

Incentivizing Investors for a Greener Economy

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Abstract

We demonstrate that investment income taxes incentivize capital allocation to the eco-friendly green sector away from the non-ecofriendly brown sector in a stylized economy. This tax reduces the arrival intensity of climate disasters, delivers the socially optimal allocation, and can be jointly implemented with a carbon tax, expanding policymakers' toolkit to reduce climate disasters. Extending the model with heterogeneous investors, we show that investment income taxes can obtain support from a political majority and thereby relax political constraints faced by a carbon tax alone.

1. Introduction

For the past 30 years, academics have coalesced around the idea that carbon taxes are the most effective way to address climate-change externalities. Such consensus was most recently reflected in a letter titled “Economists’ Statement on Carbon Dividends” (Akerlof et al. (2019)) signed by more than 3000 economists, including 28 Nobel Laureates in Economics, and 4 former Federal Reserve chairs.

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In practice, however, many legislative bodies around the world have failed to successfully implement it. As a case in point, a bipartisan effort in the United States to pass a revenue-neutral carbon tax faltered during the Trump administration. More recently (in June 2021), the Biden administration did not even try to push carbon tax legislation forward, showing no interest in passing it.

According to the latest estimates, the effective global average price of carbon is only \$2 a ton, far below even the most conservative estimates of \$38 a ton required to reach the 2015 Paris Agreement goal of keeping global temperatures within 1.5 °C above preindustrial levels (Nordhaus (2019)).

This inability to successfully implement carbon taxes, combined with the growing frequency and severity of climate disasters, has nurtured a growing debate in economic and policymaking circles as well as in the private sector around alternative mechanisms to address these challenges. In particular, the last decade has witnessed tremendous interest in exploring the role that sustainable finance and capital markets can play in mitigating the climate crisis. Such interest has been reflected in an unprecedented explosion in Environmental, Social, and Governance (ESG) investing (Halbritter and Dorfleitner (2015)), the wider use of environmentally friendly investing mandates among college endowments (Bessembinder (2016)) and sovereign wealth funds (Bolton, Samama, and Stiglitz (2012)), and the wider implementation of higher capital requirements for carbon-intensive financial activities (Esposito, Mastromatteo, and Molocchi (2019)), among others. In a recent study, Pastor, Stambaugh, and Taylor (2022) document that green assets earned high returns in recent years not because of high expected returns, but rather because of unexpectedly strong increases in environmental concerns.

Notably, the success of most of these approaches hinges on either the ability of private-market participants to coordinate efforts combined with a willingness to altruistically sacrifice returns, or on the effectiveness of regulations restricting portfolio allocations.

In light of this situation, we ask a number of questions. Are there other approaches, possibly implemented jointly with carbon taxes, that can successfully address climate-change externalities? In particular, can tax policies align the portfolio of utility-maximizing participants with the social optimum without resorting to altruistic motives? If so, what is the impact on firm value and social welfare of such a tax regime?

To address these questions, we propose a dynamic stochastic general equilibrium model featuring 2 sectors consisting of ecofriendly green firms and non-ecofriendly brown firms, with the likelihood of climate-change disasters increasing in the fraction of brown firms' capital in the economy. Each sector produces a homogeneous good in proportion to its capital stock. Firms are price-takers and adjust their capital by choosing the investment policy that maximizes their firm value. Climate-change disaster shocks adversely affect economic growth. In particular, every time a climate-change disaster occurs, a fraction of the total capital stock in the economy is obliterated.

The key innovation in our model is that the likelihood of climate change is an endogenous outcome of firm production. This is accomplished by modeling the arrival rate of climate disasters proportional to the fraction of capital operated by

brown firms in the economy. Therefore, if brown (green) firms have high (low) investment rates, climate disasters will become more (less) frequent in the future.

In our economy, capital markets are competitive. A representative investor solves a standard consumption-portfolio choice problem. He optimally chooses a consumption rate and allocates his savings to the risk-free asset, and stocks of green and brown firms. The market equilibrium is characterized by a set of prices and an allocation in which households maximize their utility, firms maximize their market value, and the goods and financial securities markets clear. Households would like the green sector in the economy to grow, since disasters cause the destruction of capital and significant welfare losses. However, when mitigating the damages associated with capital allocated to brown firms, we have to confront the climate-change externality. Because firms are atomistic, they fail to internalize their investment policies' effects on the rest of the economy.

We solve for the allocation that a social planner would choose if the investment rate of each sector could be chosen directly to maximize social welfare. Consistent with our intuition, the market equilibrium features an overinvestment (underinvestment) in the brown (green) sector relative to the first-best social optimum.

In light of this dislocation of capital due to the climate-disaster externality, we explore the normative implications of our model for the investment and valuation of both green and brown firms as well as for social welfare. Our investigation offers the following insights: First, in a stylized model with representative agents, investment income tax levied on investors can induce the first-best capital allocation as a market equilibrium, as commonly documented for carbon taxes. More importantly, we show that carbon tax and investment income tax can jointly implement the socially optimal first-best allocation. This indicates that policymakers can supplement carbon taxes on firms' output with investment income taxes on investors' returns to discourage brown firms' growth. Such taxes are redistributed in the form of output subsidies and investors' investment-income subsidies to green firms, thereby incentivizing investment and growth of green firms and reducing future climate disasters.

The economic mechanism through which investment income taxes achieve the social optimum differs from carbon taxes, which directly affect firm output. When investors' returns from brown (green) firms are taxed (subsidized), *ceteris paribus*, they rebalance their portfolios to move away from brown firms and toward green firms. This increases the demand for green firms' shares. However, in order for markets to clear, green firms will see their share price increase relative to brown firms. Hence, the cost of capital, that is, the discount factor for a firm's cash flows, becomes lower for green firms (higher for brown firms). Because investments at the firm level is inversely related to the firm's cost of capital, green (brown) firms would increase (decrease) their investments, thereby aligning the competitive investment rates with the social optimum.

Second, from a practical perspective, investment income taxes such as dividend and capital gains taxes have long been an integral part of individual income tax filing, and thus do not require a complete overhaul of the existing tax legislation on investment income tax to incentivize investors. The implementation of investment income tax to incentivize investors for a greener economy can be achieved by

focusing on classifying firms according to their ecofriendly business practices and their degree of compliance with environmental regulations and standards, then setting investment income tax rates accordingly.

Indeed, the U.S. Securities and Exchange Commission (SEC) recently unveiled its long-anticipated draft rule under which companies would disclose their own direct (Scope 1) and indirect (Scope 2) greenhouse gas emissions. If approved, this proposal can be used by regulators as the basis for implementing differential investment income taxes. To some extent, this is akin to taxing qualified dividends and long-term capital gains at a different rate than nonqualified dividends and short-term gains, which is a common practice in current investment income taxation.

The implementation would require the rating of a firm's ecofriendliness by third parties that the investing public and the policymakers trust and rely on. Coinciding with the increasing attention to the environment, the number of firms invested in assessing companies' ecofriendliness is growing. Currently, there are 6 major rating agencies providing ESG ratings on companies: Asset4 (Refinitive), MSCI KLD, MSCI IVA, Bloomberg, Sustainalytics, and RobecoSAM. Although there are substantial variations across these agencies, the variations are persistent (Avramov, Cheng, Lioui, and Tarelli (2021)), suggesting the possibility of constructing a combined rating.

Third, we extend our analysis to incorporate investor heterogeneity, in which a fraction of wealthier investors can directly invest in the stock market (stock investors), while less-wealthy investors invest only in the risk-free asset (nonstock investors). This extension allows us to examine the effect of political constraints associated with imposing investment income taxes. We show that carbon taxes can address climate-change externalities, but that they lead to an increase in the price of output produced by brown firms. Such an increase in the price of carbon-intensive output can make nonstock investors worse off, reminiscent of the protests that took place in France in the aftermath of the 2018 decision to abolish a long-standing tax advantage for diesel fuel enacted by President Macron known as the "yellow vests" movement (Gollier (2019), Grossman (2019)).

In contrast, investment income taxes can play the dual role of addressing climate change while redistributing income to nonstock investors affected by price increases of carbon-intensive goods. The idea is simple and intuitive. We propose imposing a tax on the return of brown stocks, levied on stock investors, and redistributed to the nonstock investors as a subsidy to the risk-free asset. By construction, such a tax increases the cost of capital to brown firms (addressing climate change) and is progressive (addressing political opposition from the less-well-off nonstock investors). As a result, our analysis using a heterogeneous investor model shows that investment income taxes may face fewer political constraints relative to carbon taxes levied and redistributed directly to firms.

Our study provides the first investigation on jointly implementing carbon tax and investment income tax in a dynamic stochastic general equilibrium setting to mitigate climate-disaster risk and induce socially optimal capital allocation. This represents an important step to enhance the policy toolkit available to policymakers, regulators, and environmental activists in dealing with the threat of climate change. In particular, the flexibility available in suitably combining both taxes may substantially improve policy effectiveness when facing political constraints. This is

particularly relevant if society faces insurmountable obstacles to raising carbon taxes beyond a certain point and investment income taxes face fewer political constraints relative to further increasing the carbon tax. From this perspective, we provide a mechanism and additional policy tools through which political constraints can be relaxed in order to successfully mitigate climate-change risks and improve social welfare.

Literature Review

Our article contributes to the literature on how socially responsible investing incentivizes corporations to align their investments with the objective of achieving an ecofriendly green economy. Our mechanism works through a net-return/cost-of-capital channel accomplished by differential taxes levied on investment income received by investors from ecofriendly green firms and non-ecofriendly brown firms. Most of the literature has focused on the role of mandates such as implementing emission standards. The first such model of green mandates and the cost-of-capital channel in a static CARA setting is Heinkel, Kraus, and Zechner (2001). In a related study, Hong and Kacperczyk (2009) show the impact of ethical investing mandates on sin companies. More recently, several papers (e.g., Broccardo, Hart, and Zingales (2022), Oehmke and Opp (2024)) have explored the additional mechanisms through which active mandates, such as voting for environmentally friendly policies, can affect firms' policies. Pastor, Stambaugh, and Taylor (2021) propose a model in which agents have a preference for green firms over brown firms and examine the implications for green firm returns when agents' tastes shift unexpectedly. Goldstein, Kopytov, Shen, and Xiang (2021) study the impact of ESG investment on information aggregation and price formation. Interestingly, they show that green investors and traditional investors trade in opposite directions, making the price noisier, thereby increasing the cost of capital, and potentially undermining capital allocation to green firms. We contribute to this literature by showing that incentivizing investors to optimize their capital allocations under investment income tax can achieve the social optimum as a market equilibrium, and that, moreover, implementing carbon tax and investment income tax jointly can more effectively address climate-change externalities.

A key feature of our model is the endogenous climate-disaster arrival rate. This is accomplished by modeling the likelihood of climate-change disasters as a function of the relative size of the capital stock in the ecofriendly green and non-ecofriendly brown sectors.¹ This formulation is akin to Hong, Wang, and Yang (2023b), where they focus on the role of the decarbonization sector in mitigating climate-change disasters. In a related paper, Hong, Wang, and Yang (2023a) study taxation on an individual firm's capital stock and show that capital taxes and mitigation subsidies can restore the first-best solution. Since the output is proportional to capital stock, the capital tax is tantamount to a carbon tax levied on firms. By contrast, investment income tax is levied directly on investors, who optimally allocate their investment to earn higher net-of-tax returns. This in turn affects a

¹The 2-sector setting extends the endowment economy with 2 trees developed by Cochrane, Longstaff, and Santa-Clara (2008).

firm's cost of capital. Individual firms will choose investment to maximize their firm value. Thus, introducing investment income tax in achieving the social optimum differs from and also complements existing analyses.

Finally, a large literature on Dynamic Integrated Climate Change (DICE) models, starting with Nordhaus (1994), has developed realistic scenarios integrating insights from geophysics and climate science with models of economic growth. Their main focus has been on quantitatively assessing the externalities involved and thereby aiming to pin down optimal carbon taxes. Recent contributions include Acemoglu et al. (2016), who endogenize the growth rate by explicitly modeling a firm's innovation decision, and Barnett, Brock, and Hansen (2020), who lever recent tools in decision sciences to incorporate the high degree of uncertainty around climate dynamics. By contrast, our model focuses on exploring the effect of a joint investment income tax and carbon tax on capital allocation and firm valuation in a setting with a relatively simplified specification while capturing some key features of climate-change dynamics.

II. Model

We consider a continuous-time production economy with an infinite horizon. The economy is populated with a large number of firms classified into 2 types, or sectors: ecofriendly green firms (green sector) and non-ecofriendly brown firms (brown sector). We denote by K_n, I_n , and Y_n the capital stock, investment, and output, respectively, for a representative firm of type n , where $n \in \{b, g\}$, with b representing brown sector firms and g green sector firms. The production technology for firm n is given by

$$(1) \quad Y_n(t) = \alpha_n K_n(t),$$

where $\alpha_n > 0$ is a constant. Capital accumulation follows a controlled geometric Brownian motion process

$$(2) \quad dK_n(t) = K_n(t)((\mu_n + i_n(t))dt + \sigma_n dB_n(t) - \psi dN(t)), n \in \{b, g\},$$

where μ_n is the depreciation rate of firm capital in sector n , $i_n = I_n/K_n$ is the investment-capital ratio in sector n , $\sigma_n > 0$ is the parameter governing volatility, $B_n(t)$ is a sector-specific Brownian shock, $N(t)$ represents the climate-change disaster risk, which follows a cumulative Poisson process with intensity $\lambda(t)$, and $\psi > 0$ is the fraction of capital lost after a disaster occurs.² We assume the Brownian shocks B_b and B_g are uncorrelated from each other and from the Poisson process $N(t)$. Shocks affect the accumulation dynamics specified by equation (2), as in Kogan (2004) and Cox, Ingersoll, and Ross (2005).

Importantly, we model the probability of a climate-change disaster occurrence as an increasing function of the fraction of total capital in the brown sector, that is,

²Our results are unchanged if instead of assuming that the fraction of capital ψ lost when a disaster strikes is constant we assumed that it is drawn from a distribution, as in Pindyck and Wang (2013) and Hong et al. (2023b).

$$(3) \quad \lambda(t) = \tilde{\lambda}(\eta(t)) = \lambda_g + (\lambda_b - \lambda_g)\eta(t) = \lambda_g + \eta(t)\Delta_\lambda,$$

where $0 \leq \lambda_g \leq \lambda_b$ are constants, $\Delta_\lambda = \lambda_b - \lambda_g$, and $\eta(t) = \frac{K_b(t)}{K_b(t) + K_g(t)}$ represents the fraction of total capital in the brown sector at time t . Intuitively, when green firms are the predominant sector in the economy, there is little negative impact on the environment and climate-change disasters are rare. By contrast, when brown firms are the predominant sector in the economy, climate disasters occur more frequently.³

Following Hayashi (1982), we assume a quadratic adjustment cost with homogeneity of degree one in I and K . That is, type n firm's profits net of installation costs are given by

$$(4) \quad \pi_n(t) = K_n(t) \left(\alpha_n - i_n(t) - \theta_n \frac{i_n(t)^2}{2} \right), \quad n \in \{b, g\},$$

where θ represents the coefficient of adjustment cost for type n firms.

A representative consumer has preferences over consumption streams represented by

$$(5) \quad \mathbb{E} \left[\int_0^\infty e^{-\rho t} u(C(t)) dt \right],$$

where the instantaneous utility function $u(C) = \frac{C^{1-\gamma}-1}{1-\gamma}$ features CRRA preferences and $\rho > 0$ corresponds to the subjective discount factor. The representative consumer is endowed with claims on the output produced by both types of firms.

In the market equilibrium, the representative consumer chooses consumption and portfolio policies to maximize the expected discounted lifetime utility given by expression (5); the managers of both types of firms take the equilibrium prices as given and maximize firm value; all the goods produced are either consumed or invested in either of the 2 sectors. Therefore, the goods market clearing condition

$$(6) \quad C(t) = K_b(t) \left(\alpha_b - i_b(t) - \theta_b \frac{i_b(t)^2}{2} \right) + K_g(t) \left(\alpha_g - i_g(t) - \theta_g \frac{i_g(t)^2}{2} \right)$$

holds at all times. In equilibrium, the representative consumer holds the market portfolio (i.e., claims on aggregate output for both sectors) and no risk-free asset which is in zero net supply.

³Our model nests as a special case of the Eberly and Wang (2009) economy without disaster risks when $\lambda_b = \lambda_g = 0$. Our extension is economically meaningful in at least 2 dimensions. First, our goal in adapting their framework is to study the implications for optimal taxation and asset prices in the presence of climate-change shocks. By contrast, they focus on the insights for sectoral and aggregate Tobin's Q and investment when the economy features 2 productive trees. Second, our setting induces a wedge between the competitive equilibrium and the socially optimal allocation, while in Eberly and Wang (2009) both allocations coincide. Hence, our extension endows the model with the capability of addressing normative questions regarding climate-change policies.

The market equilibrium will not coincide with the social planner's allocation, because firms fail to internalize the impact of their investment decisions on the climate-disaster risk. The social planner's goal is to maximize [expression \(5\)](#) by choosing directly the investment policies in both sectors, subject to the laws of motion of capital governed by [equation \(2\)](#) and the goods market clearing [condition \(6\)](#). In [Section III.B](#), we show that the social planner chooses a higher (lower) investment rate in green (brown) firms than the market equilibrium, because brown (green) firms in the competitive equilibrium fail to internalize the higher (lower) economy-wide exposure to climate-change disaster risks induced by their investment decisions.⁴

III. Market Equilibrium and the Social Planner's Allocation Without Taxes

A. Market Equilibrium

The state variables that capture all the relevant information in this economy are the capital stocks in the 2 sectors. We exploit homogeneity properties of the model, thereby rendering the relative size of the capital stocks $\eta = \frac{K_b}{K_b + K_g}$ as the only effective state variable. Because physical capital is non-negative, we must have $\eta \in [0, 1]$.

The evolution of η is given by

$$(7) \quad d\eta = d\left(\frac{K_b}{K_b + K_g}\right) = \Sigma(\eta)dt + \eta(1 - \eta)\sigma_b dB_b - \eta(1 - \eta)\sigma_g dB_g$$

with $\Sigma(\eta) = \eta(1 - \eta) \left[\mu_b - \mu_g + i_b(\eta) - i_g(\eta) - \eta\sigma_b^2 + (1 - \eta)\sigma_g^2 \right]$ representing the drift of the capital in the brown sector relative to the total capital stock. To ease notation, we have suppressed the dependence on t . Note that just as in the two-tree pure-endowment-economy model of Cochrane et al. (2008), the one-sector economies (either all green or all brown firms) are absorbing, since the drift and volatility of η are 0 whenever $\eta \in \{0, 1\}$. Moreover, the drift of η depends on the difference between endogenous investment rates in the 2 sectors $i_b - i_g$. This difference determines the fraction of the economy that becomes green, and as a result the exposure of the economy to climate-change disaster risks.

1. Firm Investment and Valuation

We now characterize the valuation of capital and optimal investment policies. Using the homogeneity property in our model, we have that firm value in sector n denoted by $V_n(K_n, \eta)$ satisfies

$$(8) \quad V_n(K_n, \eta) = K_n p_n(\eta), \quad n \in \{b, g\},$$

⁴We provide the exact mathematical formulations of the market equilibrium in the "Appendix for Section III.A" of the Supplementary Material and of the social planner's allocation in the "Appendix for Section III.B" of the Supplementary Material.

where $p_n(\eta)$ represents the equilibrium market value per unit of capital for firms in sector n . This value is identical to Tobin's Q of the firm under our assumption of constant returns to scale production function. A firm's optimal investment maximizes shareholder value taking equilibrium prices as given. That is, i_n is chosen to maximize the sum of the dividends plus expected capital gains:

$$(9) \quad \max_{i_n} \underbrace{K_n \left(\alpha_n - i_n(\eta) - \theta_n \frac{i_n(\eta)^2}{2} \right)}_{\text{Dividends}} + \underbrace{K_n p_n(\eta) (\mu_n + i_n(\eta))}_{\text{Capital Gains}}, \quad n \in \{b, g\}.$$

The first-order condition for $i_n(\eta)$ is given by

$$(10) \quad i_n(\eta) = \frac{1}{\theta_n} (p_n(\eta) - 1), \quad n \in \{b, g\}.$$

Intuitively, each unit of installed capital is valued at $p_n(\eta)$. Thus, the firm chooses i_n to equate the marginal benefit of investment $p_n(\eta)$ with its marginal cost $1 + \theta_n i_n(\eta)$. Because each firm is an atomistic price-taker, it does not internalize the effect that its investment policy has on the state variable η , hence on the probability of climate-change disaster risks. As we show in the next section, this externality induces a wedge between the investment policies in the competitive equilibrium and the socially optimal investment chosen by the social planner.

B. The Social Planner's Allocation

The social planner chooses consumption $c^{\text{FB}}(\eta)$ and investment policies $\{i_b^{\text{FB}}(\eta), i_g^{\text{FB}}(\eta)\}$ to maximize social welfare, where the superscript FB stands for first-best. Just like in the market equilibrium, the relevant state variable for this problem is the fraction of capital in the brown sector relative to the total capital in the economy (η). Figure 1 compares the social planner's allocation with that of the competitive equilibrium. In particular, we characterize how these 2 economies differ in their investment and dividend rates.

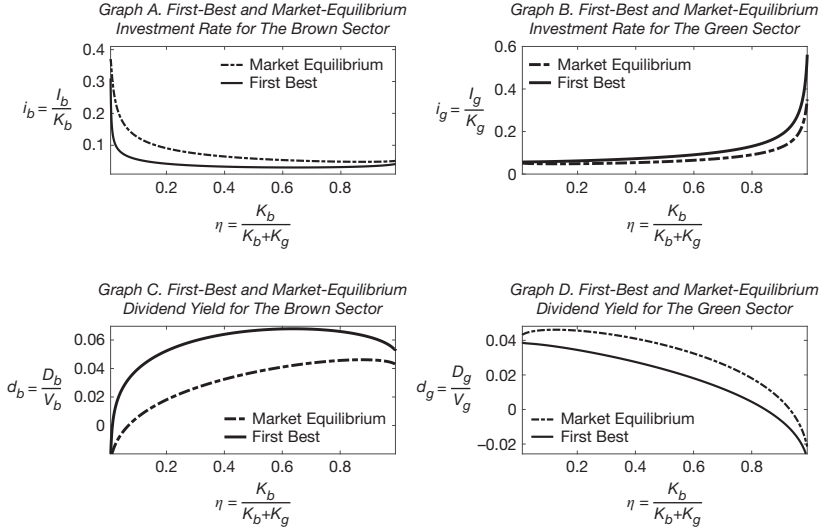
We begin our discussions with the impact of the market failure on the investment rates. Graph A of Figure 1 depicts, respectively, the investment rate in the brown sector chosen by the social planner (solid line) and the market equilibrium (dashed line), while Graph B shows the comparison for the green sector. We observe that the social planner chooses, respectively, a lower investment rate in brown firms and a higher investment rate in green firms than in their competitive equilibrium counterparts:

$$(11) \quad i_b^{\text{FB}}(\eta) \leq i_b(\eta), \quad i_g^{\text{FB}}(\eta) \geq i_g(\eta).$$

This indicates overinvestment in the brown firm sector and underinvestment in the green firm sector under the competitive market equilibrium relative to the social optimum. This misallocation is a direct consequence of the climate-change externality present in the market equilibrium. Further, the overinvestment

FIGURE 1
Investment and Dividend Rates

In Figure 1, the parameter values are $\Delta_i = 0.3, \lambda_g = 0, \psi = 0.1, \rho = 0.02, \mu_g = 0.005, \mu_b = 0.005, \sigma_g = 0.25, \sigma_b = 0.25, \alpha_g = 0.15, \alpha_b = 0.15, \theta_g = 25, \theta_b = 25,$ and $\gamma = 1$.



(underinvestment) is more severe when the brown (green) sector accounts for a small (large) fraction of the economy; that is, η is low (η is high).

Intuitively, as the brown sector initially increases from a previously entirely green economy, the marginal product of capital is high in the brown sector, leading to a high investment rate without regard to the negative externality that the social planner takes into account in deciding investment in the brown sector. As η increases, the marginal product of capital in the brown sector decreases, leading to a decrease in the investment rate. When the economy becomes one sector only, the investment rates coincide. The same explanation works for the investment rate for the green sector, as η decreases from when the economy is entirely brown.

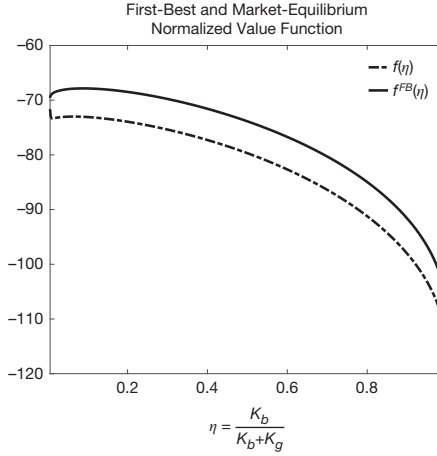
Next, we turn to the effect on the dividend rates for both types of firms. We define the dividend rates as the output per unit of capital net of investment rates and the adjustment costs per unit of capital as follows:

$$(12) \quad d_n = \alpha_n - i_n(\eta) - \theta_n \frac{i_n^2(\eta)}{2}, \quad n \in \{b, g\}.$$

Graphs C and D of Figure 1 depict the dividend rates for the brown sector d_b and the green sector d_g , respectively. For comparison, we plot the social planner's dividend rate for each sector (solid line) and the market equilibrium (dashed line). We observe that the socially optimal dividend rate is higher than that in the competitive equilibrium for brown firms. In the meantime, the socially optimal dividend rate is lower than that in the competitive equilibrium for green

FIGURE 2
Normalized Value Functions

In Figure 2, the parameter values are $\Delta_b = 0.3, \lambda_g = 0, \psi = 0.1, \rho = 0.02, \mu_g = 0.005, \mu_b = 0.005, \sigma_g = 0.25, \sigma_b = 0.25, a_g = 0.15, \alpha_b = 0.15, \theta_g = 25, \theta_b = 25,$ and $\gamma = 1$.



firms. These results are consistent with overinvestment in brown firms and underinvestment in green firms in the competitive market equilibrium relative to the social optimum. From the social planner’s perspective, the marginal utility of consumption is higher than the marginal product value of investing in brown firms, while the opposite is true for green firms. As a result, efforts to induce brown (green) firms to increase (decrease) dividend payout, thereby reducing (increasing) brown (green) firm investment, constitute a valid mechanism to mitigate climate change.

Finally, we study the implications for welfare arising from the climate-change externality. Let $F(K_b, K_g)$ and $F^{FB}(K_b, K_g)$ be the value functions in the market equilibrium and the social optimum, respectively. Exploiting homogeneity in our model, we can write

$$(13) \quad F(K_b, K_g) = f(\eta)(K_b + K_g)^{1-\gamma} + \frac{(K_b + K_g)^{1-\gamma} - 1}{\rho(1-\gamma)},$$

$$(14) \quad F^{FB}(K_b, K_g) = f^{FB}(\eta)(K_b + K_g)^{1-\gamma} + \frac{(K_b + K_g)^{1-\gamma} - 1}{\rho(1-\gamma)}.$$

We can now compare the expected utility of the representative household in the market equilibrium and at the first-best social optimum. Figure 2 depicts the normalized value function for the social planner’s allocation $f^{FB}(\eta)$ (solid line) and for the market equilibrium $f(\eta)$ (dashed line).⁵ As expected, the social planner can

⁵We normalize the value function by a factor of $(K_b + K_g)^{1-\gamma}$.

deliver higher welfare for the representative household through its ability to mitigate climate-change disaster risks, that is,

$$(15) \quad f^{\text{FB}}(\eta) \geq f(\eta).$$

In summary, because the social planner takes into account the externality when choosing the investment policy for each sector, the welfare of the representative household is higher than in the competitive market equilibrium as a result of achieving the optimum balance between reducing the probability of climate-change-induced disasters and gaining diversification by having both sectors operating.

IV. Market Equilibrium with Taxation

A. Market Equilibrium with Investment Income Tax

In this section, we introduce the investment income tax levied directly on investors and explore its implication for the social optimum. The instantaneous returns $dR_b(t)$ and $dR_g(t)$ faced by an investor upon purchasing a share of a brown or green firm are the sum of its dividend yield and its expected capital gain; these are respectively given by

$$(16) \quad dR_b = \Lambda_b(\eta)dt + \left[\sigma_b + \sigma_b\eta(1-\eta) \frac{p'_b(\eta)}{p_b(\eta)} \right] dB_b \\ - \left[\sigma_g\eta(1-\eta) \frac{p'_b(\eta)}{p_b(\eta)} \right] dB_g - \psi dN,$$

$$(17) \quad dR_g = \Lambda_g(\eta)dt + \left[\sigma_b\eta(1-\eta) \frac{p'_g(\eta)}{p_g(\eta)} \right] dB_b \\ + \left[\sigma_g - \sigma_g\eta(1-\eta) \frac{p'_g(\eta)}{p_g(\eta)} \right] dB_g - \psi dN,$$

where the explicit expressions for the expected returns on brown and green firms' shares $\Lambda_n(\eta)$, $n \in \{b, g\}$, are given in the "Appendix for Section III.A" of the Supplementary Material.

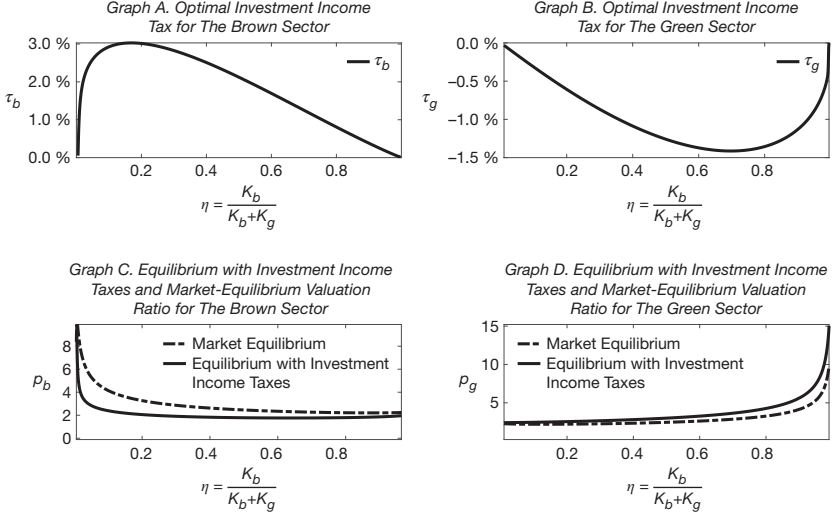
Consider levying an investment income tax $\tau_b(\eta)$ on the expected return from brown firms and $\tau_g(\eta)$ on that from green firms. As a result, the after-tax expected returns on each type of shares become

$$(18) \quad \Lambda_b(\eta) \rightarrow \Lambda_b(\eta) - \tau_b(\eta), \quad \Lambda_g(\eta) \rightarrow \Lambda_g(\eta) - \tau_g(\eta).$$

We require the government tax policy to be budget-neutral. That is, we require that the revenue collected by taxing investment income from brown firms exactly offsets the subsidies to green firms,

FIGURE 3
Optimal Investment Income Taxation

In Figure 3, the parameter values are $\Delta_i = 0.3, \lambda_g = 0, \psi = 0.1, \rho = 0.02, \mu_g = 0.005, \mu_b = 0.005, \sigma_g = 0.25, \sigma_b = 0.25, a_g = 0.15, \alpha_b = 0.15, \theta_g = 25, \theta_b = 25,$ and $\gamma = 1$.



$$(19) \quad p_b(\eta)\eta\tau_b(\eta) + p_g(\eta)(1-\eta)\tau_g(\eta) = 0,$$

where p_b and p_g denote, respectively, Tobin's Q of brown and green firms in equilibrium.

The government can then choose $\tau_b(\eta)$ and $\tau_g(\eta)$ subject to condition (19) to maximize social welfare. Optimal investment income taxation is defined to implement the first-best investment rates i_g^{FB} and i_b^{FB} as a market equilibrium.⁶

Graphs A and B of Figure 3 depict the optimal investment taxes τ_g and τ_b as a function of the fraction of brown-sector capital in the economy. Graphs C and D depict Tobin's Q with and without optimal investment income taxes.

We make a few remarks. First, consistent with intuition, it is optimal to tax investment returns on brown shares and subsidize investment returns on green shares:

$$(20) \quad \tau_g(\eta) < 0 < \tau_b(\eta).$$

⁶Our proposed investment income taxes can be thought of as either dividend taxes or a tax on unrealized capital gains. What matters is that the tax effectively changes the net-of-tax return for the investor. In practice, dividend taxes are an integral part of the tax code (Internal Revenue Code Section 316(a) and 301(c)). By contrast, unrealized capital gains taxes are actively being discussed (e.g., Kim (2023)), but in practice only realized capital gain taxes are currently implemented. Realized capital gains taxes give rise to tax-loss-harvesting issues (Chaudhuri, Burnham, and Lo (2020)) and other tax-timing issues that significantly complicate our analysis, as they require us to keep track of the price at which a given position was purchased. Such an analysis is beyond the scope of this article.

Second, the general equilibrium effect of these taxes is such that Tobin's Q of green firms increases to equal its first-best counterpart under the optimal tax, while Tobin's Q of brown firms decreases to equal its first-best counterpart; that is,

$$(21) \quad p_b^{\text{FB}}(\eta) < p_b(\eta), \quad p_g^{\text{FB}}(\eta) > p_g(\eta).$$

Because investors now demand a higher (lower) pretax return on the brown (green) shares, brown (green) shares have to become cheaper (more expensive) in equilibrium.

Third, such reduction (increase) in Tobin's Q for brown (green) firms encourages firms to invest less (more), thereby implementing the planner's allocation. We refer to this investment-income-tax implementation of the first-best allocation as an investor-capital-reallocation mechanism. Specifically, investment income tax directly affects investors' net return from investing in firm shares. In the meantime, firms utilize capital from investors to produce goods and services; and in capital market equilibrium, investors' net return is closely related to firms' cost of capital. Therefore, the investment income tax affects firms' investment decisions through their cost of capital. That is, instead of directly taxing firms' output, our mechanism taxes the suppliers (financiers) of capital. As far as the firm is concerned, it does not face any direct taxation on its profits or investments, and it conducts its business free of government intervention. We thus provide a rigorous framework to think about investment income taxes as an instrument to enrich the policy toolkit capable of tackling climate change.

In the next section, we discuss carbon taxes and show that both carbon and investment income taxes can be jointly implemented to effectively tackle the climate-change externality.

B. A Comparison and the Relation to Carbon Tax

We now consider imposing a carbon tax as a linear tax of $\delta_b(\eta)$ ($\delta_g(\eta)$) per dollar of output on brown (green) firms. The optimal investment decision of firm n can be obtained by choosing i_n to maximize the firm value given as follows:

$$(22) \quad \max_{i_n} \underbrace{K_n \left(\alpha_n - i_n - \theta_n \frac{i_n^2}{2} \right)}_{\text{Dividends}} + \underbrace{K_n p_n(\eta) (\mu_n + i_n)}_{\text{Capital Gains}} - \underbrace{\alpha_n K_n \delta_n(\eta)}_{\text{Carbon Tax}}, \quad n \in \{b, g\}.$$

The first-order condition for $i_n(\eta)$ is given by

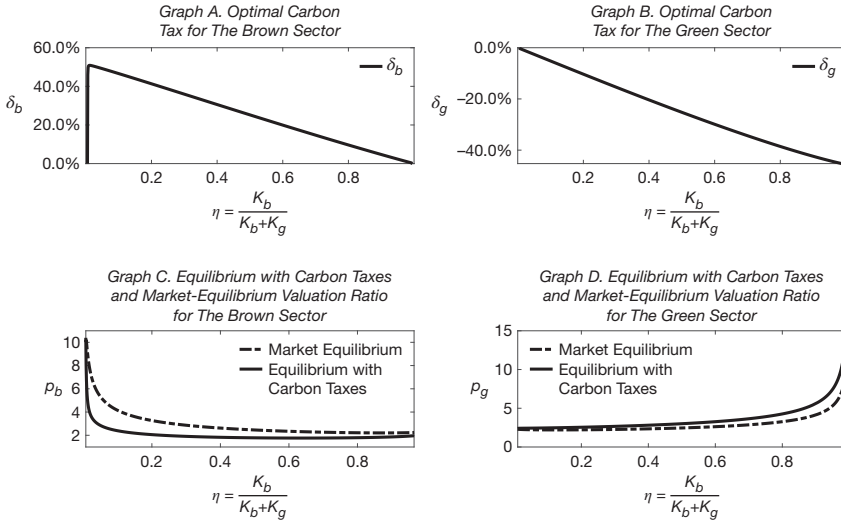
$$(23) \quad i_n(\eta) = \frac{1}{\theta_n} (p_n(\eta) - 1), \quad n \in \{b, g\}.$$

Because carbon tax is subtracted from the firm's dividend payments, the firm's Tobin's Q is affected. Since investment is proportional to Tobin's Q, as seen in equation (23), carbon tax, in turn, affects investment.

As before, we maintain the budget-neutral condition such that the revenue collected from carbon taxes on brown firms exactly offset the subsidies to the green firms, that is,

FIGURE 4
Optimal Carbon Taxation

In Figure 4, the parameter values are $\Delta_b = 0.3, \lambda_g = 0, \psi = 0.1, \rho = 0.02, \mu_g = 0.005, \mu_b = 0.005, \sigma_g = 0.25, \sigma_b = 0.25, \alpha_g = 0.15, \alpha_b = 0.15, \theta_g = 25, \theta_b = 25,$ and $\gamma = 1$.



$$(24) \quad \alpha_b \delta_b(\eta) \eta + \alpha_g \delta_g(\eta) (1 - \eta) = 0.$$

Figure 4 depicts the optimal carbon taxes that implement the social optimum. Consistent with our intuition, to achieve the social optimum, a carbon tax is levied on brown firms’ output to reduce their emissions. The tax revenue collected is then redistributed to green firms in order to increase their production:

$$(25) \quad \delta_g(\eta) < 0 < \delta_b(\eta).$$

When the brown sector accounts for a small fraction of the economy, the optimal carbon tax rate is high and the subsidy rate to green firms is low. This is because the marginal benefit of carbon tax is high in curbing brown firms’ production (the brown firm’s investment rate is high) and a low level of subsidy can increase green firms’ production. On the other hand, when the fraction of brown firms in the economy is high, the optimal carbon tax rate is low because the marginal benefit of carbon tax on reducing brown firms’ production is low and a high level of subsidy is needed to increase green firms’ production.

With carbon taxes, brown firms’ valuations decrease and green firms’ valuations increase relative to the social optimum. Further, as the fraction of capital in the brown sector increases, brown firms’ valuations decrease. The opposite is true for green firms. Therefore, introducing carbon taxes accomplishes the objective of the social planner in growing the green sector and shrinking the brown sector by directly taxing firms’ outputs.

One question worth asking is, how does the investment income tax compare with the carbon tax in terms of its effects on investment policy and firm valuation? We now show that the carbon-tax implementation described above and the investment-income-tax implementation studied in Section IV.A are equivalent with respect to firm investments and valuations. The following definition formalizes the notion of equivalence, which we will use for the subsequent discussions.

Definition. We state that tax regime j is *equivalent* to tax regime k if the following 2 conditions are satisfied:

(a) The allocations in the market equilibria induced by the 2 tax regimes are identical; that is,

$$(26) \quad i_n^j(t) = i_n^k(t), n \in \{b, g\}, \forall t \geq 0,$$

where $i_n^j(t)$ and $i_n^k(t)$ denote the investment rate of type n firms in the market equilibrium under tax regimes j and k , respectively.

(b) The asset pricing implications in the market equilibria induced by the 2 tax regimes are equivalent; that is,

$$(27) \quad p_n^j(t) = p_n^k(t), n \in \{b, g\}, \forall t \geq 0,$$

where $p_n^j(t)$ and $p_n^k(t)$ denote the Tobin's Q of type n firms in the market equilibrium under tax regimes j and k , respectively.

Under this definition, equation (26) implies that consumption under both tax regimes is identical through the market-clearing conditions. As a result, the levels of welfare for the representative household are also the same. At the same time, equation (27) implies that the equilibrium interest rates and equity premia under both tax regimes are the same. Importantly, however, we note that (b) does not follow from (a). That is, it is possible to design a tax regime that implements the first-best investment policies but has different asset-pricing implications from those of the optimal carbon tax. Our definition is, therefore, a rather definitive notion of tax regime equivalence, because it requires both allocations *and* asset prices to be identical.⁷ We formally state our results on the implications of the carbon tax and the investment income tax as follows⁸:

Proposition 1. Suppose that a carbon tax regime denoted by $\{\delta_g(\eta), \delta_b(\eta)\}$ implements the first-best allocation $\{i_b^{\text{FB}}(\eta), i_g^{\text{FB}}(\eta)\}$ as a market equilibrium. Then, there exists an equivalent investment income tax regime denoted by $\{\tau_g(\eta), \tau_b(\eta)\}$. Moreover, the relationship between these 2 tax regimes is given by

⁷An example of such a situation would be corporate investment taxes. That is, a tax per dollar of investment (e.g., a tax on CAPEX). Such a policy can deliver identical investment policies as those under the carbon tax, but it would deliver different asset-pricing implications. Details are available from the authors.

⁸We thank David Sraer for suggesting that we frame our finding as an equivalence result.

$$(28) \quad \tau_b(\eta) = \frac{\alpha_b \delta_b(\eta)}{p_b^{\text{FB}}(\eta)}, \quad \tau_g(\eta) = \frac{\alpha_g \delta_g(\eta)}{p_g^{\text{FB}}(\eta)}.$$

Proof. See the [Appendix](#) ■

The premise of [Proposition 1](#) is that there exists a carbon tax regime (taxing firms), such that market participants internalize the externalities associated with climate-change disaster risks. As a result, this market equilibrium delivers the same investment rates and, therefore, the same allocation as that of the social planner. Thus, it is possible to construct a tax regime which exclusively relies on investment income taxes (levied on investors) that is equivalent to the carbon tax regime (i.e., that delivers the same allocation and pricing implications as those of the carbon tax regime).

The intuition for this result is as follows. Recall that in the market equilibrium, firm investment is proportional to the price-capital ratio of the firm ($p_n(\eta)$), consistent with the predictions of Q-theory. As previously discussed, the socially optimal level of investment requires higher (lower) investment by green (brown) firms than in the *laissez faire* market equilibrium. Carbon taxes (subsidies) on the brown (green) firms make them less (more) profitable. Because in equilibrium households must hold both of these types of firms in their portfolio, the lower (higher) profitability of brown (green) firms must be accompanied by a reduction (increase) in their price-capital ratio. As a consequence, brown (green) firms will reduce (increase) their investment rates. Investment income taxes, by contrast, act directly on the portfolio-choice problem of the households. Taxing (subsidizing) the stock returns from brown (green) firms makes them less (more) desirable for households. Market clearing again implies that Tobin’s Q must adjust in such a way that brown (green) firms reduce (increase) their investment. Finally, [equation \(28\)](#) shows that the optimal investment income taxes are directly proportional to the carbon taxes needed to incentivize green firms to choose the first-best investment rate. The next Proposition generalizes our previous result.

Proposition 2. Suppose that a carbon tax regime denoted by $\{\delta_b(\eta), \delta_g(\eta)\}$ implements the first-best allocation $\{i_b^{\text{FB}}(\eta), i_g^{\text{FB}}(\eta)\}$ as a market equilibrium. Then, any mix of carbon and investment income taxes denoted by $\{\hat{\delta}_b(\eta), \hat{\delta}_g(\eta), \hat{\tau}_b(\eta), \hat{\tau}_g(\eta)\}$ such that

$$(29) \quad \hat{\tau}_b(\eta) + \frac{\alpha_b \hat{\delta}_b(\eta)}{p_b^{\text{FB}}(\eta)} = \frac{\alpha_b \delta_b(\eta)}{p_b^{\text{FB}}(\eta)}, \quad \hat{\tau}_g(\eta) + \frac{\alpha_g \hat{\delta}_g(\eta)}{p_g^{\text{FB}}(\eta)} = \frac{\alpha_g \delta_g(\eta)}{p_g^{\text{FB}}(\eta)}$$

is equivalent to the carbon tax $\{\delta_g(\eta), \delta_b(\eta)\}$.

Proof. See the [Appendix](#) ■

This result generalizes our finding on carbon tax and investment income tax equivalence by showing that in order to implement the first-best allocation, it is not necessary to exclusively have carbon taxes or investment income taxes in the economy. Instead, policymakers can choose from a continuous menu of options

that mix and match carbon tax and investment income tax for each type of firm and its investors, as shown in [equation \(29\)](#). Because the goal of taxation is to alter firms' Tobin's Q , any mix of carbon and investment income taxes satisfying [condition \(29\)](#) delivers this objective and implements the first-best investment level.

To summarize, [Proposition 2](#) provides policymakers with a potentially extensive set of policy prescriptions to mitigate climate-change risks above and beyond the optimal carbon tax. Given the growing urgency to tackle the climate crisis, enhancing the policymakers' toolkit with *new* and *effective* instruments is absolutely essential for our society, hence the importance of our results.

The reader may be wondering about the extent to which [Proposition 2](#) can be generalized to settings beyond the specific structure of our model. [Proposition 3](#) shows that the ability to generate equivalent allocations between carbon taxes and investment income taxes is quite general. However, we chose to illustrate this equivalence in a model with a more specific structure because it allowed us to explicitly characterize the first-best allocation that *endogenously* emerges from our characterization of the climate-change externality.

Proposition 3. Consider a general economic setting with the following 2 features:

- Firms maximize the discounted net present value of their after-tax dividends.
- There are no financial frictions.

If a carbon tax δ_t is levied as a fraction of the firm's output, then the firm's investment policy under this carbon tax is identical to that under a dividend tax $\tau_t = F(K_t)\delta_t$, where $F(K_t)$ denotes firm's output.

Proof. See the [Appendix](#) ■

Our analysis thus far has been conducted under specific modeling assumptions. In particular, a natural concern with the representative investor framework considered heretofore is the extent to which our proposed investment income tax differs from a carbon tax in its ability to gather public support when investors are heterogeneous. To that end, in [Section V](#), we extend our analysis to incorporate investor heterogeneity and assess the merits of investment income tax in mitigating political opposition to taxation. Finally, in [Section VI](#), we discuss the role played by other key modeling assumptions in delivering our results and the extent to which our results would differ in such alternative settings.

V. Heterogeneity and Political-Economy Constraints

A. Investor and Output Heterogeneity

In reality, investment income tax affects investors differently depending on their stock ownership. This differential impact will lead to different views on the deliberation of differential investment income taxes on firms' ecofriendliness.

To assess this impact, in this section, we introduce a second type of investors in our economy, whom we refer to as nonstock investors. We assume that there is a mass $(1 - \kappa) \geq \frac{1}{2}$ of nonstock investors in the economy. These investors are different

from our original stock investors, in that they do not participate in the stock market and only rely on the risk-free asset to smooth their consumption intertemporally. This assumption is motivated by the empirical observation that a large percentage of American households do not invest directly in the stock market (Badarınza, Campbell, and Ramadorai (2016)).

Importantly, without additional assumptions, as is the case in many asset pricing models with heterogeneous investors, the wealth distribution between different types of investors would not be stationary in our setting. Intuitively, because the stock investors have access to a superior investment opportunity set, they would eventually accumulate all the wealth in the economy, leading to a degenerate form of heterogeneity.

To maintain stationarity, we assume an overlapping generation (OLG) model in which investors die with rate Ω every period, and their wealth is equally distributed among the living agents upon their death. This modeling tactic restores stationarity as a result of the redistribution taking place at death, permitting a meaningful exploration of investor heterogeneity (Gârleanu and Panageas (2015)). We push this assumption one step further and study the limiting case in which the lifespan of an investor is arbitrarily short (i.e., when the death rate $\Omega \rightarrow \infty$). This assumption renders the wealth distribution between the 2 types of investors constant. This OLG model with infinitesimally short lifespans is used in the macro finance literature (e.g., He and Krishnamurthy (2013)) due to the advantage of rendering it unnecessary to keep track of the wealth distribution as an additional state variable. As a result, we can introduce heterogeneity in our setting while preserving the tractability of our model.

1. Output Heterogeneity

Furthermore, we enrich our model by introducing heterogeneity in the output of brown and green firms. That is, unlike in the preceding sections in which the output of both types of firms was indistinguishable, investors now feature different preferences for the consumption of green output C_g versus that of brown output C_b . To foreshadow, modeling heterogeneous output will allow us to meaningfully explore the impact of corrective taxation on the goods market and on the relative prices of both types of output.

The preferences of a representative stock investor are now given by

$$(30) \quad \mathbb{E} \left[\int_0^\infty e^{-\rho t} u(C_g(t), C_b(t)) dt \right],$$

where the instantaneous utility function $u(C_b, C_g) = \ln(C_b^{\varepsilon_S} C_g^{1-\varepsilon_S})$ and the parameter ε_S captures the relative preference of the investor for brown output. The preferences of the nonstock investors are identical except that the parameter for brown output is $\varepsilon_{NS} > \varepsilon_S$. This last assumption captures in reduced form the empirical facts that on both the intensive and extensive margins, stock-market participation is positively correlated with wealth (Wachter and Yogo (2010), Badarınza et al. (2016), and Chien and Morris (2017)) and that the expenditure share of carbon-intensive output is negatively associated with wealth (Dinan (2012), Carloni and Dinan (2021)).

We also note that because the output of brown firms is no longer indistinguishable from that of green firms, we need to introduce the price of brown output,

in terms of green output, which is the numeraire in our economy. We denote the price of brown output by ζ_t , which market participants take as given and will be determined in equilibrium.

In short, our economy now features investor heterogeneity with a fraction $\kappa \leq \frac{1}{2}$ of stock investors and a fraction $(1 - \kappa) \geq \frac{1}{2}$ of nonstock investors. Nonstock investors differ from stock investors in 2 ways: i) they can use only the risk-free asset to smooth their consumption; and ii) their preference for the output of brown firms, which has price $\zeta_t = \zeta(\eta(t))$, is stronger than that of stock investors.

2. Investment Goods Market

In the baseline model, when brown and green outputs were indistinguishable from each other, we assumed that if a firm wanted to increase its stock of capital by an amount $i_n K_n$, it had to forfeit $g(i_n) K_n$ units of output, as observed in [equation \(9\)](#). To clearly separate the impact of preferences for green versus brown output, without conflating their previous dual role as investment goods, we have chosen to separately model an investment goods sector. To this end, we introduce a competitive investment goods market that sells investment goods to both brown and green firms. We assume a Leontief investment-goods production function,

$$(31) \quad I = \frac{1}{2} \min \{ Z_g, Z_b \},$$

where I denotes the investment goods produced when amounts $Z_g = z_g K_g$ of green output and $Z_b = z_b K_b$ of brown output are used in the production process.⁹ The amount of brown output Z_b and green output Z_g that are allocated to the production of investment goods, and therefore not available for consumption, will be determined in equilibrium. Moreover, firms take as given the price of investment goods, which we denote by $\chi_t = \chi(\eta(t))$. As a result, we need to update our previous investment rule obtained in [equation \(9\)](#) with

$$(32) \quad \max_{i_n} K_n \left(\underbrace{\alpha_n \zeta_n(\eta) - \left(i_n + \theta_n \frac{i_n^2}{2} \right) \chi(\eta)}_{\text{Dividends}} \right) + \underbrace{K_n p_n(\eta) (\mu_n + i_n)}_{\text{Capital Gains}}, \quad n \in \{b, g\},$$

where $\zeta_b(\eta) = \zeta(\eta)$ and $\zeta_g(\eta) = 1$. The first-order condition for $i_n(\eta)$ is now given by

$$(33) \quad i_n(\eta) = \frac{p_n(\eta) - \chi(\eta)}{\theta_n \chi(\eta)}, \quad n \in \{b, g\}.$$

[Equation \(33\)](#) states that a firm's investment rate is proportional to the difference between the benefit of investment (an additional unit of capital, with market value $p_n(\eta)$) minus the cost of purchasing a unit of investment goods, whose market value is given by $\chi(\eta)$. As before, investment is inversely proportional to the firm's investment adjustment cost parameter θ .

⁹In the Supplementary Material, we show that our results are robust to a more general Cobb–Douglas production technology in the investment goods market. We use Leontief production technology here to more succinctly illustrate our economic insights.

B. Market Equilibrium

Having laid out the changes needed to incorporate heterogeneity into our setting, we can proceed to compute the market equilibrium in the absence of government intervention. Recall that the market equilibrium corresponds to an allocation in which all market participants (stock investors, nonstock investors, and firms) solve their respective optimization problems and all markets (green, brown, and investment goods, as well as brown and green stock markets) clear. In the “[Appendix for Section V](#)” of the Supplementary Material, we provide details on how to numerically characterize the market equilibrium as the solution of a system of differential-algebraic equations in the state variable η .

Henceforth, we make additional parametric assumptions in [Condition 1](#) that allow us to provide a full analytical characterization of the market equilibrium. First, we focus on the case in which the 2 sectors have symmetric technologies and productivity. Second, we shut down the Brownian risks in our economy, so that the climate-disaster risk is the only source of uncertainty in the model. Finally, we make a technical assumption by setting a lower bound for α .

Condition 1. (Parametric Restrictions).

$$(34) \quad \theta_g = \theta_b = \theta, \quad \alpha_g = \alpha_b = \alpha, \quad \mu_g = \mu_b = \mu,$$

$$(35) \quad \sigma_b = \sigma_g = 0,$$

$$(36) \quad \frac{\alpha}{\rho} > -\rho\theta + \sqrt{2\alpha\theta + \rho^2\theta^2 + 1}.$$

In such a setting, we can show the existence of a steady-state equilibrium characterized by η_{SS} such that if $\eta(t) = \eta_{SS}$, then $\eta(s) = \eta_{SS}$ for all $s \geq t$. The following proposition summarizes our findings for the steady-state market equilibrium.

Proposition 4. Under [Condition 1](#), there exists a steady-state market equilibrium such that the equilibrium fraction of brown capital in the economy $\eta_{SS} \in (0, 1)$ is given by

$$(37) \quad \eta_{SS} = \frac{1}{2} + \frac{\rho \left(-\rho\theta + \sqrt{2\alpha\theta + \rho^2\theta^2 + 1} \right) (2(1 - \kappa)\varepsilon_{NS} + 2\kappa\varepsilon_S - 1)}{2\alpha},$$

and the equilibrium price of brown output $\xi(\eta_{SS}) = 1$. Moreover, closed-form expressions for all the other equilibrium quantities $\{P_g(\eta_{SS}), P_b(\eta_{SS}), \chi(\eta_{SS}), z_g(\eta_{SS}), z_b(\eta_{SS}), r(\eta_{SS})\}$ are specified in the “[Appendix for Section V](#)” of the Supplementary Material.

[Proposition 4](#) delivers three key insights. First, the equilibrium size of the brown sector is a function of the weighted average of investors’ preferences for brown output. The term $(2(1 - \kappa)\varepsilon_{NS} + 2\kappa\varepsilon_S - 1)$ essentially states that the equilibrium size of the brown sector is proportional to how much investors value brown

output through their respective ε_S and ε_{NS} parameters and their relative importance in the economy measured by their proportion κ and $1 - \kappa$.

Second, the parameters governing the severity ψ and frequency λ of climate-change disaster shocks do not influence the steady-state market equilibrium η_{SS} . Even though climate disasters are more frequent and severe when the size of the brown sector η_{SS} is larger, the market equilibrium is unable to account for this negative externality. It is striking that the market equilibrium does not even partially account for the negative impact of climate disasters when determining the optimal share of brown capital in the economy. This result shows that the negative externality of climate-change disasters remains in the presence of investor heterogeneity with respect to stock-market participation.

Third, we note that in the steady-state market equilibrium, brown and green output command identical prices, since $\zeta(\eta_{SS}) = 1$. As we will see next, this observation is important, because corrective taxation will make the price of brown output greater than the price of green output. Since nonstock investors have a stronger preference for brown output than stock investors, their welfare will be impacted differently through the goods market channel, hence potentially preventing carbon taxes from being supported by nonstock investors.

C. Optimal Carbon Tax

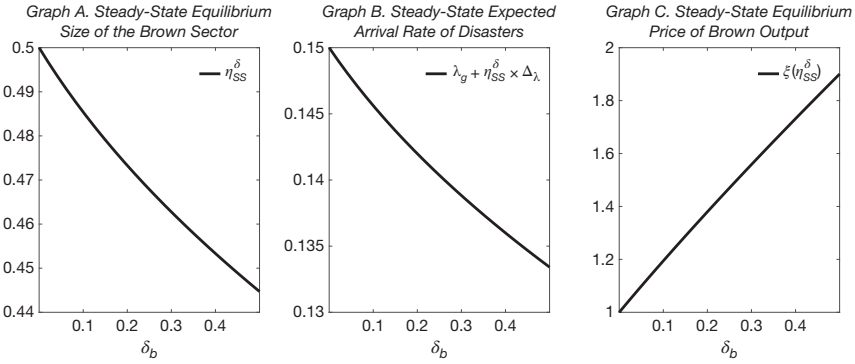
As we saw in the preceding section, the equilibrium size of the brown sector η_{SS} is independent of the severity of climate disasters and it is therefore too large relative to the socially optimal level. Thus, following the analysis of [Section IV.B](#), we reintroduce a carbon tax $\delta_b \geq 0$ levied on brown firms' output and redistributed as an output subsidy $\delta_g \leq 0$ to green firms in order to preserve budget neutrality. We also note that this tax is levied on firms and thus directly affects only the owners of these firms (i.e., the stock investors). As a result, relying exclusively on carbon taxes does not allow for the possibility of redistributing wealth from stock to nonstock investors, which can be a shortcoming from a political-economy perspective, as we will see later.

In the "[Appendix for Section V](#)" of the Supplementary Material, we show how to compute the steady-state equilibrium denoted as η_{SS}^δ arising under a given budget-neutral carbon tax scheme $\delta = (\delta_g, \delta_b)$. Graph A of [Figure 5](#) depicts comparative statics for η_{SS}^δ with respect to the carbon tax δ_b . The equilibrium size of the brown sector decreases as the carbon tax increases. Consistent with our intuition, a higher carbon tax reduces the profitability of brown firms, thereby reducing their investment rate and the size of the brown sector in the economy. Graph B shows that the expected arrival rate of climate disasters $\lambda_g + \eta_{SS}^\delta \Delta_\lambda$ is decreasing in the carbon tax δ_b . This observation is a direct consequence of higher carbon taxes reducing the relative size of the brown sector, which in turn leads to a greener economy with fewer climate-disaster shocks. Finally, Graph C shows that the steady-state equilibrium price of brown output $\zeta(\eta_{SS}^\delta)$ is increasing in δ_b . Intuitively, as the brown sector becomes smaller, brown output becomes scarcer and therefore its equilibrium price has to increase for markets to clear.

Having studied the impact of a carbon tax on the economy, we now proceed to compute the carbon tax that implements the first-best allocation. To that end, we

FIGURE 5
Steady-State Equilibrium Quantities

In Figure 5, the parameter values are $\Delta_\lambda = 0.3$, $\lambda_g = 0$, $\psi = 0.1$, $\rho = 0.02$, $\mu_g = 0.005$, $\mu_b = 0.005$, $\alpha_g = 0.15$, $\alpha_b = 0.15$, $\theta_g = 25$, $\theta_b = 25$, and $\kappa = 0.5$.



modify the planner’s problem to account for investor heterogeneity. We consider the case in which the planner maximizes the weighted sum of the values obtained by the stock and nonstock investors for a given allocation. Their respective weights are given by κ and $1 - \kappa$. By an argument similar to that in the baseline case, the value functions of both types of investors satisfy the functional form (13). Denoting the scaled value functions of the stock and nonstock investors by $f^S(\cdot)$ and $f^{NS}(\cdot)$, respectively, the social planner’s problem becomes

$$(38) \quad \max_{\delta} \kappa f^S(\eta_{SS}^\delta) + (1 - \kappa) f^{NS}(\eta_{SS}^\delta).$$

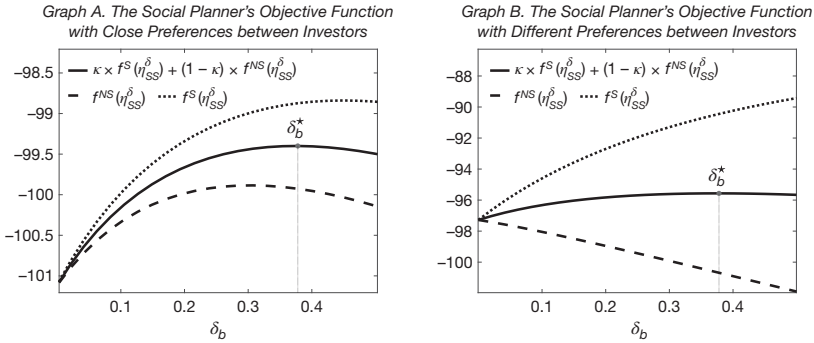
That is, the social planner considers the different steady-state allocations η_{SS}^δ induced by different (budget-neutral) carbon tax rates δ , then chooses the one that maximizes the (weighted) sum of the investors’ value functions. The solid line in Graph A of Figure 6 depicts the objective function of the social planner from expression (38). This function is concave in δ_b and reaches a maximum at the first-best carbon tax δ_b^* . The optimal carbon tax balances out the social gain from fewer climate disasters brought about by a smaller brown sector with the cost of having a lower amount of brown output available for investors’ consumption. Importantly, the second effect is more problematic for nonstock investors due to their stronger preference for brown output relative to stock investors (i.e., $\varepsilon_{NS} > \varepsilon_S$).

To see this, we depict the nonstock investor’s value function $f^{NS}(\eta_{SS}^\delta)$ (the dashed line) and the stock investor’s value function $f^S(\eta_{SS}^\delta)$ (the dotted line). We note that for Graph A’s calibration, in which their preferences are relatively similar ($\varepsilon_S = 0.48$ and $\varepsilon_{NS} = 0.52$), both types of investors are better off when the optimal carbon tax δ_b^* is implemented compared with the case of no carbon tax. As a result, a carbon tax would be approved in a referendum, since there exists a political majority to support it.

We note that the nonstock investors would favor a carbon tax below δ_b^* . On the one hand, investors benefit from the reduction in climate disasters delivered by a

FIGURE 6
Social Planner's Objective Function

In Figure 6, the parameter values are $\Delta_i = 0.3$, $\lambda_g = 0$, $\psi = 0.1$, $\rho = 0.02$, $\mu_g = 0.005$, $\mu_b = 0.005$, $\alpha_g = 0.15$, $\alpha_b = 0.15$, $\theta_g = 25$, $\theta_b = 25$, and $\kappa = 0.5$. Graph A: $\varepsilon_S = 0.48$ and $\varepsilon_{NS} = 0.52$. Graph B: $\varepsilon_S = 0.39$ and $\varepsilon_{NS} = 0.61$.



higher carbon tax (Graph B of Figure 5). On the other hand, investors suffer from the higher price of brown output $\zeta(\eta_{SS}^\delta)$ induced by the carbon tax (Graph C of Figure 5). The latter effect is stronger for nonstock investors; hence, they tend to favor a smaller carbon tax than stock investors.

Interestingly, if the difference in preferences is sufficiently large, it is possible for the optimal carbon tax to make the nonstock investors strictly worse off. That is, the impact of higher brown output prices can entirely override the benefit of fewer climate disasters for nonstock investors. The next proposition formalizes this intuition.

Proposition 5. Introducing a carbon tax would make the nonstock investor strictly worse off relative to the market equilibrium if the following parametric condition is satisfied:

$$(39) \quad \frac{\lambda \log(1 - \psi) \left(\rho \theta - \sqrt{2\alpha\theta + \rho^2 \theta^2 + 1} \right)}{4\alpha\kappa} \leq \varepsilon_{NS} - \varepsilon_S.$$

Graph B of Figure 6 illustrates a calibration ($\varepsilon_S = 0.39$ and $\varepsilon_{NS} = 0.61$) that satisfies inequality (39). In this case, the socially optimal carbon tax is strictly positive; however, nonstock investors would oppose such a tax. Therefore, carbon taxes would fail to achieve a political majority. As we see next, investment income taxes can relax these political constraints and provide a channel through which nonstock investors are willing to support environmentally friendly legislation.

D. Investment Income Tax and Redistribution

In this section, we discuss the potential for an investment income tax to relax the political constraints faced by a pure carbon tax. As shown above, carbon taxes may fail to gather the support of nonstock investors. However, because the benefit to stock investors from a carbon tax is greater than the cost to nonstock investors, a

tax scheme that mitigates the climate-change externality *and* redistributes some of the gains from stock investors to nonstock investors could gather the support of both types of investors.¹⁰

We proceed, in 2 steps, to show that a carefully designed investment income tax scheme can achieve this goal. First, consider an investment income tax $\tau_b > 0$ levied on the returns of brown stocks, exactly as in [equation \(18\)](#) of the baseline model. At the same time, departing from the baseline model, do not redistribute the proceeds in the form of a subsidy to green stock returns by setting $\tau_g = 0$. Instead, use the proceeds to subsidize the returns on the risk-free asset by an amount $\tau_r < 0$. Taken together, the after-tax expected returns on the 3 financial assets (green stocks, brown stocks, and the risk-free assets) are now respectively given by

$$(40) \quad \Lambda_b(\eta) \rightarrow \Lambda_b(\eta) - \tau_b(\eta), \quad \Lambda_g(\eta) \rightarrow \Lambda_g(\eta), \quad r(\eta) \rightarrow r(\eta) - \tau_r(\eta).$$

[Proposition 6](#) formalizes the idea that it is possible to fine-tune budget-neutral investment income taxes $(\tau_b^*, \tau_g^*, \tau_r^*)$ in order to implement the same allocation and associated steady-state η_{SS} as the one obtained under the optimal carbon tax scheme δ^* (i.e., the first-best allocation). Moreover, since $\tau_b^* > 0$, the investment income tax is levied on the stock investors, while the proceeds of the tax are redistributed to the nonstock investors, since $\tau_r^* < 0$.¹¹ As a result, the additional flexibility of investment income taxes of subsidizing the risk-free rate allows it to redistribute wealth to the nonstock investors via the only financial security they hold (i.e., the risk-free asset). Such a transfer could potentially compensate the nonstock investors for the price increase of brown output and garner their support for such a tax.

Proposition 6. Suppose that a carbon tax regime denoted by $\{\delta_g^*, \delta_b^*\}$ implements the first-best allocation of the social planner’s problem in [expression \(38\)](#). Then, the investment income tax regime $\{\tau_b^*, \tau_g^*, \tau_r^*\}$ given by

$$(41) \quad \tau_b^* = \frac{\alpha \delta_b^*}{(1 - \eta_{SS}^\delta) p_{SS}} > 0, \quad \tau_g^* = 0, \quad \tau_r^* = \frac{-\alpha \delta_b^* \eta_{SS}^\delta}{(1 - \kappa)(1 - \eta_{SS}^\delta) p_{SS}} < 0,$$

where $p_{SS} = p_g(\eta_{SS}) = p_b(\eta_{SS})$, also achieves the first-best allocation.

We clarify that redistribution from stock to nonstock investors is minimal in our setting (i.e., of order dt), due to our OLG modeling choice of infinitesimally short lifespans. Since investors have an arbitrarily short lifespan and their

¹⁰In the limiting case in which both households have identical tastes for brown output, a lump-sum rebate to all households would obtain complete support, since correcting the externality associated with disaster risks would equally benefit all households. However, when nonstock investors have a strong preference for brown output, it is necessary to redistribute a larger share of the tax proceeds to them in order to gain their support.

¹¹The goal of subsidizing the risk-free asset is to garner the support of households who do not have brokerage accounts and are not well-versed in ways of directly investing in financial markets. In practice, an effective way to implement this subsidy would be by “topping up” the return accrued in savings and checking accounts, since nearly all U.S. households have bank accounts (95.5% according to the 2021 FDIC National Survey of Unbanked and Underbanked Households).

wealth is redistributed at death, the gain for nonstock investors from the subsidy on their position in the risk-free asset has a vanishingly small effect on their value function.

However, in the realistic, but less tractable case of finite lifespans, the investment income tax regime detailed in [Proposition 6](#) would materially increase the welfare of nonstock investors. In the next section, we conclude this analysis by providing a more in-depth discussion on the practical importance of our results and how they relate to the current political debate on policy tools designed to address negative climate-change externalities.

Finally, we note that both sources of heterogeneity introduced in this section (i.e., taste and stock-market participation) play active roles in generating the above political-economy implications. As we have illustrated in this section, the main message states that a pure carbon tax cannot gather support from nonstock investors (due to heterogeneous tastes), though, by contrast, an investment income tax has the flexibility to compensate nonstock investors and gather their support (due to heterogeneous stock-market participation) via a subsidy to holding the risk-free asset.

E. Discussion

First, our results highlight an investment income tax regime capable of addressing climate-change externalities as effectively as carbon taxes. For at least a few decades, economists have lauded the desirability of carbon taxes as the most effective way of fighting climate-change risks (Akerlof et al. (2019)). Our model is entirely consistent with this economic consensus. However, our model *also* shows that investment income taxes emerge as an effective option. Therefore, our findings suggest that policymakers and environmental activists should consider jointly implementing investment income taxes and carbon taxes to fight climate change when full implementation of the latter faces political resistance and/or constraints.

Second, we have shown that investment income tax can relax political constraints emerging from the opposition of less-well-off constituents negatively affected by higher prices of carbon-intensive goods. Our proposed tax mechanism would raise taxes from the shareholders of brown firms and redistribute them to those who do not directly own firms, by subsidizing their investments in the risk-free asset. Our proposed investment income tax has the advantage that it simultaneously addresses the climate-change externality while mitigating the regressive impact of carbon taxes within a self-contained budget-neutral tax scheme.

Nonetheless, the reader may wonder about the extent to which the government could use the proceeds from carbon taxes and redistribute them directly to nonstock investors. Indeed, such an arrangement would be possible if the government were to tax brown firms and send a check to nonstock investors instead of redistributing the proceeds to green firms. Unlike in our current setup where carbon taxes were levied and redistributed “within firms” while investment income taxes were levied and redistributed “within securities,” the above tax

scheme involves taxation and redistribution across firms and securities. Without a within-firm redistribution restriction, investment income taxes would no longer be superior to carbon taxes from a political-economy perspective (as per [Proposition 6](#)). Instead, the equivalence result of [Proposition 1](#) would apply. However, restricting attention to self-contained budget-neutral policies is in practice more desirable, since it allows a given government agency to deal with firms (in the case of carbon taxes) or with household investment-income returns (in the case of investment income taxes) without the inefficiencies associated with coordinating efforts across firms and securities.

Third, our model considered the simple case in which firms were either ecofriendly or non-ecofriendly. In practice, firms operate within a spectrum, and their investment income tax would need to be adjusted accordingly. Given the existing investment income taxes, fine-tuning can be introduced such that the tax rate differs across ecofriendly firms and non-ecofriendly firms based on firms' environmental ratings and compliance with environmental regulations and standards. From that perspective, investment income taxes would likely be more convenient to implement than other measures to incentivize investment in ecofriendly green firms since they leverage existing investment income tax infrastructure.

Finally, our model does not argue that investment income tax should be viewed as a substitute for carbon tax, but rather as jointly implementable with carbon tax to achieve a greener economy. In light of [Proposition 2](#), policymakers could choose the optimal mix of the 2 tax regimes that is more likely to receive legislative approval and that faces less political opposition. As an example, one can envision that a "large" carbon tax on brown firms is politically infeasible. However, a "medium" carbon tax coupled with another "medium" investment income tax on the stock returns of brown firms is more palatable. This finding, therefore, enlarges the set of proposals worth supporting for environmental activists seeking to mitigate climate change through legislative actions.

VI. Other Considerations and Extensions

In this section, we discuss how some of the model assumptions affect the underlying mechanism for our result. Our model assumes that firms' managers act in the interest of shareholders and choose the investment policy of the firm in order to maximize shareholder value. In practice, the shareholder-manager relationship is subject to agency frictions, and compensation contracts are designed to circumvent these frictions. That being said, typically, optimal contracts do not necessarily restore the frictionless investment benchmark (DeMarzo and Sannikov (2006)). However, we do not think that this agency problem is any more problematic under the carbon tax regime or the investment income tax regime. The reason is that the effects of these schemes on the resources that are available for the managers to divert, on the volatility of cash flows, and on the observability of cash flows, are identical. Thus, we anticipate that a richer model featuring shareholder-manager conflicts would change the

optimal tax policy, but will not significantly affect the equivalence between our 2 proposed tax regimes.¹²

We also abstracted away from financial frictions faced by firms. In practice, firms are subject to various forms of financial frictions that render the value of a dollar inside the firm more valuable than that of a dollar outside the firm. Such frictions provide a rationale for firms to conduct an active cash-management policy (Bolton, Chen, and Wang (2011), Décamps, Mariotti, Rochet, and Villeneuve (2011)). Importantly, it has been shown that financial frictions have delicate implications for optimal taxation (Dávila and Hébert (2022)) and that in such settings Pigouvian carbon taxes need not be optimal (Heider and Inderst (2022), Döttling and Rola-Janicka (2023)).

Furthermore, it is unclear whether carbon taxes, while suboptimal, remain equivalent to investment income taxes in the presence of financial frictions, the way they do in our baseline setting. Because carbon taxes are paid out of the firm's cash reserves, while investment income taxes are paid out of the household's savings, our equivalence result would not directly hold in such a setting. The extent to which this wedge between the 2 tax regimes is significant depends on the severity of financial frictions in the economy. As a result, we speculate that in countries with more-developed financial markets, investment income taxes can provide a close approximation to a carbon tax. By contrast, in countries with less-developed financial markets, and more-prevalent financial frictions, such approximation would be less accurate.

Finally, our model does not feature financial intermediation and instead relies on households' direct participation in equity markets. In reality, households face limited stock-market participation and financial intermediaries play a critical role in allocating capital from households to firms (Diamond (1984)). Such intermediation complicates the implementation of a differential investment income tax for green firms because, for example, a fund could invest a fraction of its holdings in green firms and another fraction in brown firms. If this fund enters and exits various positions on a regular basis, which investment income tax should the ultimate investor (household) face? For taxable mutual funds, the tax pass-through status may require these funds to specify the sources of capital gains or dividends in the same way as those stocks directly acquired by households. For tax-deferred funds such as pension funds, one approach would be to ask funds to label themselves according to their commitment to green investments. A fund that commits to allocating at least 50% of its portfolio to green stocks would be subject to an investment income tax of 50% that of green firms and 50% that of brown firms. We conjecture this type of implementation could approximate the allocative efficiency of a pure carbon tax; however, a rigorous analysis incorporating the challenges of modeling tax-exempt institutions is beyond the scope of this article.

¹²Indeed, Oehmke and Opp (2024) explore a setting in which moral hazard limits the cash flows that can be pledged by the entrepreneurs. This consideration pushes green firms significantly below their optimal size and affects the magnitude of the optimal intervention needed relative to the frictionless benchmark. In fact, when green firms are financially constrained, rebating some of the proceeds from carbon taxes to subsidize their production can increase welfare by relaxing financial constraints.

VII. Conclusion

We investigate firm investment and household consumption-portfolio choice decisions in an economy with climate-change disaster risk. Our economy consists of ecofriendly (green) and non-ecofriendly (brown) firms. The climate disaster obliterates capital in the economy and is modeled to be proportional to the relative size of the non-ecofriendly (brown) sector. Households optimize their consumption and investment by allocating capital to risky stocks of these 2 types of firms and the risk-free asset. The market equilibrium fails to achieve social optimum because brown (green) firms fail to internalize the increase (decrease) in climate-disaster risk caused by their investment policies. As a result, brown (green) firms overinvest (underinvest) relative to the socially optimal investment policies.

We show that an investment income tax, aiming to incentivize investors to reallocate capital from brown to green firms, can be jointly implemented with an existing carbon tax to achieve the social optimum. Moreover, investment income tax can relax political constraints because it can rely on a subsidy to risk-free assets such as savings and checking accounts, held by a majority of households, to mitigate the regressivity of higher good prices from non-ecofriendly firms. Consequently, our findings expand the toolkit that policy-makers and environmental activists may use in their fight to mitigate climate-change risks.

In practice, many features outside of our model would determine the optimal joint implementation of investment income and carbon taxes. We highlight 4 important ones here: uncertainty about the distribution of climate shocks (Barnett et al. (2020)), the presence of political constraints due to the influence of lobbyists in the political deliberation process and their opposition to specific tax regimes (Jenkins (2014)), the heterogeneity in exposure and responses to climate shocks in the cross section of firms (Li, Shan, Tang, and Yao (2020)), and the heterogeneity of beliefs about the severity of climate-change risks across the political spectrum (Bernstein et al. (2022)). Incorporating these features in a tractable climate-change model is the subject of our future research.

Appendix

Proof of Proposition 1. Suppose there is a Carbon tax regime $\{\delta_g(\eta), \delta_b(\eta)\}$ that implements the First-Best allocation $\{i_b^{FB}(\eta), i_g^{FB}(\eta)\}$ as a market equilibrium. First, we show that under such carbon tax system, in equilibrium, the First-Best price-to-capital ratios satisfy a specific system of equations. The dynamics of returns from the allocations in the brown (green) firm are:

$$dR_b = \frac{Div_b}{V_b} dt + \frac{dV_b}{V_b} = \left[\Lambda_b(\eta) - \frac{\alpha_b \delta_b(\eta)}{p_b(\eta)} \right] dt + \Gamma_b(\eta) dB_b - \Delta_b(\eta) dB_g - \psi dN,$$

$$dR_g = \frac{Div_g}{V_g} dt + \frac{dV_g}{V_g} = \left[\Lambda_g(\eta) - \frac{\alpha_g \delta_g(\eta)}{p_g(\eta)} \right] dt + \Gamma_g(\eta) dB_b + \Delta_g(\eta) dB_g - \psi dN;$$

where:

$Div_b = K_b \left(\alpha_b - i_b(\eta) - \frac{\theta}{2}(i_b(\eta))^2 - \alpha_b \delta_b(\eta) \right)$ and $Div_g = K_g \left(\alpha_g - i_g(\eta) - \frac{\theta}{2}(i_g(\eta))^2 - \alpha_g \delta_g(\eta) \right)$ correspond to the dividend payments made by brown/green firms.

Denoting the fractions of wealth invested in the brown (green) firm as π_b (π_g), and the fractions of wealth consumed as c , we derive the wealth process as follows:

$$\begin{aligned} dW = & \left[r(\eta)W - c(\eta)W + \pi_b(\eta)W_t \left(\Lambda_b(\eta) - \frac{\alpha_b \delta_b(\eta)}{p_b(\eta)} - r(\eta) \right) \right. \\ & \left. + \pi_g(\eta)W \left(\Lambda_g(\eta) - \frac{\alpha_g \delta_g(\eta)}{p_g(\eta)} - r(\eta) \right) \right] dt \\ & + W [\pi_b(\eta)\Gamma_b(\eta) + \pi_g(\eta)\Gamma_g(\eta)] dB_b \\ & + W [-\pi_b(\eta)\Delta_b(\eta) + \pi_g(\eta)\Delta_g(\eta)] dB_g - W\psi dN. \end{aligned}$$

Denoting the value function by $F(W; \eta)$, it satisfies the HJB equation:

$$0 = \max_{c(\eta), \pi_b(\eta), \pi_g(\eta)} \left\{ \begin{aligned} & -\rho F(W; \eta) + u(c(\eta)W) \\ & + F_W(W; \eta)W \left[r(\eta) - c(\eta) + \pi_b(\eta) \left(\Lambda_b(\eta) - \frac{\alpha_b \delta_b(\eta)}{p_b(\eta)} - r(\eta) \right) \right. \\ & \quad \left. + \pi_g(\eta) \left(\Lambda_g(\eta) - \frac{\alpha_g \delta_g(\eta)}{p_g(\eta)} - r(\eta) \right) \right] \\ & + \frac{1}{2} F_{WW}(W; \eta)W^2 \left[(\pi_b(\eta)\Gamma_b(\eta) + \pi_g(\eta)\Gamma_g(\eta))^2 \right. \\ & \quad \left. + (-\pi_b(\eta)\Delta_b(\eta) + \pi_g(\eta)\Delta_g(\eta))^2 \right] \\ & + F_\eta(W; \eta)\Sigma(\eta) + \frac{1}{2} F_{\eta\eta}(W; \eta)\eta^2(1-\eta)^2(\sigma_b^2 + \sigma_g^2) \\ & + F_{W\eta}(W; \eta)W\eta(1-\eta) [\sigma_b(\pi_b(\eta)\Gamma_b(\eta) + \pi_g(\eta)\Gamma_g(\eta)) \\ & \quad + \sigma_g(\pi_b(\eta)\Delta_b(\eta) - \pi_g(\eta)\Delta_g(\eta))] \\ & + \lambda\eta [F(W - W\psi(\pi_b(\eta) + \pi_g(\eta)); \eta) - F(W; \eta)] \end{aligned} \right\}. \quad (A-1)$$

We guess (and subsequently verify) that in this case the value function is given by:

$$F(W; \eta) = I(\eta)W^{1-\gamma} + \frac{W^{1-\gamma} - 1}{\rho(1-\gamma)}. \quad (A-2)$$

The optimal consumption and allocations satisfy:

$$c(\eta) = \left[(1-\gamma)I(\eta) + \frac{1}{\rho} \right]^{-\frac{1}{\gamma}}; \quad (A-3)$$

and

$$(A-4) \quad \begin{aligned} \pi_b(\eta) &= \frac{\lambda\eta\psi \left[(1-\gamma)I(\eta) + \frac{1}{\rho} \right] (1-\psi)^{-\gamma} - A - B\pi_g(\eta) - D}{E}; \\ \pi_g(\eta) &= \frac{\lambda\eta\psi \left[(1-\gamma)I(\eta) + \frac{1}{\rho} \right] (1-\psi)^{-\gamma} - A' - B\pi_b(\eta) - D'}{E'}; \end{aligned}$$

where

$$\begin{aligned} A &= \left[(1-\gamma)I(\eta) + \frac{1}{\rho} \right] \left[\Lambda_b(\eta) - \frac{\alpha_b\delta_b(\eta)}{p_b(\eta_t)} - r(\eta) \right]; \\ B &= (-\gamma) \left[(1-\gamma)I(\eta) + \frac{1}{\rho} \right] \left[\Gamma_g(\eta)\Gamma_b(\eta) - \Delta_g(\eta)\Delta_b(\eta) \right]; \\ D &= (1-\gamma)I'(\eta)\eta(1-\eta) \left[\sigma_b\Gamma_b(\eta) + \sigma_g\Delta_b(\eta) \right]; \\ E &= (-\gamma)(1-\gamma)I(\eta) \left[\Gamma_b^2(\eta) + \Delta_b^2(\eta) \right]; \\ A' &= \left[(1-\gamma)I(\eta) + \frac{1}{\rho} \right] \left[\Lambda_g(\eta) - \frac{\alpha_g\delta_g(\eta)}{p_g(\eta_t)} - r(\eta) \right]; \\ D' &= (1-\gamma)I'(\eta)\eta(1-\eta) \left[\sigma_b\Gamma_g(\eta) - \sigma_g\Delta_g(\eta) \right]; \\ E' &= (-\gamma)(1-\gamma)I(\eta) \left[\Gamma_g^2(\eta) + \Delta_g^2(\eta) \right]. \end{aligned}$$

Replacing equations (A-2)–(A-4) into equation (A-1) we obtain:

$$(A-5) \quad \begin{aligned} 0 &= \left[(1-\gamma)I(\eta) + \frac{1}{\rho} \right] \left\{ r(\eta) + \pi_b(\eta) \left(\Lambda_b(\eta) - \frac{\alpha_b\delta_b(\eta)}{p_b(\eta)} - r(\eta) \right) \right. \\ &\quad \left. + \pi_g(\eta) \left(\Lambda_g(\eta) - \frac{\alpha_g\delta_g(\eta)}{p_g(\eta)} - r(\eta) \right) + \frac{\lambda\eta \left((1-\psi)^{1-\gamma} - 1 \right) - \rho}{(1-\gamma)} \right. \\ &\quad \left. - \frac{\gamma}{2} \left[\left(\pi_b(\eta)\Gamma_b(\eta) + \pi_g(\eta)\Gamma_g(\eta) \right)^2 + \left(-\pi_b(\eta)\Delta_b(\eta) + \pi_g(\eta)\Delta_g(\eta) \right)^2 \right] \right\} \\ &\quad + I'(\eta) \left\{ (1-\gamma)\eta(1-\eta) \left[\sigma_b(\pi_b(\eta)\Gamma_b(\eta) + \pi_g(\eta)\Gamma_g(\eta)) \right. \right. \\ &\quad \left. \left. + \sigma_g(\pi_b(\eta)\Delta_b(\eta) - \pi_g(\eta)\Delta_g(\eta)) \right] + \Sigma(\eta) \right\} \\ &\quad + \frac{1}{2} I''(\eta)\eta^2(1-\eta)^2 \left(\sigma_b^2 + \sigma_g^2 \right) + \frac{\gamma \left[(1-\gamma)I(\eta) + \frac{1}{\rho} \right]^{-(1-\gamma)}}{1-\gamma}. \end{aligned}$$

In equilibrium, the following conditions should also hold:

- (I) Balanced budget: $\alpha_g\delta_g(\eta)(1-\eta) + \alpha_b\delta_b(\eta)\eta = 0$.
- (II) Goods market clearing: $c_t(\eta)W = Div_b + Div_g$.
- (III) The agent holds both trees in equilibrium and none of the bond: $\pi_b(\eta) = \frac{K_b p_b(\eta)}{W}$, $\pi_g(\eta) = \frac{K_g p_g(\eta)}{W}$, and $\pi_b(\eta) + \pi_g(\eta) = 1$.

Equating the optimal allocations in equation (A-4) with condition (III), we have that in equilibrium the price-to-capital ratios and risk-free interest rate satisfy:

$$(A-6) \quad \frac{\eta p_b(\eta)}{\eta p_b(\eta) + (1-\eta)p_g(\eta)} = \frac{-1}{\gamma[\Gamma_b^2(\eta) + \Delta_b^2(\eta)]} \left\{ \lambda\eta\psi(1-\psi)^{-\gamma} - \left[\Lambda_b(\eta) - \frac{\alpha_b\delta_b(\eta)}{p_b(\eta)} - r(\eta) \right] + [\Gamma_g(\eta)\Gamma_b(\eta) - \Delta_g(\eta)\Delta_b(\eta)] \frac{\gamma(1-\eta)p_g(\eta)}{\eta p_b(\eta) + (1-\eta)p_g(\eta)} - \frac{(1-\gamma)I'(\eta)}{(1-\gamma)I(\eta) + \frac{1}{\rho}} \eta(1-\eta) [\sigma_b\Gamma_b(\eta) + \sigma_g\Delta_b(\eta)] \right\};$$

and

$$(A-7) \quad \frac{(1-\eta)p_g(\eta)}{\eta p_b(\eta) + (1-\eta)p_g(\eta)} = \frac{-1}{\gamma[\Gamma_g^2(\eta) + \Delta_g^2(\eta)]} \left\{ \lambda\eta\psi(1-\psi)^{-\gamma} - \left[\Lambda_g(\eta) - \frac{\alpha_g\delta_g(\eta)}{p_g(\eta)} - r(\eta) \right] + [\Gamma_g(\eta)\Gamma_b(\eta) - \Delta_g(\eta)\Delta_b(\eta)] \frac{\gamma\eta p_b(\eta)}{\eta p_b(\eta) + (1-\eta)p_g(\eta)} - \frac{(1-\gamma)I'(\eta)}{(1-\gamma)I(\eta) + \frac{1}{\rho}} \eta(1-\eta) [\sigma_b\Gamma_g(\eta) - \sigma_g\Delta_g(\eta)] \right\}.$$

In addition, using the optimal consumption in [equation \(A-3\)](#) together with the firms' investment decisions in [equation \(23\)](#) and the balanced budget condition (I), we can rewrite the market clearing condition (II) as:

$$\begin{aligned} & \left[(1-\gamma)I(\eta) + \frac{1}{\rho} \right]^{\frac{-1}{\gamma}} [\eta p_b(\eta) + (1-\eta)p_g(\eta)] \\ &= \eta \left[\alpha_b - \left(\frac{1}{\theta}(p_b(\eta) - 1) \right) - \frac{\theta}{2} \left(\frac{1}{\theta}(p_b(\eta) - 1) \right)^2 - \alpha_b\delta_b(\eta) \right] \\ &+ (1-\eta) \left[\alpha_g - \left(\frac{1}{\theta}(p_g(\eta) - 1) \right) - \frac{\theta}{2} \left(\frac{1}{\theta}(p_g(\eta) - 1) \right)^2 - \alpha_g\delta_g(\eta) \right], \end{aligned}$$

which is equivalent to:

$$\begin{aligned}
 \text{(A-8)} \quad & \left[(1-\gamma)I(\eta) + \frac{1}{\rho} \right]^{\frac{-1}{\gamma}} \left[\eta p_b(\eta) + (1-\eta)p_g(\eta) \right] \\
 & = \eta \left[\alpha_b - \left(\frac{1}{\theta}(p_b(\eta) - 1) \right) - \frac{\theta}{2} \left(\frac{1}{\theta}(p_b(\eta) - 1) \right)^2 \right] \\
 & \quad + (1-\eta) \left[\alpha_g - \left(\frac{1}{\theta}(p_g(\eta) - 1) \right) - \frac{\theta}{2} \left(\frac{1}{\theta}(p_g(\eta) - 1) \right)^2 \right].
 \end{aligned}$$

Therefore, in equilibrium, $p_g^{FB}(\eta), p_b^{FB}(\eta)$ satisfy the system of differential-algebraic equations (A-5)–(A-8).

Next, suppose that there is an investment tax regime such that:

$$\tau_b(\eta) = \frac{\alpha_b \delta_b(\eta)}{p_b^{inv}(\eta)}, \quad \tau_g(\eta) = \frac{\alpha_g \delta_g(\eta)}{p_g^{inv}(\eta)},$$

where $p_b^{inv}(\eta)$ and $p_g^{inv}(\eta)$ are the equilibrium price-to-capital ratios.

Denoting the value function under this investment tax regime by $G(W; \eta)$, the HJB equation is:

$$\text{(A-9)} \quad 0 = \max_{c(\eta), \pi_b(\eta), \pi_g(\eta)} \left\{ \begin{aligned} & -\rho G(W; \eta) + u(c^{inv}(\eta)W) \\ & + G_W(W; \eta)W [r(\eta) - c^{inv}(\eta) + \pi_b^{inv}(\eta)(\Lambda_b^{inv}(\eta) - \tau_b(\eta) - r^{inv}(\eta)) \\ & \quad + \pi_g^{inv}(\eta)(\Lambda_g^{inv}(\eta) - \tau_g(\eta) - r^{inv}(\eta))] \\ & + \frac{1}{2} G_{WW}(W; \eta)W^2 \left[\left(\pi_b^{inv}(\eta)\Gamma_b^{inv}(\eta) + \pi_g^{inv}(\eta)\Gamma_g^{inv}(\eta) \right)^2 \right. \\ & \quad \left. + \left(-\pi_b^{inv}(\eta)\Delta_b^{inv}(\eta) + \pi_g^{inv}(\eta)\Delta_g^{inv}(\eta) \right)^2 \right] \\ & + G_\eta(W; \eta)\Sigma^{inv}(\eta) + \frac{1}{2} G_{\eta\eta}(W; \eta)\eta^2(1-\eta)^2(\sigma_b^2 + \sigma_g^2) \\ & + G_{W\eta}(W; \eta)W\eta(1-\eta) \left[\sigma_b \left(\pi_b^{inv}(\eta)\Gamma_b^{inv}(\eta) + \pi_g^{inv}(\eta)\Gamma_g^{inv}(\eta) \right) \right. \\ & \quad \left. + \sigma_g \left(\pi_b^{inv}(\eta)\Delta_b^{inv}(\eta) - \pi_g^{inv}(\eta)\Delta_g^{inv}(\eta) \right) \right] \\ & + \lambda\eta [G(W - W\psi(\pi_b(\eta) + \pi_g(\eta)); \eta) - G(W; \eta)] \end{aligned} \right\}.$$

We guess (and verify) that in this case the value function is given by:

$$\text{(A-10)} \quad G(W; \eta) = J(\eta)W^{1-\gamma} + \frac{W^{1-\gamma} - 1}{\rho(1-\gamma)}.$$

The optimal consumption and allocations satisfy:

$$\text{(A-11)} \quad c_t^{inv} = \left[(1-\gamma)J(\eta) + \frac{1}{\rho} \right]^{\frac{-1}{\gamma}};$$

$$(A-12) \quad \pi_b^{inv}(\eta) = \frac{\lambda\eta\psi \left[(1-\gamma)J(\eta) + \frac{1}{\rho} \right] (1-\psi)^{-\gamma} - A_2 - B_2\pi_g^{inv}(\eta) - D_2}{E_2};$$

$$\pi_g^{inv}(\eta) = \frac{\lambda\eta\psi \left[(1-\gamma)J(\eta) + \frac{1}{\rho} \right] (1-\psi)^{-\gamma} - A'_2 - B_2\pi_b^{inv}(\eta) - D'_2}{E'_2};$$

where

$$A_2 = \left[(1-\gamma)J(\eta) + \frac{1}{\rho} \right] [\Lambda_b^{inv}(\eta) - \tau_b(\eta) - r^{inv}(\eta)];$$

$$A'_2 = \left[(1-\gamma)J(\eta) + \frac{1}{\rho} \right] [\Lambda_g^{inv}(\eta) - \tau_g(\eta) - r^{inv}(\eta)].$$

$B_2, D_2, E_2, D'_2, E'_2, \Lambda_b^{inv}, \Lambda_g^{inv}, \Gamma_b^{inv}, \Gamma_g^{inv}, \Delta_b^{inv}, \Delta_g^{inv}$ are obtained by replacing $p_g(\eta), p_b(\eta), i_g(\eta), i_b(\eta), I(\eta)$ with $p_g^{inv}(\eta), p_b^{inv}(\eta), i_g^{inv}(\eta), i_b^{inv}(\eta), J(\eta)$ in their counterparts from the Carbon tax regimes.

Replacing equations (A-10)–(A-12) into equation (A-9) we obtain:

$$(A-13) \quad 0 = \left[(1-\gamma)J(\eta) + \frac{1}{\rho} \right] \left\{ r^{inv}(\eta) + \pi_b^{inv}(\eta) (\Lambda_b^{inv}(\eta) - \tau_b(\eta) - r^{inv}(\eta)) \right. \\ \left. + \pi_g^{inv}(\eta) (\Lambda_g^{inv}(\eta) - \tau_g(\eta) - r^{inv}(\eta)) + \frac{\lambda\eta \left((1-\psi)^{1-\gamma} - 1 \right) - \rho}{(1-\gamma)} \right. \\ \left. - \frac{\gamma}{2} \left[\left(\pi_b^{inv}(\eta) \Gamma_b^{inv}(\eta) + \pi_g^{inv}(\eta) \Gamma_g^{inv}(\eta) \right)^2 + \left(-\pi_b^{inv}(\eta) \Delta_b^{inv}(\eta) + \pi_g^{inv}(\eta) \Delta_g^{inv}(\eta) \right)^2 \right] \right\} \\ + J'(\eta) \left\{ (1-\gamma)\eta(1-\eta) \left[\sigma_b \left(\pi_b^{inv}(\eta) \Gamma_b^{inv}(\eta) + \pi_g^{inv}(\eta) \Gamma_g^{inv}(\eta) \right) \right. \right. \\ \left. \left. + \sigma_g \left(\pi_b^{inv}(\eta) \Delta_b^{inv}(\eta) - \pi_g^{inv}(\eta) \Delta_g^{inv}(\eta) \right) \right] + \Sigma^{inv}(\eta) \right\} \\ + \frac{1}{2} J''(\eta) \eta^2 (1-\eta)^2 \left(\sigma_b^2 + \sigma_g^2 \right) + \frac{\gamma \left[(1-\gamma)J(\eta) + \frac{1}{\rho} \right]^{-(1-\gamma)}}{1-\gamma}.$$

In equilibrium, the following conditions must hold:

(I'). Balanced budget: $(1-\eta)\tau_g(\eta)p_g^{inv}(\eta) + \eta\tau_b(\eta)p_b^{inv}(\eta) = 0..$

(II'). Goods market clearing: $c_t^{inv}(\eta)W = Div_b^{inv} + Div_g^{inv} + Tax_b^{inv} + Tax_g^{inv}$, where Tax_b^{inv}/Tax_g^{inv} denote the investment taxes imposed on the investors of the brown/green firms.

(III'). The agent holds both trees in equilibrium and none of the bond:

$$\pi_b^{inv}(\eta) = \frac{K_b p_b^{inv}(\eta)}{W}, \quad \pi_g^{inv}(\eta) = \frac{K_g p_g^{inv}(\eta)}{W}, \quad \text{and} \quad \pi_b^{inv}(\eta) + \pi_g^{inv}(\eta) = 1.$$

From condition (III') and equations (A-12), the equilibrium price-to-capital ratios and risk-free interest rate must satisfy:

$$(A-14) \quad \frac{\eta p_b^{inv}(\eta)}{\eta p_b^{inv}(\eta) + (1-\eta) p_g^{inv}(\eta)}$$

$$= \frac{-1}{\gamma \left[\Gamma_b^{inv^2}(\eta) + \Delta_b^{inv^2}(\eta) \right]} \left\{ \lambda \eta \psi (1-\psi)^{-\gamma} - \left[\Lambda_b^{inv}(\eta) - \tau_b(\eta) - r^{inv}(\eta) \right] \right.$$

$$+ \left[\Gamma_g^{inv}(\eta) \Gamma_b^{inv}(\eta) - \Delta_g^{inv}(\eta) \Delta_b^{inv}(\eta) \right] \frac{\gamma (1-\eta) p_g^{inv}(\eta)}{\eta p_b^{inv}(\eta) + (1-\eta) p_g^{inv}(\eta)}$$

$$\left. - \frac{(1-\gamma) J'(\eta)}{\left[(1-\gamma) J(\eta) + \frac{1}{\rho} \right]} \eta (1-\eta) \left[\sigma_b \Gamma_b^{inv}(\eta) + \sigma_g \Delta_b^{inv}(\eta) \right] \right\};$$

and

$$(A-15) \quad \frac{(1-\eta) p_g^{inv}(\eta)}{\eta p_b^{inv}(\eta) + (1-\eta) p_g^{inv}(\eta)}$$

$$= \frac{-1}{\gamma \left[\Gamma_g^{inv^2}(\eta) + \Delta_g^{inv^2}(\eta) \right]} \left\{ \lambda \eta \psi (1-\psi)^{-\gamma} - \left[\Lambda_g^{inv}(\eta) - \tau_g(\eta) - r^{inv}(\eta) \right] \right.$$

$$+ \left[\Gamma_g^{inv}(\eta) \Gamma_b^{inv}(\eta) - \Delta_g^{inv}(\eta) \Delta_b^{inv}(\eta) \right] \frac{\gamma \eta p_b^{inv}(\eta)}{\eta p_b^{inv}(\eta) + (1-\eta) p_g^{inv}(\eta)}$$

$$\left. - \frac{(1-\gamma) J'(\eta)}{\left[(1-\gamma) J(\eta) + \frac{1}{\rho} \right]} \eta (1-\eta) \left[\sigma_b \Gamma_g^{inv}(\eta) - \sigma_g \Delta_g^{inv}(\eta) \right] \right\}.$$

For the market clearing condition to hold, we need:

$$c_t^{inv}(\eta) W = Div_b^{inv} + Div_g^{inv} + Tax_b^{inv} + Tax_g^{inv},$$

where

$$Div_b^{inv} = K_b \left(\alpha_b - i_b^{inv}(\eta) - \frac{\theta}{2} \left(i_b^{inv}(\eta) \right)^2 \right), Div_g^{inv} = K_g \left(\alpha_g - i_g^{inv}(\eta) - \frac{\theta}{2} \left(i_g^{inv}(\eta) \right)^2 \right),$$

$$Tax_b^{inv} = V_b \tau_b(\eta), Tax_g^{inv} = V_g \tau_g(\eta).$$

Thus, the market clearing condition is equivalent to:

$$\left[(1-\gamma) J(\eta) + \frac{1}{\rho} \right]^{\frac{-1}{\gamma}} \left[\eta p_b^{inv}(\eta) + (1-\eta) p_g^{inv}(\eta) \right]$$

$$= \eta \left[\alpha_b - \left(\frac{1}{\theta} (p_b^{inv}(\eta) - 1) \right) - \frac{\theta}{2} \left(\frac{1}{\theta} (p_b^{inv}(\eta) - 1) \right)^2 \right]$$

$$+ (1-\eta) \left[\alpha_g - \left(\frac{1}{\theta} (p_g^{inv}(\eta) - 1) \right) - \frac{\theta}{2} \left(\frac{1}{\theta} (p_g^{inv}(\eta) - 1) \right)^2 \right]$$

$$+ (1-\eta) \tau_g(\eta) p_g^{inv}(\eta) + \eta \tau_b(\eta) p_b^{inv}(\eta).$$

Using the balanced budget condition, we can rewrite it as:

$$(A-16) \quad \begin{aligned} & \left[(1-\gamma)J(\eta) + \frac{1}{\rho} \right]^{\frac{1}{\gamma}} \left[\eta p_b^{inv}(\eta) + (1-\eta)p_g^{inv} \right] \\ & = \eta \left[\alpha_b - \left(\frac{1}{\theta} (p_b^{inv}(\eta) - 1) \right) - \frac{\theta}{2} \left(\frac{1}{\theta} (p_b^{inv}(\eta) - 1) \right)^2 \right] \\ & + (1-\eta) \left[\alpha_g - \left(\frac{1}{\theta} (p_g^{inv}(\eta) - 1) \right) - \frac{\theta}{2} \left(\frac{1}{\theta} (p_g^{inv}(\eta) - 1) \right)^2 \right]. \end{aligned}$$

If we have:

$$\tau_b(\eta) = \frac{\alpha_b \delta_b(\eta)}{p_b^{inv}(\eta)}, \quad \tau_g(\eta) = \frac{\alpha_g \delta_g(\eta)}{p_g^{inv}(\eta)},$$

then the system of equations (A-13)–(A-16) is identical to (A-5)–(A-8). Thus, the First-Best price-to-capital ratios will satisfy the system of equations (A-13)–(A-16). Therefore, $p_b^{inv}(\eta) = p_b(\eta) = p_b^{FB}(\eta)$, $p_g^{inv}(\eta) = p_g(\eta) = p_g^{FB}(\eta)$. Moreover, because under both tax regimes $i_n(\eta) = \frac{1}{\theta}(p_n(\eta) - 1)$, the investment rates of brown (green) firms in equilibrium are identical under the 2 tax regimes. Finally, we also have $\tau_b(\eta) = \frac{\alpha_b \delta_b(\eta)}{p_b^{inv}(\eta)} = \frac{\alpha_b \delta_b(\eta)}{p_b(\eta)} = \frac{\alpha_b \delta_b(\eta)}{p_b^{FB}(\eta)}$, and $\tau_g(\eta) = \frac{\alpha_g \delta_g(\eta)}{p_g^{inv}(\eta)} = \frac{\alpha_g \delta_g(\eta)}{p_g(\eta)} = \frac{\alpha_g \delta_g(\eta)}{p_g^{FB}(\eta)}$.

Proof of Proposition 2. Suppose that a Carbon tax regime denoted by $\{\delta_g(\eta), \delta_b(\eta)\}$ implements the First-Best allocation $\{i_b^{FB}(\eta), i_g^{FB}(\eta)\}$ as a market equilibrium, and that there is a mixed system of Carbon and Investment Income Taxes denoted by $\{\hat{\delta}_g(\eta), \hat{\delta}_b(\eta), \hat{\tau}_g(\eta), \hat{\tau}_b(\eta)\}$ such that:

$$(A-17) \quad \hat{\tau}_b(\eta) + \frac{\alpha_b \hat{\delta}_b(\eta)}{p_b^{mix}(\eta)} = \frac{\alpha_b \delta_b(\eta)}{p_b^{mix}(\eta)}, \quad \hat{\tau}_g(\eta) + \frac{\alpha_g \hat{\delta}_g(\eta)}{p_g^{mix}(\eta)} = \frac{\alpha_g \delta_g(\eta)}{p_g^{mix}(\eta)},$$

where $p_b^{mix}(\eta)$ and $p_g^{mix}(\eta)$ are the equilibrium price-to-capital ratios under this tax regime.

The value function in this case is:

$$H(W; \eta) = K(\eta)W^{1-\gamma} + \frac{W^{1-\gamma} - 1}{\rho(1-\gamma)}.$$

Solving for the optimal consumption and allocation under the mixed tax regime, replacing them and the above guess into the HJB equation, and using the condition that the agent holds both trees in equilibrium, we have the equilibrium price-capital ratios and risk-free rate fulfilling:

$$\begin{aligned}
 \text{(A-18)} \quad 0 = & \left[(1-\gamma)K(\eta) + \frac{1}{\rho} \right] \left\{ \pi_b^{mix}(\eta) \left(\Lambda_b^{mix}(\eta) - \frac{\alpha_b \hat{\delta}_b(\eta)}{P_b^{mix}(\eta)} - \hat{\tau}_b(\eta) \right) \right. \\
 & + \pi_g^{mix}(\eta) \left(\Lambda_g^{mix}(\eta) - \frac{\alpha_g \hat{\delta}_g(\eta)}{P_g^{mix}(\eta)} - \hat{\tau}_g(\eta) \right) + \frac{\lambda \eta \left((1-\psi)^{1-\gamma} - 1 \right) - \rho}{(1-\gamma)} \\
 & \left. - \frac{\gamma}{2} \left[\left(\pi_b^{mix}(\eta) \Gamma_b^{mix}(\eta) + \pi_g^{mix}(\eta) \Gamma_g^{mix}(\eta) \right)^2 + \left(-\pi_b^{mix}(\eta) \Delta_b^{mix}(\eta) + \pi_g^{mix}(\eta) \Delta_g^{mix}(\eta) \right)^2 \right] \right\} \\
 & + K'(\eta) \left\{ (1-\gamma)\eta(1-\eta) \left[\sigma_b \left(\pi_b^{mix}(\eta) \Gamma_b^{mix}(\eta) + \pi_g^{mix}(\eta) \Gamma_g^{mix}(\eta) \right) \right. \right. \\
 & \left. \left. + \sigma_g \left(\pi_b^{mix}(\eta) \Delta_b^{mix}(\eta) - \pi_g^{mix}(\eta) \Delta_g^{mix}(\eta) \right) \right] + \Sigma^{mix}(\eta) \right\} \\
 & + \frac{1}{2} K''(\eta) \eta^2 (1-\eta)^2 \left(\sigma_b^2 + \sigma_g^2 \right) + \frac{\gamma \left[(1-\gamma)K(\eta) + \frac{1}{\rho} \right]^{\frac{-(1-\gamma)}{\gamma}}}{1-\gamma};
 \end{aligned}$$

and

$$\begin{aligned}
 \text{(A-19)} \quad \pi_b^{mix} = & \frac{\eta P_b^{mix}(\eta)}{\eta P_b^{mix}(\eta) + (1-\eta) P_g^{mix}(\eta)} \\
 = & \frac{-1}{\gamma \left[\Gamma_b^{mix^2}(\eta) + \Delta_b^{mix^2}(\eta) \right]} \left\{ \lambda \eta \psi (1-\psi)^{-\gamma} - \left[\Lambda_b^{mix}(\eta) - \frac{\alpha_b \hat{\delta}_b(\eta)}{P_b^{mix}(\eta)} - \hat{\tau}_b(\eta) - r^{mix}(\eta) \right] \right. \\
 & + \left[\Gamma_g^{mix}(\eta) \Gamma_b^{mix}(\eta) - \Delta_g^{mix}(\eta) \Delta_b^{mix}(\eta) \right] \frac{\gamma (1-\eta) P_g^{mix}(\eta)}{\eta P_b^{mix}(\eta) + (1-\eta) P_g^{mix}(\eta)} \\
 & \left. - \frac{(1-\gamma)K'(\eta)}{\left[(1-\gamma)K(\eta) + \frac{1}{\rho} \right]} \eta (1-\eta) \left[\sigma_b \Gamma_b^{mix}(\eta) + \sigma_g \Delta_b^{mix}(\eta) \right] \right\};
 \end{aligned}$$

and

$$\begin{aligned}
 \text{(A-20)} \quad \pi_g^{mix} = & \frac{(1-\eta) P_g^{mix}(\eta)}{\eta P_b^{mix}(\eta) + (1-\eta) P_g^{mix}(\eta)} \\
 = & \frac{-1}{\gamma \left[\Gamma_g^{mix^2}(\eta) + \Delta_g^{mix^2}(\eta) \right]} \left\{ \lambda \eta \psi (1-\psi)^{-\gamma} - \left[\Lambda_g^{mix}(\eta) - \frac{\alpha_g \hat{\delta}_g(\eta)}{P_g^{mix}(\eta)} - \hat{\tau}_g(\eta) - r^{mix}(\eta) \right] \right. \\
 & + \left[\Gamma_g^{mix}(\eta) \Gamma_b^{mix}(\eta) - \Delta_g^{mix}(\eta) \Delta_b^{mix}(\eta) \right] \frac{\gamma \eta P_b^{mix}(\eta)}{\eta P_b^{mix}(\eta) + (1-\eta) P_g^{mix}(\eta)} \\
 & \left. - \frac{(1-\gamma)K'(\eta)}{\left[(1-\gamma)K(\eta) + \frac{1}{\rho} \right]} \eta (1-\eta) \left[\sigma_b \Gamma_g^{mix}(\eta) - \sigma_g \Delta_g^{mix}(\eta) \right] \right\};
 \end{aligned}$$

where $\Lambda_b^{mix}, \Lambda_g^{mix}, \Gamma_b^{mix}, \Gamma_g^{mix}, \Delta_b^{mix}, \Delta_g^{mix}$ are obtained by replacing $p_g(\eta), p_b(\eta), i_g(\eta), i_b(\eta)$ with $P_g^{mix}(\eta), P_b^{mix}(\eta), i_g^{mix}(\eta), i_b^{mix}(\eta)$, the equilibrium price-capital and investment-

capital ratios under the mixed tax regime. Using the market clearing condition, we also have:

$$\begin{aligned} & \left[(1-\gamma)K(\eta) + \frac{1}{\rho} \right]^{\frac{1}{\gamma}} \left[\eta p_b^{\text{mix}}(\eta) + (1-\eta)p_g^{\text{mix}}(\eta) \right] \\ &= \eta \left[\alpha_b - \left(\frac{1}{\theta} (p_b^{\text{mix}}(\eta) - 1) \right) - \frac{\theta}{2} \left(\frac{1}{\theta} (p_b^{\text{mix}}(\eta) - 1) \right)^2 - \alpha_b \hat{\delta}_b(\eta) \right] \\ &+ (1-\eta) \left[\alpha_g - \left(\frac{1}{\theta} (p_g^{\text{mix}}(\eta) - 1) \right) - \frac{\theta}{2} \left(\frac{1}{\theta} (p_g^{\text{mix}}(\eta) - 1) \right)^2 - \alpha_g \hat{\delta}_g(\eta) \right] \\ &+ (1-\eta)\tau_g(\eta)p_g^{\text{mix}}(\eta) + \eta\tau_b(\eta)p_b^{\text{mix}}(\eta). \end{aligned}$$

Utilizing the balanced budget condition (A-17), together with equations (A-17), we can rewrite the clearing condition as:

$$\begin{aligned} \text{(A-21)} \quad & \left[(1-\gamma)K(\eta) + \frac{1}{\rho} \right]^{\frac{1}{\gamma}} \left[\eta p_b^{\text{mix}}(\eta) + (1-\eta)p_g^{\text{mix}}(\eta) \right] \\ &= \eta \left[\alpha_b - \left(\frac{1}{\theta} (p_b^{\text{mix}}(\eta) - 1) \right) - \frac{\theta}{2} \left(\frac{1}{\theta} (p_b^{\text{mix}}(\eta) - 1) \right)^2 \right] \\ &+ (1-\eta) \left[\alpha_g - \left(\frac{1}{\theta} (p_g^{\text{mix}}(\eta) - 1) \right) - \frac{\theta}{2} \left(\frac{1}{\theta} (p_g^{\text{mix}}(\eta) - 1) \right)^2 \right]. \end{aligned}$$

If we have:

$$\hat{\tau}_b(\eta) + \frac{\alpha_b \hat{\delta}_b(\eta)}{p_b^{\text{mix}}(\eta)} = \frac{\alpha_b \delta_b(\eta)}{p_b^{\text{mix}}(\eta)}, \quad \hat{\tau}_g(\eta) + \frac{\alpha_g \hat{\delta}_g(\eta)}{p_g^{\text{mix}}(\eta)} = \frac{\alpha_g \delta_g(\eta)}{p_g^{\text{mix}}(\eta)},$$

then the system of equations (A-18)–(A-21) is identical to (A-5)–(A-8).

Thus, the First-Best price-to-capital ratios will satisfy the system of equations (A-18)–(A-21). Hence, $p_b^{\text{mix}}(\eta) = p_b(\eta) = p_b^{\text{FB}}(\eta)$, $p_g^{\text{mix}}(\eta) = p_g(\eta) = p_g^{\text{FB}}(\eta)$ and $i_b^{\text{mix}}(\eta) = i_b(\eta) = i_b^{\text{FB}}(\eta)$, $i_g^{\text{mix}}(\eta) = i_g(\eta) = i_g^{\text{FB}}(\eta)$. Finally, we also have:

$$\hat{\tau}_b(\eta) + \frac{\alpha_b \hat{\delta}_b(\eta)}{p_b^{\text{FB}}(\eta)} = \frac{\alpha_b \delta_b(\eta)}{p_b^{\text{FB}}(\eta)}, \quad \hat{\tau}_g(\eta) + \frac{\alpha_g \hat{\delta}_g(\eta)}{p_g^{\text{FB}}(\eta)} = \frac{\alpha_g \delta_g(\eta)}{p_g^{\text{FB}}(\eta)}.$$

Proof of Proposition 3. By assumption, the firm's problem consists of maximizing the discounted net present value of after-tax dividends received by the household denoted by Div_t . That is:

$$\text{(A-22)} \quad \max E^* \left[\int_0^\infty e^{-rt} Div_t dt \right]$$

subject to an initial K_0 , the law of motion of capital, and where the expectation E^* is taken with respect to the risk-neutral pricing measure. Consider first the after-tax dividends paid by a firm facing carbon taxes:

$$(A-23) \quad Div_t^\delta = \underbrace{[(1 - \delta_t)F(K_t) - g(i_t, K_t) - \Delta Cash_t]}_{z_t} + \mathbf{1}_{z_t < 0} h(z_t).$$

The first term in the brackets corresponds to the revenue from production net of the carbon tax. The second term corresponds to the cost of investment and the third term is the change in cash reserves of the firm. The last term captures financial frictions as an arbitrary cost $h(\cdot) \leq 0$ of paying negative dividends (i.e., of bringing a dollar from outside the firm inside the firm). By assumption, the firm does not need to keep cash reserves as it can costlessly raise fresh cash (i.e., $h(\cdot) = 0$) due to the absence of financial frictions. Moreover, cash is not a state variable, and $\Delta Cash_t = 0$. Therefore, the after-tax dividends simplify to $Div_t^\delta = (1 - \delta_t)F(K_t) - g(i_t, K_t)$.

Next, consider the case of a dollar dividend tax faced by households. In this case:

$$(A-24) \quad Div_t^\tau = \underbrace{[F(K_t) - g(i_t, K_t) - \Delta Cash_t]}_{z_t} + \mathbf{1}_{z_t < 0} h(z_t) - \tau_t.$$

Again, in the absence of financial frictions, $h(\cdot) = 0$, and it is optimal for the firm to set $\Delta Cash_t = 0$. Therefore, $Div_t^\tau = Div_t^\delta$ if and only if $\tau_t = F(K_t)\delta_t$. As a result, the firm would be solving the same optimization problem and therefore would choose the same investment policy. Finally, we note that in our article, in order to endogenously derive the first-best investment rate, we imposed more structure: CRTS in production $F(K)$, homogeneity of degree one in investment costs $g(i, K)$, and the investment income tax was in percentage. Under such additional structure, firm value is linear in K , and the equivalence result from equation (28) in the body of the article follows from $\tau_t = F(K_t)\delta_t$ obtained above.

Supplementary Material

To view supplementary material for this article, please visit <http://doi.org/10.1017/S0022109024000267>.

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