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A characterization of alternating links in thickened surfaces – CORRIGENDUM

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Recall that a spanning surface for a link L is by assumption a connected unoriented surface with boundary equal to L.

Theorem 1.1 and Corollary 4.9 from the paper are incorrect as stated. For example, one can construct counterexamples to Theorem 1.1 using links L contained in 3-balls, so-called *local links*. Let $L \subset B^3$ be a link with an alternating projection on $S^2 = \partial B^3$. Under inclusion $B^3 \subset \Sigma \times I$, we obtain a local link L in $\Sigma \times I$ which bounds definite spanning surfaces of opposite sign. However, if the genus $g(\Sigma) > 0$, then L does not have minimal genus.

To correct for this issue, we need to add the assumption that L is not a local link in the case $g(\Sigma) > 0$. The corrected statement of the theorem is as follows.

Theorem 1.1. *Let L be a link in $\Sigma \times I$, and assume that L bounds a positive definite spanning surface and a negative definite spanning surface. Then $L \subset \Sigma \times I$ is a non-split alternating link which either has minimal genus or is contained in a 3-ball.*

A few remarks on the proof are in order. For $g(\Sigma) = 0$, the proof is the same as before. For $g(\Sigma) \geqslant 1$, then arguing as before, we see that P and N are not S^*-equivalent, unless the core surface S of $\nu(P \cup N)$ is a 2-sphere. In the latter case, L is contained in a 3-ball, since $\Sigma \times I$ is irreducible, and L has a connected alternating diagram on S, implying that L is non-split and has an alternating diagram on Σ.

Otherwise, assuming that P and N are not S^*-equivalent, then the argument goes through as before.

Below is a corrected statement of the corollary.

Corollary 4.9. *A link $L \subset \Sigma \times I$ in a thickened surface of positive genus is alternating and has minimal genus if and only if L bounds definite spanning surfaces of opposite sign and is not contained in a 3-ball.*

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