J. Fluid Mech. (2024), *vol.* 1000, F3, doi:10.1017/jfm.2024.871

JFM FOCUS ON FLUIDS

Asking the right questions on Rayleigh–Bénard turbulence

Detlef Lohse†

Physics of Fluids Department and Max-Planck Center Twente for Complex Fluid Dynamics, Faculty of Science and Technology, J.M. Burgers Center for Fluid Dynamics, University of Twente, PO Box 217, 7500 AE Enschede, The Netherlands

(Received 19 June 2024; accepted 29 August 2024)

The paper by Castaing *et al.* (*J. Fluid Mech.*, vol. 204, 1989, pp. 1–30) on turbulent Rayleigh–Bénard convection has been one of the most impactful papers on the subject – not by giving the right and complete answers but by developing versatile concepts and by asking the right questions, namely: (i) What is the overall flow organization? (ii) What is the dependence of the Nusselt number Nu (the dimensionless heat transport) on the Rayleigh number Ra (the thermal driving strength)? (iii) What is the ultimate state of turbulence for extremely large Ra? Thanks to Castaing *et al.* having asked the right questions, the field has made tremendous progress over the last 35 years.

Key words: bénard convection

1. Rayleigh–Bénard convection: its simplicity, relevance and the historical context

Rayleigh–Bénard (RB) convection, the flow in a container heated from below and cooled from above, is one of the paradigmatic systems in fluid dynamics and the physics of fluids. It is – so to say – the *Drosophila* of the physics of fluids, as many new concepts in fluid dynamics have been tested with this system, be it linear instabilities, pattern formation, turbulence or the so-called ultimate regime of turbulence for very strong thermal or buoyant driving. The reason for this is the conceptual simplicity of the system, with high symmetries, the dynamical equations and their boundary conditions being well known, and the accessibility, both experimentally and more recently also numerically.

One of the most impactful papers in the history of RB research is the paper 'Scaling of hard thermal turbulence in Rayleigh–Bénard convection', by B. Castaing, G. Gunaratne, F. Heslot, L.P. Kadanoff, A. Libchaber, S. Thomae, X.-Z. Wu, S. Zaleski and G. Zanetti (Castaing *et al.* 1989), the 'Chicago Nine', as they have been called, alluding to the 'Chicago Seven' trial against seven civil right activists in 1969. This paper also has had a

† Email address for correspondence: d.lohse@utwente.nl

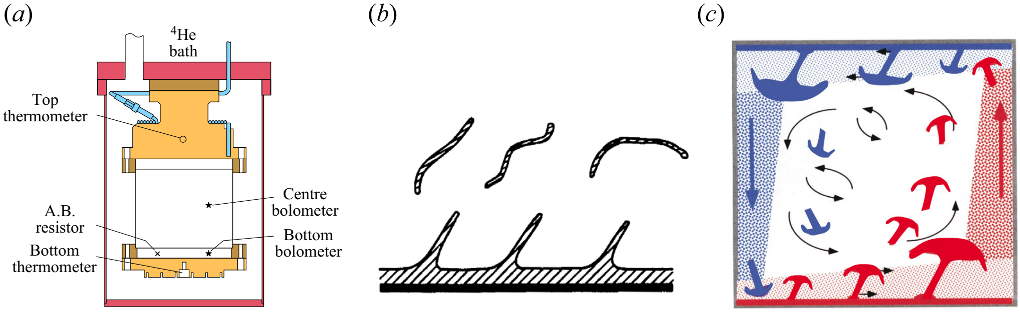

Figure 1. (*a*) Experimental set-up of the highly controlled RB cell. With the bolometers, time series of the local temperature were measured. (*b*) Hand-drawing by Leo Kadanoff: visualization of the plumes detaching from the laminar boundary layer and driving the large-scale flow, one of the central ideas of Castaing *et al.* (1989), from where (*a*,*b*) are taken, with (*a*) being coloured for this article for clarity. (*c*) Later hand-drawing by Leo Kadanoff of how the 'wind of turbulence' would evolve in the RB cylinder (taken from Kadanoff (2001)). Some years later Shang *et al.* (2003) and Sun, Xia & Tong (2005) found that the tilting of the wind of turbulence is along the other diagonal; for more details on the flow organization as presently known, the reader is referred to the review by Xia (2013).

tremendous impact on me personally, having first read it in 1990 as a first-year PhD student. In the authors' words, the popularity of the RB system lies in the fact that 'this system is in many respects of unsurpassed simplicity for the study of the irregular and complex motions in fluids and hence is well-suited for a fundamental study', that 'it has an advantage over open flow systems because of the greater ease with which very well-defined boundary conditions can be realized experimentally' and that 'in a cavity one only needs to control the temperature of the walls and this can be done to a remarkable precision and with great stability over a long period of time' (see figure 1*a*).

Let us sketch the research landscape in the physics community in the 1970s and 1980s. The fields of nonlinear dynamics and deterministic chaos experienced a boost, as very nicely described in the textbook by Strogatz (1994) or in the more popular book by Gleick (1988). Rayleigh–Bénard flow played a very central role therein, already starting with Lorenz (1963), who developed very simplistic model equations for thermal convection just beyond its onset (now called Lorenz equations), and continuing with Ahlers (1974) and Maurer & Libchaber (1979), who both found new routes to chaos in the RB system by analysing the spectra of time series of the heat transport. The thermal driving strength in both cases was weak, i.e. the Rayleigh number Ra (the non-dimensionalized temperature difference Δ between the hot bottom and cold top plates, defined by $Ra \equiv \beta g L^3 \Delta/(\nu\kappa)$, where L is the distance between the plates, β the thermal expansion coefficient, g the gravitational acceleration, ν the kinematic viscosity and κ the thermal diffusivity) was relatively low, just beyond the onset of convection. Both Ahlers (1974) and Maurer & Libchaber (1979) did these RB studies with cryogenic helium as the working fluid, having had a background in the field of critical phenomena and in the knowledge of how well such a system can be controlled. With these new insights it was very natural to ask what would happen for stronger thermal driving, i.e. larger Ra, so that the flow inside the container loses its spatial coherence and becomes turbulent, rather than simply being chaotic. Threlfall (1975) was the first to perform turbulent convection experiments in that regime in low-temperature gaseous helium.

Castaing *et al.* (1989) picked up the idea of these experiments, but now with a much higher degree of precision and control and with much stronger thermal driving, up to

$Ra \sim 10^{13}$, so clearly in the turbulent regime. These high Ra numbers were possible with a relatively small cylinder height of only 8.7 cm (and the same diameter), as at the low operating temperatures (around 5 K) and operating pressures up to 2 atm the kinematic viscosity and thermal diffusivity of helium are very small. Moreover, Castaing *et al.* (1989) varied Ra over eight orders of magnitude, by changing the operating pressure. The main objective of the paper was 'to provide some phenomenological explanation' of the results. In retrospect, one can say that the reason for the huge impact of the paper on the community and on me personally is not that it provided the right answers – in fact it partially did not – but as it stated the problem, developed versatile concepts and asked the right questions, which have stimulated the field for decades.

2. Asking the right questions

The questions which Castaing *et al.* (1989) asked were: (i) What is the overall flow organization? (ii) What is the dependence of the Nusselt number Nu (the dimensionless heat transfer, i.e. the heat transfer normalized by the purely conductive heat transfer) on the Rayleigh number Ra? (iii) Is there, for very large Ra, an ultimate regime of turbulence, with different scaling properties? Correctly answering these questions has always been of utmost importance, in order to develop a theory of the flow to be able to predict the heat transfer and to extrapolate the results to the much larger Ra relevant in geophysical and astrophysical contexts.

The answer to question (i) on the overall flow organization was in those days – in the times prior to elaborate particle image velocimetry and detailed direct numerical simulations – much more difficult to find than it is nowadays. To get some idea, Castaing *et al.* (1989) placed thermal probes (bolometers) into the flow at different locations and detected temperature time series, thus partially illuminating the fluid dynamics inside the non-transparent pressurized container. Inspired by such time series and parallel RB visualization experiments in water at much lower Ra (Zocchi, Moses & Libchaber 1990), Castaing *et al.* (1989) developed the picture of thermal plumes detaching from a thermal boundary layer (figure 1*b*) into a mixing layer (coloured regions in figure 1*c*). These thermal plumes, with a thickness typical of the boundary layer, self-organize on a larger scale and ultimately form the so-called 'wind of turbulence' on the scale of the container, see figure 1(*c*), a key notion that Castaing *et al.* came up with.

This picture then inspired Castaing *et al.* (1989) to develop their so-called mixing zone theory, in which they postulated the existence of a mixing layer between the thermal boundary layer and the bulk. With this central picture in mind, they tried to rationalize their observed scaling behaviour for various quantities, most importantly for Nu (the dimensionless heat transport through the cylinder) as a function of Ra; cf. above question (ii). The result was $Nu \sim Ra^{2/7}$, which seemingly was consistent with Castaing *et al.*'s experimental data; see figure 2(*a*). In fact, the best fit to an overall scaling law gave an exponent of 0.282 ± 0.006, very close to $2/7 \approx 0.286$. Moreover, the mixing zone theory of Castaing *et al.* (1989) gave the scaling laws $\Delta_c/\Delta \sim Ra^{-1/7}$ for the typical temperature fluctuations in the bulk, $Re_c \sim Ra^{3/7}$ for the typical (non-dimensionalized) velocity fluctuations in the bulk and $Re \sim Ra^{1/2}$ for the (non-dimensionalized) velocity of the large-scale wind of turbulence, which develops out of the plumes. All these scaling laws were consistent with Castaing *et al.*'s experimental data.

The final question (iii), that on the ultimate regime, was put forward, but not answered in Castaing *et al.* (1989). They distinguished 'soft turbulence' with Gaussian temperature fluctuations for small Ra up to $Ra \sim 4 \times 10^7$ and 'hard turbulence' with temperature

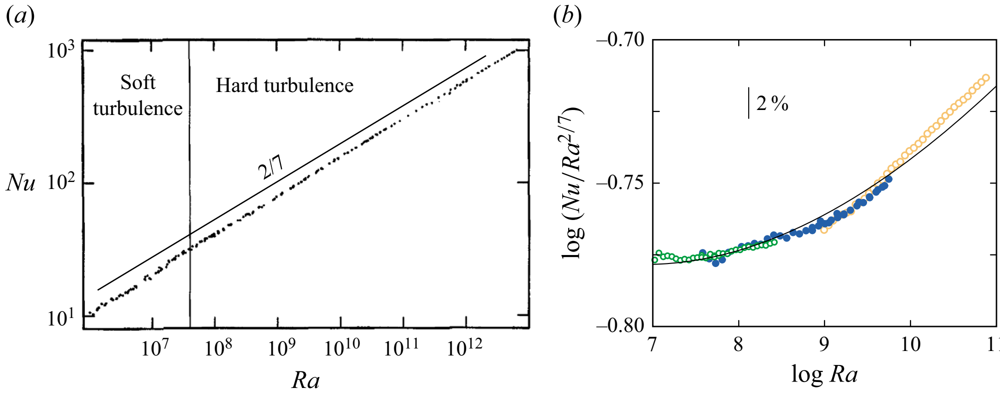

Figure 2. (*a*) Plot of $Nu(Ra)$ as originally shown in Castaing *et al.* (1989). The slope 2/7 (solid line) has been added for this Focus on Fluids article. (*b*) A better way to see details of the scaling is a compensated plot, here done in the form $Nu/Ra^{2/7}$ versus Ra for the heat transfer data of Ahlers & Xu (2001). Now a smooth transition between different scaling dependence is revealed, consistent with the earlier prediction of the GL theory (solid line) (Grossmann & Lohse 2000, 2001).

fluctuations with a stretched exponential distribution for larger Ra; see figure 2(*a*). Whether there is an ultimate regime for very large Ra, in the sense as suggested by Kraichnan (1962), they left open, writing: 'We do not know. Perhaps the hard turbulence extends to infinitely high values of Ra, perhaps not'.

3. The answers to these questions today

Castaing *et al.* (1989) began their conclusions with the statement: 'Clearly, our work here is a beginning, not an end'. And indeed, it was. Later in the conclusions Castaing *et al.* (1989) themselves identified various weaknesses of the paper and the mixing zone theory, one of them being the Prandtl number ($Pr \equiv \nu/\kappa$, i.e. a material parameter characteristic for the fluid) dependence resulting from the mixing zone theory. This dependence was not spelled out in Castaing *et al.* (1989) (neither was it measured), but only implicitly present. It was spelled out a few years later by Cioni, Ciliberto & Sommeria (1997), giving $Nu \sim Ra^{2/7}Pr^{-1/7}$ for moderate Pr and $Nu \sim Ra^{2/7}Pr^{2/7}$ for small Pr. The heat transfer measurements of Cioni *et al.* (1997) for RB convection in liquid mercury ($Pr \approx 0.02$) indeed found a strong increase of Nu with increasing Pr. It was this experimental work by Cioni *et al.* (1997) of which I became aware during my visit to Sergio Ciliberto in Lyon in October 1997, which triggered the development of a unifying theory for the scaling in RB convection (Grossmann & Lohse 2000, 2001). This theory can explain the dependences $Nu(Ra, Pr)$ and $Re(Ra, Pr)$, consistent with the experimental and numerical results. It is now known as GL theory and reviewed in Ahlers, Grossmann & Lohse (2009), Stevens *et al.* (2013) and Lohse & Shishkina (2024). The key insight of that theory is that there are no pure scaling laws, but smooth transitions between different regimes, dominated by either the boundary layer contributions or the bulk contributions to the kinetic and thermal dissipation rates. The effective scaling law $Nu \sim Ra^{2/7}$ found by Castaing *et al.* (1989) can in fact then be understood as a smooth transition between $Nu \sim Ra^{1/4}$ for small Ra (boundary layer dominance) to $Nu \sim Ra^{1/3}$ (bulk dominance). The smooth increase of the scaling exponent was confirmed in Ahlers & Xu (2001), who had higher precision in their heat transfer measurements, a nearly constant Prandtl number and, in contrast to Castaing *et al.*, plotted their precise heat transfer data in a compensated way, namely as $Nu/Ra^{2/7}$ versus Ra, in which much more details of the dependence can be seen; see figure 2(*b*). I emphasize that a key ingredient in the GL theory is the existence of the wind of turbulence, which first had been identified by Castaing *et al.* (1989).

Roughly a decade after Castaing *et al.*'s pioneering paper, also the Pr dependence of Nu had become clear. In addition to the experiments by Cioni *et al.* (1997) for small Pr, the pioneering direct numerical simulations of Verzicco & Camussi (1999) had revealed an increase of $Nu \sim Pr^{0.14}$ (for fixed $Ra = 6 \times 10^5$) up to $Pr \approx 0.3$ and then a saturation and for larger Pr beyond $Pr \approx 7$ even a slight decrease, and the experiments by Xia, Lam & Zhou (2002) in the up to then totally unexplored large-Prandtl-number range $4 \leq Pr \leq 1350$ confirmed and quantified that decrease in a much larger Ra range, all consistent with the predictions of the GL theory.

The answer to Castaing *et al.*'s question on the ultimate regime has taken much longer and the research is in fact still ongoing. The first who found the transition to a much steeper increase (as had been suggested by Kraichnan (1962)) in the Nu versus Ra relation were Chavanne *et al.* (1997), namely an effective scaling of $Nu \sim Ra^{0.38}$ beyond $Ra \sim 10^{11}$. These results could not be reproduced by Niemela *et al.* (2000) and Urban *et al.* (2014), but were reproduced by Chavanne *et al.* (2001) and Roche *et al.* (2010), who provided many details on the transition. Also He *et al.* (2012*b*,*a*) did find a transition, though at larger $Ra \sim 10^{14}$. A much more detailed discussion on the various experiments on the transition to the ultimate regime and their seemingly conflicting results is provided in Lohse & Shishkina (2024).

These various seemingly conflicting results can be reconciled by the insight that the transition to the ultimate regime is of non-normal–nonlinear nature (Roche 2020; Lohse & Shishkina 2023, 2024). Such type of transition is typical in shear flow (Trefethen *et al.* 1993; Avila, Barkley & Hof 2023). Here the strongly sheared (by the wind of turbulence) boundary layers undergo such a transition from laminar type to turbulent type, thus strongly enhancing the heat flux. The nature of such a transition is subcritical, with a strong sensitivity to distortions (such as provided by the thermal probes in the boundary layer in Chavanne *et al.* (1997), which were absent in other experiments) and other details of the flow and the set-up. A detailed discussion of the suggested non-normal–nonlinear transition in strongly driven RB turbulence can again be found in Lohse & Shishkina (2024). In that paper we also suggest various experiments and numerical simulations which should be done to further elucidate the transition to the ultimate regime of RB turbulence and its suggested subcritical and non-normal–nonlinear nature. A very promising line of research is to apply controlled distortions to the boundary layer flow at a Rayleigh number Ra around the onset to the ultimate regime, and to study how the temporal development of these distortions depends on this driving strength Ra and on the history of the system, similarly to what had been done in pipe flow in order to study the transition from laminar to turbulent flow (Avila *et al.* 2023).

In summary, remarkable progress in the understanding of RB turbulence has been achieved since the pioneering work by Castaing *et al.* (1989). This also includes huge progress in the direct numerical simulations of the RB system (for the state of the art, again see Lohse & Shishkina (2024)), in which $Ra \sim 10^{13}$ (the largest experimental value of Castaing *et al.* (1989)) has now become achievable, though only for $\sim$20 large-eddy turnover times and for a cylinder aspect ratio of $\Gamma = 1/2$, rather than $\Gamma = 1$ (Stevens, Lohse & Verzicco 2020). All this progress could also be achieved because of Castaing *et al.* (1989) asking the right questions 35 years ago. Clearly, for me personally the paper had a tremendous impact.

Acknowledgement. The author thanks his PhD advisor Siegfried Grossmann for having introduced him in 1990 to the great problem of RB convection and his postdoctoral advisor Leo Kadanoff for immersing him in the versatile scientific environment of the James Franck Institute at the University of Chicago and both for many years of inspiring joint scientific work and for all I learnt from them. I also thank all my later collaborators on this wonderful scientific problem.

Declaration of interests. The author reports no conflict of interest.

Author ORCID.
Detlef Lohse https://orcid.org/0000-0003-4138-2255.

References

AHLERS, G. 1974 Low temperature studies of the Rayleigh–Bénard instability and turbulence. *Phys. Rev. Lett.* **33**, 1185–1188.

AHLERS, G., GROSSMANN, S. & LOHSE, D. 2009 Heat transfer and large scale dynamics in turbulent Rayleigh–Bénard convection. *Rev. Mod. Phys.* **81**, 503–537.

AHLERS, G. & XU, X. 2001 Prandtl-number dependence of heat transport in turbulent Rayleigh–Bénard convection. *Phys. Rev. Lett.* **86**, 3320–3323.

AVILA, M., BARKLEY, D. & HOF, B. 2023 Transition to turbulence in pipe flow. *Annu. Rev. Fluid Mech.* **55**, 575–602.

CASTAING, B., GUNARATNE, G., HESLOT, F., KADANOFF, L., LIBCHABER, A., THOMAE, S., WU, X.-Z., ZALESKI, S. & ZANETTI, G. 1989 Scaling of hard thermal turbulence in Rayleigh–Bénard convection. *J. Fluid Mech.* **204**, 1–30.

CHAVANNE, X., CHILLÀ, F., CASTAING, B., HEBRAL, B., CHABAUD, B. & CHAUSSY, J. 1997 Observation of the ultimate regime in Rayleigh–Bénard convection. *Phys. Rev. Lett.* **79**, 3648–3651.

CHAVANNE, X., CHILLÀ, F., CHABAUD, B., CASTAING, B. & HÉBRAL, B. 2001 Turbulent Rayleigh–Bénard convection in gaseous and liquid He. *Phys. Fluids* **13**, 1300–1320.

CIONI, S., CILIBERTO, S. & SOMMERIA, J. 1997 Strongly turbulent Rayleigh–Bénard convection in mercury: comparison with results at moderate Prandtl number. *J. Fluid Mech.* **335**, 111–140.

GLEICK, J. 1988 *Chaos: Making a New Science*. Penguin.

GROSSMANN, S. & LOHSE, D. 2000 Scaling in thermal convection: a unifying theory. *J. Fluid Mech.* **407**, 27–56.

GROSSMANN, S. & LOHSE, D. 2001 Thermal convection for large Prandtl numbers. *Phys. Rev. Lett.* **86**, 3316–3319.

HE, X., FUNFSCHILLING, D., BODENSCHATZ, E. & AHLERS, G. 2012*a* Heat transport by turbulent Rayleigh–Bénard convection for $Pr \sim 0.8$ and $4 \times 10^{11} \lesssim Ra \lesssim 2 \times 10^{14}$: ultimate-state transition for aspect ratio $\varGamma = 1.00$. *New J. Phys.* **14**, 063030.

HE, X., FUNFSCHILLING, D., NOBACH, H., BODENSCHATZ, E. & AHLERS, G. 2012*b* Transition to the ultimate state of turbulent Rayleigh–Bénard convection. *Phys. Rev. Lett.* **108**, 024502.

KADANOFF, L.P. 2001 Turbulent heat flow: structures and scaling. *Phys. Today* **54** (8), 34–39.

KRAICHNAN, R. 1962 Turbulent thermal convection at arbitrary Prandtl number. *Phys. Fluids* **5**, 1374–1389.

LOHSE, D. & SHISHKINA, O. 2023 Ultimate turbulent thermal convection. *Phys. Today* **76** (11), 26–32.

LOHSE, D. & SHISHKINA, O. 2024 Ultimate Rayleigh–Bénard turbulence. *Rev. Mod. Phys.* **96**, 035001.

LORENZ, E.N. 1963 Deterministic nonperiodic flow. *J. Atmos. Sci.* **20**, 130–141.

MAURER, J. & LIBCHABER, A. 1979 Rayleigh–Bénard experiment in liquid helium; frequency locking and the onset of turbulence. *J. Phys. Lett.* **40**, L419–423.

NIEMELA, J.J., SKRBEK, L., SREENIVASAN, K.R. & DONNELLY, R.J. 2000 Turbulent convection at very high Rayleigh numbers. *Nature* **404**, 837–841.

ROCHE, P.-E. 2020 The ultimate state of convection: a unifying picture of very high Rayleigh numbers experiments. *New J. Phys.* **22** (7), 073056.

ROCHE, P.-E., GAUTHIER, F., KAISER, R. & SALORT, J. 2010 On the triggering of the ultimate regime of convection. *New J. Phys.* **12** (8), 085014.

SHANG, X.-D., QIU, X.-L., TONG, P. & XIA, K.-Q. 2003 Measured local heat transport in turbulent Rayleigh–Bénard convection. *Phys. Rev. Lett.* **90** (7), 074501.

STEVENS, R.J.A.M., LOHSE, D. & VERZICCO, R. 2020 Toward DNS of the ultimate regime of Rayleigh–Bénard convection. In *Direct and Large Eddy Simulations 12,* ERCOFTAC Series, vol. 27, pp. 215–224. Springer.

STEVENS, R.J.A.M., VAN DER POEL, E.P., GROSSMANN, S. & LOHSE, D. 2013 The unifying theory of scaling in thermal convection: the updated prefactors. *J. Fluid Mech.* **730**, 295–308.

STROGATZ, S.H. 1994 *Nonlinear Dynamics and Chaos*. Perseus.

SUN, C., XIA, K.-Q. & TONG, P. 2005 Three-dimensional flow structures and dynamics of turbulent thermal convection in a cylindrical cell. *Phys. Rev.* E **72**, 026302.

THRELFALL, D.C. 1975 Free convection in low-temperature gaseous helium. *J. Fluid Mech.* **67**, 17–28.

TREFETHEN, L.N., TREFETHEN, A.E., REDDY, S.C. & DRISCOL, T.A. 1993 Hydrodynamic stability without eigenvalues. *Science* **261**, 578–584.

URBAN, P., HANZELKA, P., MUSILOVÁ, V., KRÁLÍK, T., MANTIA, M.L., SRNKA, A. & SKRBEK, L. 2014 Heat transfer in cryogenic helium gas by turbulent Rayleigh–Bénard convection in a cylindrical cell of aspect ratio 1. *New J. Phys.* **16**, 053042.

VERZICCO, R. & CAMUSSI, R. 1999 Prandtl number effects in convective turbulence. *J. Fluid Mech.* **383**, 55–73.

XIA, K.-Q 2013 Current trends and future directions in turbulent thermal convection. *Theor. Appl. Mech. Lett.* **3** (5), 052001.

XIA, K.-Q., LAM, S. & ZHOU, S.Q. 2002 Heat-flux measurement in high-Prandtl-number turbulent Rayleigh–Bénard convection. *Phys. Rev. Lett.* **88**, 064501.

ZOCCHI, G., MOSES, E. & LIBCHABER, A. 1990 Coherent structures in turbulent convection, an experimental study. *Physica* A **166**, 387–407.