equiv 1$, then using $h^2 = (\kappa-1)\phi^2$, which is valid on any (κ,μ)-contact metric manifold, we get $h = 0$ and so by (3.1) and (2.10) M is Sasakian manifold.

So we have proved the following Theorem.

Theorem 3.5 *On a non Sasakian, generalized (κ,μ)-contact metric manifold M^{2m+1} with $m > 1$, the functions κ, μ are constant, i.e., M^{2m+1} is a (κ,μ)-contact metric manifold.*

Using Lemma 3.3, for the 3-dimensional case, and working as in the case $m > 1$, we easily prove the following Theorem.

Theorem 3.6 *Let M be a non Sasakian, generalized (κ,μ)-contact metric manifold. If κ, μ satisfy the condition $a\kappa + b\mu = c$ (a, b, c, constant), then κ, μ are constant.*

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Remarks 1. If $\kappa = \mu = 0$, then $R(x, y)\xi = 0$ and such a contact metric manifold M^{2m+1} is locally the product of a flat $(m+1)$-dimensional manifold and an m-dimensional manifold of constant curvature 4 [2].

2. Recently, we have been informed by D. E. Blair, that (κ, μ)-contact metric manifolds have been classified [4]. For the 3-dimensional case see also [3].

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