Iterative methods for the solution of equations, by J.F. Traub. Prentice Hall, Inc. Publishers, Englewood Cliffs, N.J., 1964. xviii + 310 pages.

Iteration algorithms have been of interest to mathematicians from the theoretical as well as from the practical point of view for at least 250 years. The original problem is for a given real equation $f(x) = 0$ to find a function $\varphi(x)$ such that the iterative (or recursive) sequence $x_{n+1} = \varphi(x_n)$ converges to a certain real, in most cases simple root $x$ which then is a fixed point of the function $\varphi(x)$ . Newton and Raphson were probably the first who solved the problem by the now familiar tangent formula $\varphi(x) = x - \frac{f(x)}{f'(x)}$ which appears as an example in most calculus texts. Later it was found that Newton's solution was neither unique nor the best possible, rather that there was a gold mine open for exploration. This became evident mainly through the important work by E. Schröder (Math. Annalen 2, (1870), 317-365) which in a way may be considered as a forerunner of the present book. During the last 150 years a great number of mathematicians took up the problem and so it happened that many iteration formulae have been discovered several times independently. A new stimulus for iteration procedures arose from the arrival of high-speed computation facilities; they encouraged the use of formerly rejected formulae and the development of many new ones.

This is probably the first book devoted entirely to the theory and practice of iteration algorithms for the numerical solution of equations and (very briefly also) systems of equations excluding systems of linear algebraic equations. Iteration functions $\varphi$ are classified and systematized (Chap. 12 is a compilation of iteration functions) and new classes of computationally effective iteration functions are introduced with the use of computing machinery in mind. In this direction the author goes beyond anything attempted so far and while a general or casual reader will probably enjoy the early chapters of this book he will certainly, while being impressed by the mass of orderly composed material, gradually lose interest in details however important they may be for the computing professional.

In his attempt to classify iteration formulae the author has introduced many new notions and names which should help to keep apart rather similar looking cases. Thus Newton's formula is a typical example of a "one-point" iteration; the so-called secant formula (regula falsi) $\varphi(x, y) = x - f(x) \frac{y - x}{f(y) - f(x)}$ whereby $x_{n+1} = \varphi(x_n, x_{n-1})$ is a "one-point iteration with memory" although two initial points are involved. In the case of several initial points, to determine the next approximation to the root the author speaks of "multi-point iteration" or "multi-point iteration with memory" according to whether for none or for some of the points old "information" is used.

The quality of an iteration formula is measured by its order: the process $\varphi$ has the order p if

$$\frac{x_{n+1} - x^*}{(x_n - x^*)^p} \to C ,$$

the asymptotic error constant. The order theoretically determines the speed of the convergence of the iteration sequence.

In some respect a more rounded theory might have been obtained if complex variables had been taken into consideration.

Well known mathematicians like Newton, Euler, Laguerre in the past, Ostrowski, Householder, Kantorovitch and many others have contributed to the subject. The simple Newton formula and the convergence of its iteration have been investigated from several points of view during the last 30 years. The formula has been generalized in many different ways already by Schröder. It has been extended to the solution of a system of equations.

This takes us near to another matter which has been excluded from the present work. Indeed iteration procedures, in particular Newton's formula, have been generalized to be applicable to equations in Banach spaces and in topological spaces by means of differentiation in the sense of Frechet and Gateaux and their close connection with general fixed point theorems has been recognized. Thus the subject has developed into a part of modern functional analysis.

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<u>Numerical solution of partial differential equations,</u> by G.D. Smith. Oxford, London, 1965. viii + 179 pages. $3.50.

This book is intended as a textbook for undergraduates in mathematics, physics, and engineering. It is well but concisely written. The good student should have little difficulty supplying the required details and should appreciate the numerous well-chosen numerical examples. The problems are very good and most have at least the outline of a solution supplied.

There are 5 chapters in the book. Independent chapters (2, 4 and 5) on parabolic, hyperbolic, and elliptic equations require but a limited knowledge of matric algebra and no prior knowledge of the calculus of finite differences (which is discussed in chapter 1). The important chapter 3 on "Convergence, Stability, and Systematic Iterative Methods" group together those topics requiring a knowledge of matric algebra. This results in some redundancy in chapters 2, 4, and 5, and possibly a somewhat unnatural grouping of topics in chapter 3.

There are some flaws in the work. On page 14, the comparison of the error with $(\delta t)^p$ where the method is $0(\delta t)^p$ is extremely mis-