PART II

RR LYRAE VARIABLES IN POPULATION II SYSTEMS

OBSERVATIONAL ASPECTS OF RR LYRAE VARIABLES IN GLOBULAR CLUSTERS

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1. Introduction

RR Lyrae variables play a prominent role in many of the problems of globular clusters, and from several points of view. In the first place they can be considered as pretty good indicators of population and distance; although they do not form a completely homogeneous set of stars, the knowledge of their mean absolute magnitude gives a powerful means of establishing distances within and outside the Galaxy, and hence of determining the form and size of the Galaxy itself. Moreover, the number of RR Lyrae stars in clusters, the relative frequency of RR_c and RR_{ab} types, the length of the transition periods, the array of colors, when correctly interpreted, give important information on the degree of evolution, age and chemical composition of the clusters. Placed as they are in a peculiar region of the H−R diagram of Population II, the RR Lyr variables can be used as a good test of the theories of advanced evolution or the models of pulsating stars.

This explains why in the last ten years studies of the RR Lyr variables in globular clusters and nearby galaxies (Draco, Sculptor, Leo II), as well as of RR Lyr stars in the galactic bulge, have acquired so great an importance, and why they have been the object of observational research at many Observatories.

A survey of the present status of knowledge of RR Lyr variables in globular clusters from the observational point of view can be developed along the following lines:

(1) Frequency of RR Lyrae stars in globular clusters.
(2) Determination of magnitudes and colors.
(3) Periods and light curves; amplitudes.
(4) Secular and periodic variations of period and form of the light curves.
(5) Properties of RR Lyrae stars in different globular clusters.
(6) Absolute magnitude, composition, mass, age.

2. Frequency of RR Lyrae Stars in Globular Clusters

Of 122 known galactic globular clusters, about one-hundred have been searched for variables. However, not all of them have been thoroughly and extensively searched. An increase in the total number of variables may still be expected in some clusters, although it is very likely that the general situation will not be changed. A very important contribution in the survey for variables of southern globular clusters has been made by Fourcade and Laborde, of the Cordoba Observatory, who examined most of the clusters south of Declination −29° with the 60-in. telescope of Bosque Alegre.

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Their Catalogue and Atlas (Fourcade and Laborde, 1966) are of extreme value for future investigations.

Sixteen globular clusters remain still unexplored (Table I). Some of them are Palomar clusters, very faint because of their large distance or strong interstellar absorption. The others are clusters in the direction of the galactic center, between 17 and 19 h in RA and −8° to −26° in Declination, strongly reddened. It is likely that most of these clusters, which belong to the nucleus-disk system and have an advanced spectral type, are poor in RR Lyrae variables. A survey should be opportune.

TABLE I

Globular clusters not examined for variable stars

Cluster	R.A.	Dec.	Class	$(m-M)_{\mathrm{app}}$.
Pal 6	$17^{\mathrm{h}}40^{\mathrm{m}}.6$	$-26°12'$	XI	21.5
Pal 8	18 38.5	−19 52	X	20.4
Pal 14	16 8.8	+15 5	–	–
Pal 15	16 57.6	− 0 28	–	–
Arp 2	19 25.6	−30 27	–	–
NGC 6316	17 13.4	−28 5	III	–
6325	17 15.0	−23 42	IV	–
6342	17 18.2	−19 32	IV	–
6355	17 20.9	−26 19	–	–
6440	17 45.9	−20 21	V	–
6517	17 59.1	− 8 57	IV	–
6544	18 4.3	−25 1	–	–
6638	18 27.9	−25 32	VI	–
6642	18 28.4	−23 30	–	–
6684	18 46.5	−65 12	–	–
6717	18 52.1	−22 47	VIII	17.1

Of one hundred clusters examined for variables only three contain more than 150 variables, mostly of RR Lyr-type. Two are the well known clusters NGC 5139 (ω Cen) with about 171 variables and NGC 5273 (M3) with 189 variables. The third, IC 4499, is a globular cluster studied by Fourcade and Laborde (1969) who found the system's exceptional richness in variables.

The sixteen globular clusters with more than 35 variables, the richest in the Galaxy, are listed in Table II. They contain nearly 1400 variables, of which more than 75% are RR Lyrae stars. Since the total number of variables in one hundred globular clusters is about 2060, the 16 clusters of Table II represent 70% of the total. It is therefore interesting to examine the properties of these clusters.

First they are all very rich in stellar population, as shown by their integrated absolute magnitudes (from −10 to −7.5) and as proved by counts of stars. However this is a necessary, but not a sufficient condition. We know in fact that extremely rich clusters, such as 47 Tuc and M13, are poor in variables.

Table II also indicates that, in clusters with many variables, most of the variables are RR Lyrae stars. Now, the occurrence of RR Lyrae stars depends on the density

TABLE II

Globular clusters with more than 35 variables

Cluster NGC (*IC*)	Var	RR	%RR	Sp.	Morgan class	Q	V	Oost. class	n_b/n
3201	82	72	88	–	–	−0.32	–	I	–
5139 (ω Cen)	171	138	81	F7	II	−0.39	2.9:	II	0.85
5272 (M3)	189	173	92	F7	II	−0.41	2.64	I	0.53
4499	170	–	–	–	–	–	–	–	–
5904 (M5)	97	92	95	F5	II	−0.39	2.58	I	0.72
6266 (M62)	89	74	83	F8	–	−0.31	–	I	–
6402 (M14)	77	63	82	F8	IV	−0.31	–	I	–
6715 (M54)	80	63	79	F7	III	−0.35	–	I	–
7006	72	67	93	F3–4	II	−0.40	2.6	I	–
7078 (M15)	102	74	74	F3	I	−0.44	3.1	II	0.73
5024 (M53)	47	33	70	F4	II	−0.37	3.1	II	0.85:
6121 (M4)	43	41	95	–	–	−0.31	2.5	I	0.40:
6934	51	44	86	F7	–	−0.35	–	–	–
6981 (M72)	39	39	100	G0	II	−0.32	2.6	I	0.60
2419	36	27	75	F5	–	−0.40	–	–	–
4590 (M68)	38	35	92	F2	–	−0.43	3.0:	II	–

and distribution of stars along the horizontal branch. The best condition seems to be reached when there is a moderate excess of blue over red components (ω Cen, M5) or an even distribution of stars (M3). There is only one cluster, NGC 7006, which is very rich in RR Lyrae variables and yet shows an excess of components to the red side of the gap. The peculiarities of the color-magnitude diagram of this remote cluster have been pointed out by Sandage and Wildey (1967).

Clusters rich in RR Lyrae variables have another property: most of them belong to the halo and have a relatively low metal abundance. With only one exception (M72, Sp. G3), their integrated spectral types are earlier than F8, and the corresponding Q is less than −0.31, the average value being −0.37. Most of these clusters belong to Morgan's class I–III with the sole exception of M4 (class IV).

It may be interesting to compare clusters rich in RR Lyrae stars with those in which no or few RR Lyrae stars have been found, notwithstanding the intrinsic richness of stars. Some of these variable poor clusters are listed in Table III. They can be divided into two classes: (a) clusters, like NGC 6254 (M10) or NGC 6205 (M13) which have

TABLE III

Rich clusters without or with few RR Lyr variables

Cluster NGC	Var	RR	%RR	Sp.	Morgan class	Q	V	Oost. class	n_b/n
104 (47 Tuc)	14	2?	–	G3	–	−0.26	2.15	–	0
6205 (M13)	15	2:	–	F5	III	−0.44	2.55	–	1
6218 (M12)	1	0	–	F7	–	−0.40	2.8	–	1
6254 (M10)	3	0	–	G0	IV	−0.41	2.85	–	0.9
6838 (M71)	4	0	–	G2	VI	−0.23	2.1	–	0
6637 (M69)	10	2?	–	G5	VII	−0.21	1.0	–	0

almost all the horizontal branch components on the blue side of the gap. The absence of RR Lyrae stars depends on the absence of stars in other parts of the horizontal branch besides the extreme blue. (b) The second group consists of clusters with a high degree of metallicity, which have most of the components on the red side of the gap, so that the horizontal branch is reduced to a short red stub. 47 Tuc and many of the nucleus disk clusters are typical members of this group. It seems apparent that metal abundance rather than other parameters determines the strange structure of the horizontal branch in these clusters, and therefore the absence or scarcity of RR Lyrae variables. Age, however, may play an important role.

Connected with the frequency of RR Lyrae stars in globular clusters is the problem of their distribution within and around the clusters. Are there stars escaping from globular clusters ? It would be difficult to decide this with non-variable stars, since it would be hard to recognize them among the field stars. With RR Lyrae stars, on the other hand, the observational control becomes relatively easy, since if they escaped from clusters their mean magnitude would be about the same as for the RR Lyrae stars inside the clusters. According to Kukarkin (1961), researches on RR Lyrae stars near globular clusters have produced the following conclusions: (1) RR Lyrae stars exist at very great distances from the globular clusters, the majority of them coming from the clusters. (2) Preliminary estimates indicate that about $\frac{1}{3}$ of all RR Lyrae stars, not *now* connected with globular clusters, have originated in globular clusters.

Researches with wide field telescopes of large size should be continued to ascertain the relation of the field RR Lyrae stars to the globular clusters.

3. Determination of Magnitudes and Colors

The next step in the study of the RR Lyrae variables in globular clusters is the determination of magnitudes and colors. Knowledge of the mean magnitudes allows a classification of the RR Lyrae stars even before periods and light curves are obtained.

In the past, the magnitudes were mostly given in the pg system, without great accuracy. There is now a tendency to improve the precision and to use correct photometric systems, generally *UBV*, deriving also, whenever possible, $U-B$ and $B-V$ color indices. Photometric observations in different colors are very important for understanding the evolution of RR Lyrae variables with temperature, and to find correlations of the temperature with other parameters (amplitude, period, luminosity). For variables of known period a relatively small number of observations is sufficient to derive amplitudes and mean colors, and in particular to obtain their position in the $U-B$, $B-V$ diagram, together with the interstellar reddening and the ultraviolet excess. According to recent observations of Sandage (1969) the $B-V$ boundaries of the instability strip are pretty well determined. Apparently they do not vary from cluster to cluster when due corrections for absorption and blanketing are made: the boundaries are 0.175 on the blue side and 0.420 on the red. Therefore the positions of the RR Lyrae stars in a given cluster enable us to find the amount of interstellar absorption, if any, or the ultraviolet excess or other anomalies in the cluster.

Besides the sharp separation in color between RR Lyrae variables and non-variable stars along the horizontal branch, there may be also a complete separation in color between *ab*- and *c*-type variables. This separation, which is well-defined in some clusters (M3, NGC 6171, M72) where the *c*-type variables are systematically bluer than the *ab*, is not so sharp, however, in other clusters of lower metal content, such as ω Cen and M15, which show an overlap in color between the two types of RR Lyrae stars. Accurate determinations of magnitudes and colors of RR Lyrae variables have assumed great importance; however, despite the increasing attention paid to RR Lyrae variables in recent years, our knowledge of them is still unsatisfactory. Precise, possibly photoelectric, observations of selected samples of RR Lyrae stars in different clusters should be the objective of future researches.

4. Periods and Light Curves

After the RR Lyrae variables in clusters have been identified and approximate magnitudes given, a further step is the determination of periods. There are no intrinsic difficulties in the search for periods when a sufficient number of well distributed observations are available. However, the taking and reduction of the material requires time. Much assistance in the finding of periods has come from the use of electronic computers. Computer programs have been written by various people and are now frequently used to produce elements and obtain light curves of RR Lyrae variables in clusters. Although there is still much to do in this field, the situation is rapidly improving. In one of her last reports on variable stars in clusters Dr Sawyer Hogg has listed about 1150 variables with known periods. They represent more than one-half of the variables in globular clusters, and already provide good material for statistical investigations. Periods of the other 900 variables will certainly be determined in the near future.

Light curves in different colors are also of great importance for the study of RR Lyrae stars in clusters. The variation of magnitudes and colors at different phases allows the determination of radii and luminosities using the Wesselink (1969) method. If the surface gravity is introduced into the calculations, even masses can be derived (Woolley and Savage, 1971).

5. Variations in Periods. Blazhko Effect

A still more advanced phase in the study of RR Lyrae variables in clusters is attained when the secular or periodic changes of elements and light curves are investigated. Modern researches have shown a definite trend in this direction, and a number of papers dealing with period variations have been published during the last years. Among many other excellent researches, I would like to recall here those made by Martin (1938) and Belserene (1952) in ω Cen, by Oosterhoff (1941) in M3, Margoni in M53 (1965, 1967), Coutts and Sawyer Hogg (1969) in M5.

It is well known that the epoch of maximum, or any other phase, can be obtained in regular RR Lyrae variables from the simple formula $T_i = T_0 + iP$. It was pointed out a long time ago, however, that when early observations are discussed, the $O-C$, observed minus calculated phases, show in many cases systematic deviations which cannot be accounted for by observational errors. Since the RR Lyrae variables are evolving through the instability strip, and this means a change in the magnitude and color and therefore in the density and period, the occurrence of secular period changes was not surprising and gave the possibility of establishing the direction of evolution of RR Lyrae stars along the horizontal branch. Secular changes of period of the type $P = P_0 + \beta(t - T_0)$ give to the $O-C$ diagrams a parabolic form of the type: $O-C = \beta(t-T_0)^2/2P_0^2$. Parabolic curves for the $O-C$ were obtained by different observers, who derived values of β (variation of period in days per day) of the order of 10^{-10} ($0^{\mathrm{d}}.03$ per 10^{-6} yr). Up to now, about eleven clusters rich in RR Lyrae stars have been examined for period variations. The results have not always been in agreement with the theories of horizontal branch evolution. In fact, it was noticed that in the same cluster some periods were increasing while others, without distinction of color, magnitude or period, were decreasing. For instance, examining 112 RR Lyr stars in Messier 3, Szeidl found that the periods of 22 variables were increasing with a rate of $0^{\mathrm{d}}.18$ per 10^{-6} yr while 25 other variables showed periods decreasing at a rate of $-0^{\mathrm{d}}.20$ per 10^{-6} yr.

In addition, some variables in M3 display $O-C$ curves which are by no means parabolic, but which indicate instead a periodic variation of the period (Figure 1.).

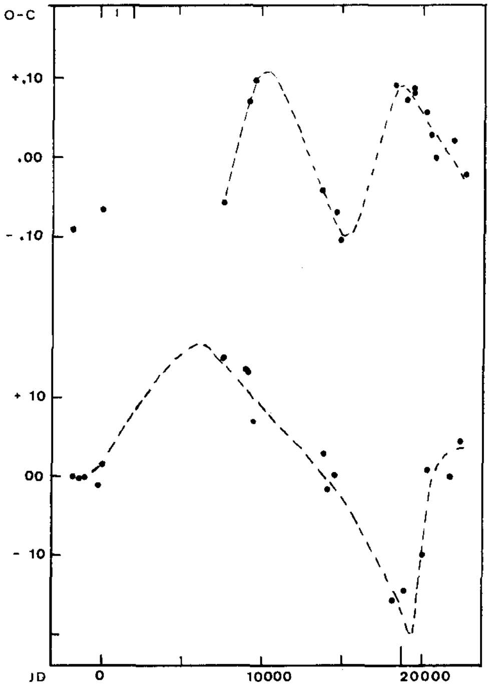

Fig. 1. Irregular $O-C$ curves observed by Szeidl (1965) in RR Lyrae variables of M3.

A somewhat different result was found by Belserene (1964) in an analysis of the RR Lyrae variables in Omega Centauri. She observed that 70% of the RR Lyrae variables show secular increases of period, while the other periods showed decreases, fluctuations, or remained constant. In M5, Coutts and Sawyer Hogg (1969) found 18 RR Lyrae variables with a constant period, 20 with increasing period ($0\overset{d}{.}05 \pm \pm 0\overset{d}{.}02\ 10^{-6}\ \mathrm{yr}^{-1}$) and 12 with decreasing period. In M15 the stars with increasing periods outnumber those with decreasing period, but the tendency to increase is not as clearcut as in ω Cen.

Variations of periods in opposite directions are not easily accounted for, although Faulkner and Iben (1966) have found, by theoretical considerations, that stars *do* change direction of evolution in the horizontal branch. Moreover, by assuming that the stars do not spend more than 10^8 yr in the RR Lyrae strip, it is easy to show that the observed changes of period are sometimes at least one order of magnitude greater than expected. So, there is now a widely held opinion among observers that changes of period in RR Lyrae variables are not directly connected with the evolution of these stars, that they are not secular but periodic. The researches of Szeidl, Margoni and others have shown that periodic variations of the periods in the RR Lyrae variables of globular clusters are more frequent than previously believed. Abrupt changes of period or random changes may also occur so that the problem becomes even more complicated.

Periodic variations of period are closely connected with periodic changes in the form of the light curve. It is well known to variable star observers that the light curves of some RR Lyrae do not repeat exactly from cycle to cycle. While there are stars which undergo perfectly regular variations, others show changes in the phase of the rising branch, and in the shape and amplitude of the light curve (Blazhko effect). Szeidl (1965) has noticed, for instance, that of 112 variables in M3, at least 36 show the Blazhko effect. Stars with variable light curves are encountered more frequently in M3 among the RR Lyrae stars with periods between $0\overset{d}{.}47$ and $0\overset{d}{.}56$. Some years ago, Detre (1961) pointed out that there are two different types of Blazhko effect. In the first type the lower part of the rising branch is constant in time and phase, while the upper part changes from one cycle to another in phase and amplitude. In stars of this type phase variations are greater at maximum than at minimum. In the second type, on the other hand, phase variations are small near maximum, but pronounced in the lower part of the light curve. This type is more frequently encountered among variables having a long-term secondary period. Different cycles may differ in length and amplitude in the variables affected by the Blazhko effect. An example is shown by the RR Variable No. 30 in M53 studied by Margoni and also by Wachmann (1968). Periodic changes of the light curve and brightness at maximum occur in this variable with a period of about 37 days (Figure 2).

Work on the period variations and Blazhko effect in the RR Lyrae stars of globular clusters is just beginning. It should be extended and refined with photoelectric observations in different colors, for their obvious implications in pulsation theory and stellar models. Particular care should be taken in the search for incipient or final stages of

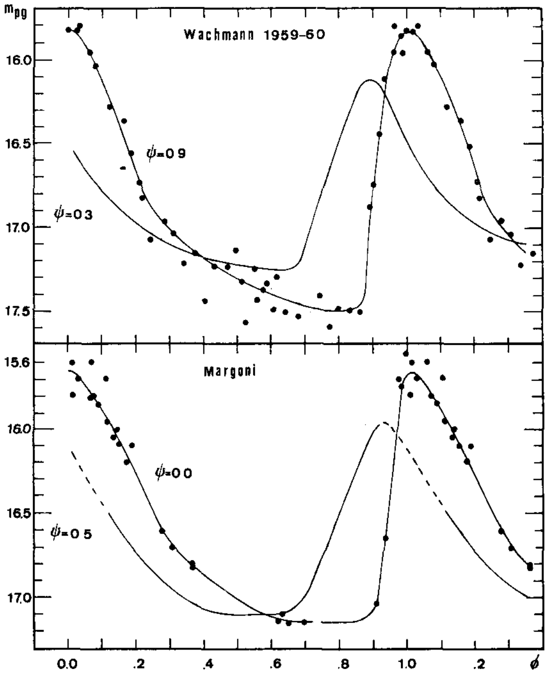

Fig. 2. Variations in the light curve and amplitude of variable No.30 in M15 due to the Blazhko effect, observed by Margoni and Wachmann (1968) at different phases of the 37-day period.

variability at the boundaries of the instability strip, and in the observation of stars near the transition from the fundamental to the first overtone mode.

6. General Properties of RR Lyrae Variables in Globular Clusters

It is well known that globular clusters may differ among themselves not only in age, concentration and richness of stars, but also in chemical composition, particularly for the relative abundance of hydrogen, helium and heavy elements. Theoretical work on unmixed horizontal branch models have shown that metal abundance may strongly affect the size and extension of the horizontal branch, and hence also the color and period of the RR variables. Also other parameters exert a strong influence on the horizontal branch. An observational approach to the theory should therefore follow these lines: (a) Establish properties of the RR Lyrae variables within rich clusters, as for instance ω Cen, M3, M5, M15. (b) Compare the RR Lyrae in clusters of different characteristics and see how their properties depend on the parameters of the cluster.

RR Lyrae stars in clusters are distributed in two classes: RR_c stars with periods less than $0\overset{\mathrm{d}}{.}4$, small amplitudes, light curves of sinusoidal type, and RR_{ab} stars with periods longer than $0\overset{\mathrm{d}}{.}4$, amplitudes up to $1\overset{\mathrm{m}}{.}7$ and even more, asymmetric light curves with steep rise and slow decline. It is generally assumed that *ab* variables vibrate in the fundamental mode and *c* variables in the first overtone, the ratio of the fundamental to the first overtone period being about $\frac{4}{3}$. The two classes of stars appear neatly separated on a *period-amplitude* diagram. The RR_c stars maintain more or less the same amplitude (about $0^{\mathrm{m}}.5$ in ω Cen) while the RR_{ab} stars show a decrease in amplitude with increasing period. RR_c variables have been further subdivided into two subtypes: those with periods from $0^{\mathrm{d}}.20$ to $0^{\mathrm{d}}.36$ and those with periods from $0^{\mathrm{d}}.36$ to about $0^{\mathrm{d}}.40$. In the *period-color* diagram the two subtypes occupy different positions, the shortest period group being to the left, near the blue edge of the strip. The RR_{ab} stars have also been classified into two subtypes: those with $P < 0^{\mathrm{d}}.6$ and $P > 0^{\mathrm{d}}.6$. The long period group has, at equal period, a larger amplitude and a bluer color than the short period group. The appearance of the diagrams suggests that a cluster may contain both groups of RR_{ab} variables, but in different proportions (Figure 3).

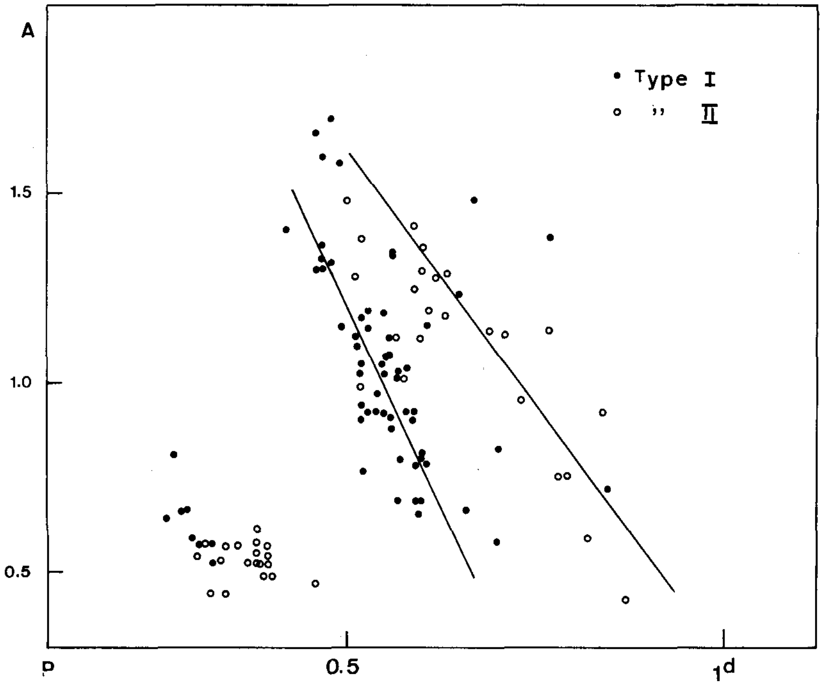

Fig. 3. Period-amplitude relation for RR Lyr variables in globular clusters of Oosterhoff type I and II.

The *period-luminosity* relation of RR Lyrae stars is also very interesting. At first sight, all RR Lyrae variables in a given cluster have about the same luminosity. However, when precise measures of the mean magnitudes are made, it is apparent that both RR_c and RR_{ab} increase slightly in brightness with period (Figure 4). The positions of the two types in the color-magnitude diagram have already been discussed. The separation in color of the two types may or may not be sharp (Figure 5). In general, however, the color index increases with period, the RR_c stars being in the

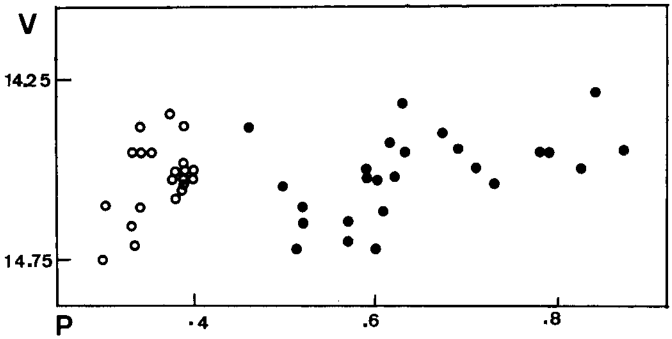

Fig. 4. Period-luminosity relation for RR Lyr variables of ω Cen (Dickens and Saunders, 1965).

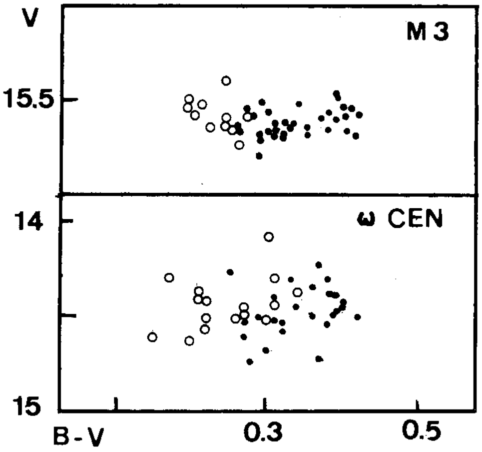

Fig. 5. Color-magnitude diagram for RR Lyr variables in M3 and ω Cen (Geyer and Szeidl, 1970).

average bluer than the *ab* stars. The period-amplitude, period-color and color-amplitude relations indicate some differences from cluster to cluster. When, however, the frequency distribution of the two types and the distribution of their periods are considered, these differences suddenly become very important. It is well known (Oosterhoff, 1939, 1944; Sawyer Hogg, 1944) that galactic globular clusters can be divided into two well-separated groups, according to the number N and mean period $\langle P\rangle$ of the RR_{ab} and RR_c types. In group I the mean period of RR_{ab} stars is nearly $0\overset{d}{.}53$ and that of RR_c stars is $0\overset{d}{.}32$ (Figure 6). The ratio $r = N_c/(N_c + N_{ab})$ is on the average 0.18. In group II the corresponding periods are respectively $0\overset{d}{.}65$ and $0\overset{d}{.}37$ and the ratio $r \sim 0.44$. In other words, in group I there is a predominance of RR_{ab} over RR_c stars and the mean periods are shorter than in group II, where RR_c stars are about as numerous as the RR_{ab} ones. Table IV illustrates the characteristics of the two groups according to the most recent data. Some extreme cases are presented by NGC 6981 with 27 RR_{ab} stars and only one RR_c star and in M68 with 21 RR_c stars and only 14 RR_{ab} types. Selection effects do not influence the relative abundance of

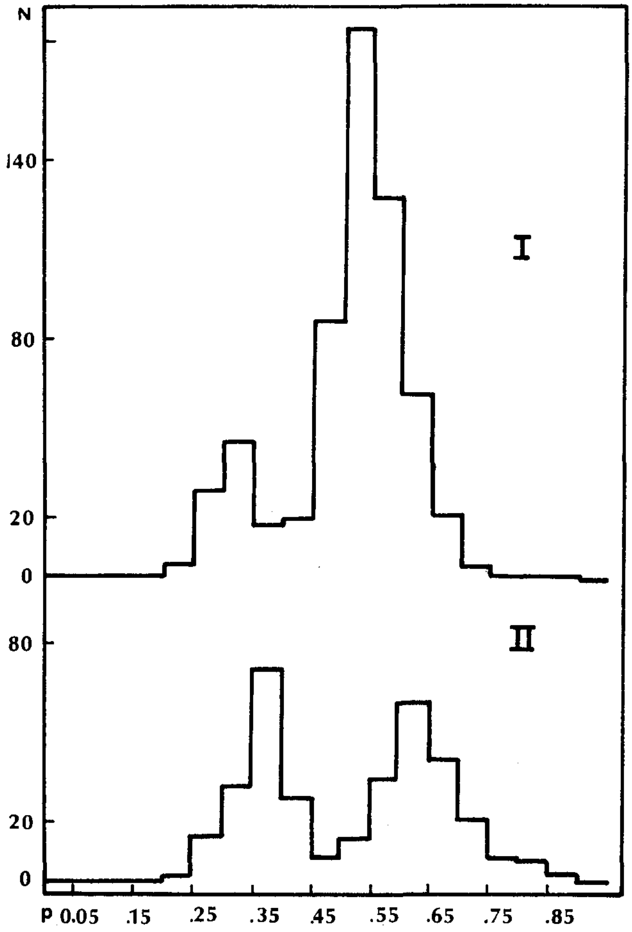

Fig. 6. Period distribution of RR Lyr variables in Oosterhoff groups I and II.

RR_c stars in the two groups, since the amplitude is about the same and the probability of discovery depends on the amplitude. A unique case is that of the loose cluster Pal 5 (not included in Table IV) which has five RR Lyrae variables, all of type *c*.

The classification of the globular clusters into two groups can be refined. Castellani *et al.* (1970) have proposed a multidimensional classification of the clusters into four classes, considering more strictly the ratio of type *ab* to *c*. Group I is formed by subclasses AI with a predominance of *ab*'s over *c*'s, and CI with about an equal number of *ab*'s and *c*'s (NGC 4147, 6362); group II is subdivided into CII with about the same number of *ab*'s and *c*'s, and AII with a predominance of *ab*'s over *c*'s (M9, M2). Besides the quantity *S* of Hartwick (1968) and the color index of the junction point $(B-V)_{0,g}$, they introduce as significant parameters the ratio N_c/N_{ab} and the absorption free parameter *Q*. This last parameter is shown in Table VI, together with the integrated spectral type of the cluster and the parameter ΔV, measured at $(B-V)_0=$

TABLE IV

Globular clusters in Oosterhoff's groups I and II

Group I

NGC	$\langle P_c \rangle$	N_c	$\langle P_{ab} \rangle$	N_{ab}	$\langle P_F \rangle$	Sp.	ΔV	Q	P_{tr}
362	–	–	$0^{\mathrm{d}}542$	7	$0^{\mathrm{d}}542$	F8	2.0	−0.42	$0^{\mathrm{d}}48$
3201	$0^{\mathrm{d}}305$	5	0.553	66	0.543	–	–	−0.32	0.48
4147	0.342	10	0.525	5	0.478	A5	–	−0.42	0.49
5272 (M3)	0.325	28	0.550	143	0.530	F7	2.64	−0.41	0.496
5904 (M5)	0.319	23	0.546	67	0.515	F5	2.58	−0.39	0.455
6121 (M4)	0.301	10	0.548	31	0.512	–	2.5	−0.31	0.44
6171	0.292	8	0.539	14	0.484	G2	1.8	−0.35	0.423
6229	0.293	2	0.525	9	0.501	F7	–	−0.39	0.466
6266	0.299	12	0.544	62	0.520	F8	–	−0.31	0.44
6362	0.345	8	0.535	7	0.495	–	–	−0.35	0.456
6402 (M14)	0.364	2	0.564	31	0.559	F8	–	−0.31	0.47
6712	–	–	0.544	7	0.544	F7	–	−0.31	0.50
6715 (M54)	0.356	2	0.548	32	0.544	F7	–	−0.35	0.45
6723	–	–	0.515	17	0.515	G3	–	−0.28	0.43
6981 (M72)	0.353	1	0.549	27	0.548	G2	2.6	−0.32	0.46
7006	–	–	0.571	27	0.571	F3–4	2.6	−0.32	0.49
	0.319	111	0.548	552	0.528				0.464
Group II									
4590 (M68)	0.376	21	0.625	14	0.550	F2	3.0	−0.46	0.55
4833	–	–	0.684	6	0.684	–	–	−0.41	0.63
5024 (M53)	0.364	19	0.642	23	0.571	F4	3.10	−0.37	0.53
5053	0.365	5	0.673	5	0.579	–	3.1	–	0.59
5139 (ω Cen)	0.372	59	0.654	77	0.585	F7	2.9	−0.39	0.57
5466	0.350	10	0.636	8	0.541	–	–	–	0.57
5824	0.35?	1	0.639	8	0.620	F5	–	−0.41	0.58
6333 (M9)	0.330	4	0.614	7	0.550	F2	–	−0.41	0.57
6341 (M92)	0.365	3	0.626	9	0.591	F2	–	−0.43	0.59
6426	0.346	6	0.665	4	0.542	–	–	–	–
6656	0.367	10	0.651	8	0.561	F6	–	−0.45	0.61
7078 (M15)	0.381	29	0.645	31	0.579	F3	3.1	−0.44	0.57
7089 (M2)	–	–	0.629	11	0.629	F3	–	−0.41	0.52
	0.367	167	0.646	211	0.578				0.57

= 1.4. Finally, column 6 in the Table gives for each cluster the mean value of the fundamental period obtained after multiplying the *c* periods by the factor $\frac{4}{3}$.

The position and extension of the horizontal branch strongly depend on the age of the cluster, the heavy element abundance, and the helium to hydrogen ratio. On the other hand, the period of a pulsating star is determined by its luminosity, mass, effective temperature and mode of pulsation. Changes in some of these parameters cause changes in the period, and, if the RR Lyrae stars are numerous, in the period distribution.

Without entering into a discussion of the different interpretations given to the phenomenon of the dichotomy in galactic globular clusters, it can be observed that, as shown in Table IV, clusters of the Oosterhoff group II have lower metal abundance than clusters of group I. On the average, the spectra of group II are earlier, the ΔV higher, the Q lower, than in group I. The separation is not sharp, and if it is true that the metal content is in general higher in clusters of group I, in some cases the degree of metallicity in clusters of the two groups is comparable, as for instance in ω Cen and M3. Perhaps the best interpretation of the observed dichotomy is still that given by Sandage (1958), which, however, leaves open the problem of the ratio N_c/N_{ab} in the two groups. From the relation $P\sqrt{\varrho}=C$, under some general assumptions, the following equation can be derived: $M_v=a(B-V)-b\log P/C+c$. For a given value of C, the length of the period depends on the absolute magnitude and temperature. On the other hand, Sandage (1970) has shown that the gap of instability in the horizontal branch of unreddened clusters is always limited between $B-V=0.175$ and 0.420. Therefore, the difference in the mean periods between Oosterhoff groups is not due to temperature, but only depends on the mean luminosity of the RR Lyrae stars in the two groups. And since the brightness of the RR Lyrae variables in clusters of group II is higher than in group I, as proved by the pulsation theory, also their periods, in the average, must be longer. Some questions, however, still remain open.

The first is the absence of groups with intermediate mean period between $0\overset{d}{.}65$ and $0\overset{d}{.}55$, which is one of the main points of the dichotomy phenomenon. However Castellani *et al.* (1972) have observed that the discontinuity in period between groups I and II almost disappears if the mean values of the periods of RR_{ab} and RR_c types are taken, after transformation of the periods of the RR_c stars to fundamental periods. The fundamental periods in RR Lyrae stars of group II, however, still remain higher than in group I, with perhaps some small overlap.

The second point concerns the absence in the galactic globular clusters of a third group of RR Lyrae, with periods concentrated around $0\overset{d}{.}44$, which undoubtedly exists in the direction of the galactic bulge among field stars. Tentative searches for clusters with RR Lyrae stars having a period distribution of this type have been made. Clusters with a relatively high degree of metallicity, as NGC 6712, NGC 6171, and NGC 6981 have been examined, but without great success from this point of view. Only NGC 6171, studied by Dickens (1970), shows some signs of a trend toward shorter periods, although scarcely significant, in comparison with the values found for the RR Lyrae stars in the galactic bulge.

Finally, as has been already remarked, a further difference between RR Lyrae stars in globular clusters of type I and II is found when secular variations of period are investigated. Among RR Lyrae stars in type I clusters the number of variables with increasing period are about the same as those with decreasing periods. In the type II clusters there seems to be a prevalence of variables with increasing period. The effect, however, has not been confirmed among the variables of M15, another cluster with low metal abundance, where the tendency to increase is not as marked as in ω Cen. So this effect, which should be further investigated, remains somewhat doubtful.

7. Absolute Magnitudes, Masses and Radii of RR Lyrae Stars in Globular Clusters

For many years RR Lyrae variables have been considered as the best distance indicators within the Galaxy. It was assumed that all RR Lyrae variables, wherever they were, had the same absolute photographic magnitude, equal to zero. However, after the main sequences of some globular clusters were identified, it became apparent that this assumption was incorrect. The fitting of the main sequences, even when due corrections for interstellar absorption and blanketing were applied, gave a dispersion of the mean absolute magnitudes of the horizontal branches which was significant. Additionally, a difference in the absolute magnitudes of the RR Lyrae variables was also a prevision of the theories of stellar evolution, considering that the position of the horizontal branch depends on the age and composition of the cluster concerned.

An approach to the determination of the absolute magnitudes of RR Lyrae variables was made from various directions. Statistical parallaxes were determined for galactic RR Lyrae stars in moving groups by Eggen and Sandage (1959, 1962), who obtained an average value $\langle M_v \rangle = +0.6$. More recently Woolley and Savage (1971), from a discussion of the secular parallaxes of a number of galactic RR Lyrae stars, derived values of the visual absolute magnitude ranging from +0.62 for RR_c types with $P > 0^d.36$ to +0.46 for RR Lyrae *ab* types with $P > 0^d.44$.

In 1965, after a careful determination of reddening and color excess, Sandage (1969) arrived at the following values of the visual absolute magnitude of RR Lyrae stars in different clusters by a main sequence fitting procedure:

M92	+0.47
M15	+0.51
M13	−0.09
M3	+0.38
47 Tuc	+0.44.

These values were later improved by Sandage (1970) using field subdwarfs with known trigonometric parallaxes for calibration of the main sequences. The final result was $M_v = +0.6 \pm 0.2$ for the mean absolute magnitude of the RR Lyrae variables in M3 (type I), M15 and M92 (type II). The result, however, was somewhat disappointing, since the RR Lyrae stars in M3 were found to be about $0^m.3$ brighter than the RR Lyrae stars of M92 and M15, while the pulsation theory gives a difference of $0^m.3$, but in the opposite direction. So, the observations have come to exactly the opposite result than theory. The RR Lyrae variables in M13 were found to be much brighter ($M_v = +0.05$), than the others but this fact was easily explained by assuming that they might be associated with the asymptotic branch rather than the horizontal branch.

There are other methods, based on pulsation theory, for deriving the absolute magnitudes of RR Lyrae variables in globular clusters. Christy (1966) has found that the transition period from the fundamental mode to the first overtone is dependent on the luminosity according to the equation:

$$P_{\rm tr} = 0.057(L/L_\odot)^{0.6},$$

which can be reduced to the following (Dickens, 1970):

$$M_{\rm v} = -0.46 - 4.17 \log P_{\rm tr}.$$

Using this formula, and with the values of the transition period reported in the last column of Table IV, Dickens derived from a grouping of the clusters in the Deutsch-Kinman A, B, C classes, the following mean values for the mean absolute magnitudes:

Very low metal abundance (class C)	$M_{\rm v} = +0.51$
Low metal abundance (class B)	$+0.69$
Moderate metal abundance (class A)	$+0.96$

By grouping the clusters into the Oosterhoff classes I and II, the following values of the mean absolute visual magnitude have been found:

Group I	$M_{\rm v} = +0.926$
Group II	$.559$

with a difference $\Delta M_{\rm v} = +0.367$ between group I and group II. It should be kept in mind that these results assume the full validity of the pulsation theory. If the observational errors in the direct determination of the absolute magnitudes of the RR Lyrae stars could be reduced, or if the distances of the systems containing RR Lyrae could be determined by other methods there would be the possibility of obtaining more reliable values for the absolute magnitudes, and, at the same time, of obtaining further controls on the pulsation theory. At the present moment, the only conclusion which can be drawn from direct observations is that the mean visual absolute magnitude of RR Lyrae stars in globular clusters is about $M_{\rm v} = +0.6 \pm 0.2$.

Another point concerns the dispersion of the magnitudes of the RR Lyrae stars within the same cluster, which can reach as much as $0^{\rm m}.3$. Part of this dispersion is certainly due to observational errors or the Blazhko effect, but in part it is intrinsic. The dispersion may be ascribed to differing stages of evolution or different composition among the stars of the horizontal branch.

Radii and masses of galactic RR Lyrae stars have been derived recently by Woolley and Savage (1971) through an extension of the Baade-Wesselink method, taking into account the surface gravity and its variations. For RR$_{ab}$ stars with $P > 0^{\rm d}.44$, absolute magnitude $+0.40$, they have found $R \sim 5.5 R_\odot$, $\mathcal{M} \sim 0.5 \mathcal{M}_\odot$; for RR$_c$ stars with $P > 0^{\rm d}.36$, $M_{\rm v} = +0.8$, $R \sim 4.5 R_\odot$, $\mathcal{M} \sim 0.6 \mathcal{M}_\odot$. From these parameters, values of the pulsation constant are found to be in fairly good agreement with those given by the pulsation theory.

Masses can also be derived from the pulsation theory, from knowledge of $(B-V)_{\rm trans.}$ which gives the T_e at the transition point, and from the high temperature boundary of the instability strip, given by the color index of the bluest *c*-type variables. This last parameter also gives the helium content. The values derived by Dickens for three globular clusters: NGC 6171, M3, ω Cen are: Masses from 0.48 to 0.43 solar masses; M_V from 0.57 to 1.10; helium content from 0.33 to 0.46. These values confirm those found by Sandage, who gives a helium content from 0.3 to 0.35 and masses of the order of 0.5. The ages of globular clusters, derived from the magnitude of the turn-off point are, according to Sandage, about 10×10^9 yr, in good agreement with cosmological theories.

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DISCUSSION

Schwarzschild: Is is not possible to discover very small amplitude variables by careful photographic photometry?

Rosino: Discovery of variables in globular clusters with an amplitude less than $0^{m}_{.}2$ is very difficult when the usual techniques (blink, negative on positive, and so on) are employed. However, if a given star is, for some reason, suspected of variability and special and extensive observations of the particular star are made then even a variation less than $0^{m}_{.}2$ can be detected.

Dickens: Typical standard errors (per plate) are about 0.03 mag. at best so that variables with amplitudes less than about 0.2 mag. would be difficult to find, except perhaps with rather specialized and extensive measurements.

Cox: Is the colour boundary extremely sharp between *c* type variables and non-variables?

Dickens: Only in M3 are the observations adequate to test this and indeed the separation between variables and non-variables is quite sharp, as shown many years ago by Roberts and Sandage.

Graham: Bright field horizontal branch stars can be picked out by Stromgren *uvby* photometry alone. Careful examination with photoelectric photometry of those stars whose colors indicate that they are near to the instability strip could possibly detect small amplitude RR Lyrae stars. Examination of one such star, HD 161817 by Graham and Zinter some years ago failed to reveal any variations as great as $0^{m}_{\cdot}1$.

Jones: I know of no small amplitude RR Lyrae in the field. The small amplitude variables have shorter periods and only a few have weak lines.

Cox: I believe that non-linear pulsation theory predicts a very sharp blue edge to the pulsation instability strip. Linear theory certainly does. In a given cluster, where stars exist to check this point, how sharp is this blue edge observed to be?

Rosino: I think it is of the order of one tenth of a magnitude, but it strongly depends on the observational technique.