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Magnetic Field & Helicity in the Sun & Heliosphere

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Magnetic Helicity Conservation

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Abstract. Magnetic Helicity measures basic structural properties of magnetic fields such as twist, shear, linking, writhe, and handedness. It is conserved in ideal MHD and approximately conserved during reconnection. The minimum energy state of a field with a given magnetic helicity is a linear force free field. Helicity plays an important role in MHD turbulence and dynamo theory, and provides a valuable observational tool in solar and space physics. Helicity conservation can be tracked from the solar dynamo to active regions to coronal mass ejections to magnetic clouds in interplanetary space.

1. Magnetic Helicity: Basic Properties

We can define the magnetic helicity of all space as a double integral over the magnetic field:

$$H = \frac{1}{4\pi} \int \int \mathbf{B}(\mathbf{x}) \cdot \frac{\mathbf{r}}{r^3} \times \mathbf{B}(\mathbf{y}) \ \mathrm{d}^3 y \ \mathrm{d}^3 x \qquad (\mathbf{r} = \mathbf{y} - \mathbf{x}). \tag{1}$$

This formula also works inside a volume bounded by a magnetic surface S, where $\mathbf{B} \cdot \hat{n}|_S = 0$ (Moffatt 1969). Nobody computes with this formula, but it has the advantage of being independent of gauge, and it has a similar form to the Gauss linking integral. For ease of use, we remove one of the integrals via the Coulomb gauge (Biot–Savart) vector potential (Cantarella et al. 2001)

$$\mathbf{A}(\mathbf{x}) = \frac{1}{4\pi} \int \frac{\mathbf{r}}{r^3} \times \mathbf{B}(\mathbf{y}) \ \mathrm{d}^3 y, \tag{2}$$

which gives the more familiar form $H = \int \mathbf{A} \cdot \mathbf{B} \ \mathrm{d}^3 x$.

For volumes $\mathcal{V}$ not bounded by a magnetic surface the helicity is measured relative to that of the potential field P, where $\nabla \times \mathbf{P} = 0$ and $\mathbf{P} \cdot \hat{n}|_S = \mathbf{B} \cdot \hat{n}|_S$:

$$H_{\mathcal{V}} = \int_{corona} (\mathbf{A} + \mathbf{A}_P) \cdot (\mathbf{B} - \mathbf{B}_P) \ \mathrm{d}^3 x \tag{3}$$

(Berger & Field 1984; Finn & Antonsen 1985). This formula gives the same result for any choice of gauge; however a particularly convenient choice satisfies

$$\nabla \times \mathbf{A}_P = \mathbf{P}, \qquad \hat{\mathbf{n}} \cdot \nabla \times \mathbf{A}_P = B_n, \tag{4}$$

$$\nabla \cdot \mathbf{A}_P = 0, \qquad \mathbf{A}_P \cdot \hat{\mathbf{n}} = 0. \tag{5}$$

With the above expressions, we can compute time derivatives due to internal dissipation and to flow through boundaries. For example, the time derivative of

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the magnetic helicity H_{corona} of the solar corona is

$$\frac{\mathrm{d}H_{corona}}{\mathrm{d}t} = -2\int_{corona} \mathbf{E}\cdot\mathbf{B}\,\mathrm{d}^3x - 2\oint_{photosphere} \mathbf{A}_P \times \mathbf{E}\cdot\hat{\mathbf{n}}\,\mathrm{d}^2x. \qquad (6)$$

The volume integral of $\mathbf{E}\cdot\mathbf{B}$ gives helicity dissipation. In ideal MHD the parallel electric field vanishes, so magnetic helicity is conserved, and only changes due to flow through the boundary. For finite resistivity, an inequality relates helicity dissipation to energy dissipation (Berger 1984). If we let the magnetic energy $W = \frac{1}{2}\int B^2\,\mathrm{d}^3x$, then we can define a length scale $L \equiv |H|/2W$. The corresponding dissipation time is $\tau_d = L^2/\eta$. Consider a reconnection occurring over a time Δt. Then the inequality gives

$$\left|\frac{\Delta H}{H}\right| \leq \sqrt{\frac{\Delta t}{\tau_d}}. \qquad (7)$$

For solar flares, if we choose $\Delta t \sim 10^3 s$, $L \sim 10^6\,\mathrm{m}$, $\eta \sim 1\,\mathrm{m}^2 s^{-1}$, then $\tau_d \sim 10^{12} s$ and $|\Delta H/H| < 3\times 10^{-5}$.

These results can be generalized to situations where a simple Ohm's law is not appropriate (Berger 1984). Also we should note that helicity is not conserved (and not even well defined) in a periodic box with net flux (Berger 1997). The approximate conservation of magnetic helicity during reconnection can generate twist in coronal loops by changing mutual helicity into self helicity (Berger 1982; Song & Lysak 1989; Wright & Berger 1989).

The minimum energy state for a given helicity is a linear force free field (Woltjer 1958; Dixon et al. 1989). Taylor (1986) pointed out that reconnection would be necessary to achieve this minimum energy state. Heyvaerts & Priest (1984) applied the theory of relaxation to force-free fields to coronal heating theory.

2. Helicity Observations

Northern coronal structures tend to have negative helicity; Southern structures tend to have positive helicity (Hale 1927; Seehafer 1990; Rust & Kumar 1994; Martin et al. 1994; Pevtsov et al. 1995; Canfield et al. 1999). This tells us the *sign* of helicity, and already gives us important clues to the nature of solar magnetism. However, new work emphasizes *quantitative* measurements of helicity. Can we detail the total magnetic helicity balance of the sun and heliosphere? (Rust & Kumar 1994; Rust & Kumar 1996).

On the largest scale, rotation of the solar dipole injects positive helicity into the Southern Parker spiral (Bieber et al. 1987). On the scale of the entire sun, differential rotation of the solar field injects positive helicity into the Southern interior, with consequences for the solar dynamo (Berger & Ruzmaikin 2000).

The helicity of coronal fields has been measured in several different ways. Vector magnetogram studies give part of the current helicity (j_z/B_z) (Abremenko et al. 1997; Bao & Zhang 1998; Leka & Skumanich 1999; Pevtsov et al. 1995). Differential rotation can inject helicity into active regions (van Ballegooijen et al. 1998; DeVore 2000, Demoulin et al. 2002). Magnetograms can be

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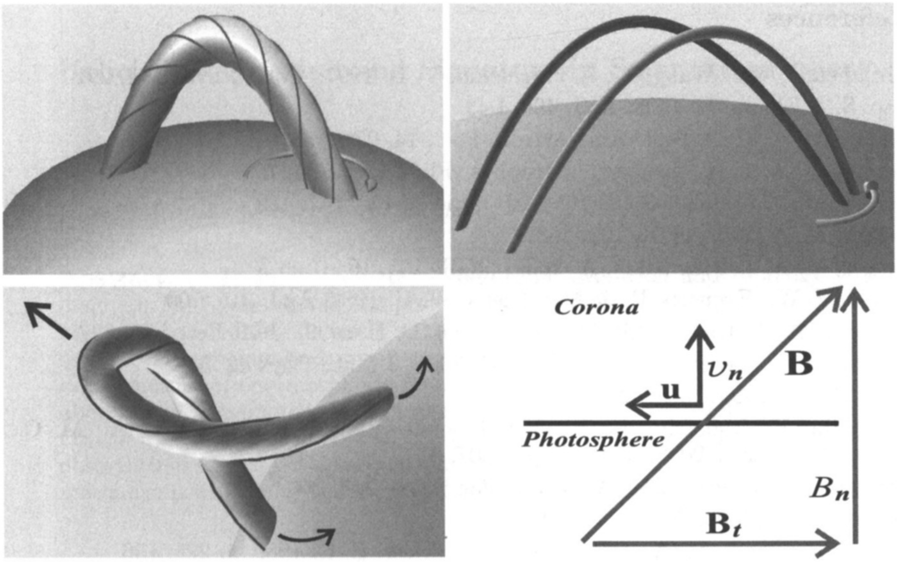

Figure 1. Top: photospheric motions can twist, shear, or braid corona fields. Bottom: The footpoints of a rising flux tube display an apparent transverse motion $\mathbf{u}$. Geometrically, $\mathbf{u}/v_n = -\mathbf{B}_t/B_n$.

combined with best fit force-free extrapolations to estimate the helicity (Green et al. 2002).

Let the photospheric plasma velocity be $\mathbf{v} = \mathbf{v}_t + v_n\hat{r}$ in terms of its tangential and normal (radial) components. Then the flow of helicity through the photosphere (6) is

$$\frac{\mathrm{d}H_{corona}}{\mathrm{d}t} = 2\oint_{photosphere} ((\mathbf{A}_P \cdot \mathbf{B}_t)\mathbf{v}_n - (\mathbf{A}_P \cdot \mathbf{v}_t)B_n)\ \mathrm{d}^2x \tag{8}$$

(Kusano et al. 2002). We can infer the components of $\mathbf{v}$ from Döppler measurements. Alternatively, local correlation tracking velocity techniques give the motion $\mathbf{u}$ of a magnetic element across the photosphere (Chae 2001; Nindos et al. 2003). A rising field line has an *apparent* transverse motion $-(v_n/B_n)\mathbf{B}_t$. Adding the plasma transverse motion, we obtain $\mathbf{u} = \mathbf{u}_t - (v_n/B_n)\mathbf{B}_t$. This makes the helicity flow equation especially simple (Démoulin & Berger 2003):

$$\frac{\mathrm{d}H_{corona}}{\mathrm{d}t} = -2\oint_{photosphere} (\mathbf{A}_P \cdot \mathbf{u})B_n\ \mathrm{d}^2x. \tag{9}$$

Energy flow also can be expressed in terms of the tracking velocity:

$$\frac{\mathrm{d}E_{corona}}{\mathrm{d}t} = -\frac{1}{\mu_0}\oint_{photosphere} (\mathbf{B} \cdot \mathbf{u})B_n\ \mathrm{d}^2x. \tag{10}$$

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