PRECESSION AND NUTATIONS OF MARS CALCULATED WITH KINOSHITA'S MODEL

S. BOUQUILLON AND J. SOUCHAY
Observatoire de Paris
Paris, France

Abstract. This paper presents the theoretical study of precession and nutation of Mars in a rigourous way. For this work we choose a natural reference system, based on the concept of non-rotating origin, and the appropriate canonical variables. Then, we solve the equations of the problem by taking into account the effects of the Sun, the Earth, Jupiter, and satellites of Mars, Phobos and Deimos.

1. Introduction

Theoretical study of the rotation of planets, other than the Earth, has started rather recently (1950). Their measurements are also much less accurate than for the Earth. On the contrary, the Earth's rotation has been studied since a long time (precession motion has been known since the time of Hipparchus), and, now, its rotation parameters are known at the submilliarcsecond level (Kinoshita and Souchay, 1990). That is why we are interested in applying the Earth's model of rotation to the other planets. This work might help to determine some physical values of Mars as the term Mr^2/C.

We have some reasons to consider initially the rotation of Mars. First, it is a telluric planet like the Earth. Second, thanks to Viking data, Mars is the planet whose rotation elements are the best known. In the following, we present the results obtained for Mars with Kinoshita's theory (1977) for a rigid Earth.

2. Method

2.1. REFERENCE SYSTEM AND VARIABLES

Our study is based on the following choices :

• We take as a reference plane for the measurement of precession-nutation the mean plane of Mars' orbit. Conventionally, the IAU reference sytem for

I. M. Wytrzyszczak, J. H. Lieske and R. A. Feldman (eds.),
Dynamics and Astrometry of Natural and Artificial Celestial Bodies, 325, 1997.

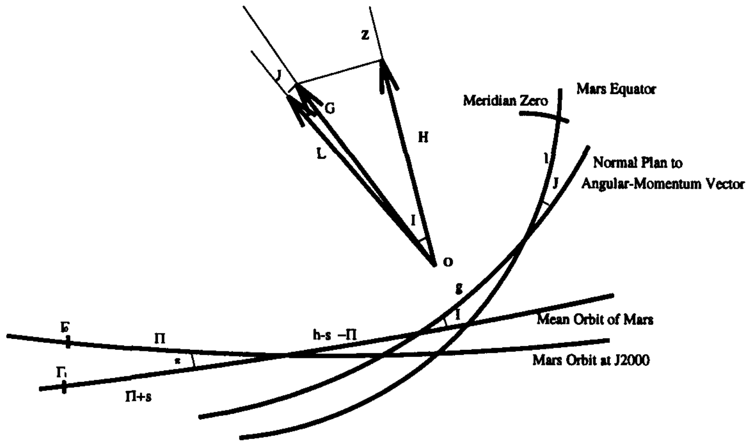

Figure 1. Reference system.

the Earth's precession is the mean orbit of the Sun (mean ecliptic). So, we choose the equivalent plane for Mars. Its main advantage is to give a physical sense to our results – that is to say our precession-nutation terms show the variations between the direction of the martian pole and the plane of the apparent orbit of the Sun at the same time. These variations are for example necessary for the climatic study of Mars. The choice of this non-inertial plane introduces an additional term in the Hamiltonian of the system without complicating significantly the calculations.

• We choose as an inertial plane Mars' orbit plane at the date J2000. This choice reduces the amplitude of the additional term E.

• We take as an origin on the reference plane the point called "non-rotating origin" (Guinot, 1979; Capitaine *et al.*, 1986), which we call Γ_t. This point determines a triad ($O\Gamma_t$, OY, OZ) verifying the condition of non rotation around OZ, at any instant t. So, it is the ideal point to measure any displacement along the mean orbit of Mars, since it is the displacement of the plane of Mars' equator which characterizes the precession-nutation of the planet.

Our variables are the three angle variables (g, h, l) defined on Figure 1 and the three action variables (G, H, L) defined as follows :

G is the amplitude of the angular momentum.

H is the component of the angular momentum along the axis perpendicular to Mars' mean orbit plane, $(H = G\cos I)$.

L is the component of the angular momentum along the axis perpendicular to Mars' equator plane, $(L = G\cos J)$.

These variables are canonical and conjugate. We are particulary interested in the two variables h and I, which determine the motion of precession-nutation respectively in longitude and in obliquity.

2.2. HAMILTONIAN AND EQUATIONS OF MOVEMENT

The Hamiltonian K can be written with the canonical variables defined on Figure 1:

$$K = F + U_i + E \tag{1}$$

with

$$F = \frac{1}{2}\left(\frac{\sin l^2}{A} + \frac{\cos l^2}{B}\right)(G^2 - L^2) + \frac{1}{2C}L^2 \tag{2}$$

$$U_i = \frac{k^2 M_i}{r_i^3}\left[\left(\frac{2C - A - B}{2}\right) P_2(\sin\delta_i) + \left(\frac{A-B}{4}\right) P_2^2(\sin\delta_i)\cos 2\alpha_i\right] \tag{3}$$

$$E = G\sin I\left(\sin\pi\cos(h-\Pi)\dot{\Pi} - \sin(h-\Pi)\dot{\pi}\right) \tag{4}$$

The terms with the index i are related to the disturbing body i. r_i is the distance between the barycenter of the disturbing body and the barycenter of Mars. δ_i and α_i are respectively the latitude and the longitude marscocentrique of these bodies. A, B and C are the principal moments of inertia of Mars. The other terms are defined on Figure 1.

The free motion of Mars' rotation is related to F, which has no important effect on the angles h and I (Kinoshita, 1972). On the other hand we will take into account the additional term E and the term U_i, which have effects on the precession-nutation given by these two angles. The term U_i, due to the external body i, characterizes the forced motion.

After the determination of U_i and E, the angles h and I can be obtained by the integration of the following equations :

$$\dot{h} = \frac{\partial K}{\partial H} = -\frac{1}{G\sin I}\frac{\partial K}{\partial I} \simeq -\frac{1}{G\sin I}\left(\frac{\partial E}{\partial I} + \frac{\partial U_i}{\partial I}\right) \tag{5}$$

$$\dot{I} = \frac{1}{G}\left(\frac{\partial K}{\partial h}\frac{1}{\sin I} - \frac{\partial K}{\partial g}\frac{1}{\tan I}\right) \simeq \frac{1}{G\sin I}\left(\frac{\partial E}{\partial h} + \frac{\partial U_i}{\partial h}\right) \tag{6}$$

3. Analytical Calculation and Results

3.1. EFFECTS DUE TO THE SUN

The Sun is the main external body, which acts on the martian rotation. By using ephemerides of Mars (here VSOP87 rectangular), the term U_{sun} can be expressed as a function of the variable h defined on Figure 1 and of the λ_j, which are the mean longitudes of the eight main planets. Then, we can resolve a part of equations (5) and (6). The results are in Table 1.

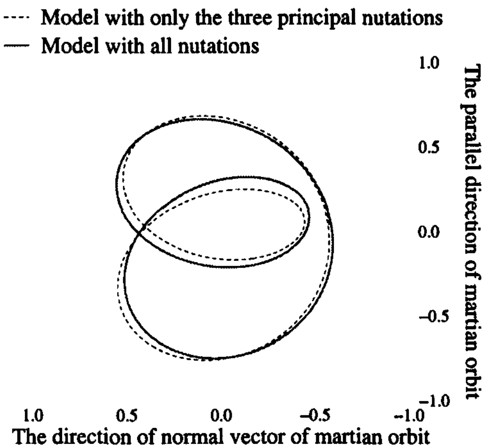

Figure 2. Two-dimensional motion of the martian pole without precession.

3.2. EFFECTS DUE TO THE ADDITIONAL TERM E

As the equations (5) and (6) point it out, the additional term E has to be considered. E depends on π and Π (4). Thanks to the elements Ω and i (respectively the longitude of the node of Mars' orbit on the J2000 ecliptic and its inclination) we can determine the values π and Π. Ω and i are themselves determined with the help of the ephemerides of Mars (Bretagnon, 1982). So, we obtain the following results in seconds per century:

$$\pi = 45.118T + 0.0519785T^2 + 0.0000915799T^3 \tag{7}$$

$$\Pi = -598213.01 + 815.904T + 1.38842T^2 \tag{8}$$

and the partial derivative of E with respect to h and I are:

$$\frac{\partial E}{\partial I} = -G \cos I \left(\dot{\pi} \sin (h - \Pi) - \dot{\Pi} \sin \pi \cos (h - \Pi) \right) \tag{9}$$

$$\frac{\partial E}{\partial h} = \left(-\dot{\Pi} \sin \pi \sin (h - \Pi) - \dot{\pi} \cos (h - \Pi) \right) G \sin I \tag{10}$$

The variations of all these angles, necessary to calculate $\dot{h}$ and $\dot{I}$, [see equations (5), (6)], are very small: h, π and Π have the respective periods 1.71×10^5, 2.87×10^6 and 1.59×10^5 years. So, these terms can be considered as constant for a short period of time. With this approximation we can calculate the influence of the additional term E on the angles h and I. Equations (11) and (12) express respectively the variation of longitude and the variation of obliquity of Mars with respect to the moving Mars orbit of t.

TABLE 1. Precession-nutation of Mars obtained with semi-analytical way.

Nutation of Mars due to the Sun			
arguments	period [days]	Δh ["] (sin)	ΔI["] (cos)
$2M^{(1)} + 2\Lambda^{(2)}$	343.49	1.09512	0.51532
M	687.00	−0.63358	0.
$3M + 2\Lambda$	228.99	0.23932	0.11261
$M + 2\Lambda$	686.99	−0.10446	−0.04915
2M	343.50	−0.04425	0.
$4M + 2\Lambda$	171.75	0.04069	0.01915
$5M + 2\Lambda$	137.40	0.00616	0.00296
3M	229.00	−0.00404	0.
$6M + 2\Lambda$	114.50	0.00092	0.00043
$\lambda_{Ma} - \lambda_{Ju}$	816.43	−0.000412	0
$4\lambda_{Ea} - 10\lambda_{Ma} + 3\lambda_{Ju}$	−343.3	0	0.000142(sin)
Precession and quadratic variation			
precession term		$-7.57845T - 0.00101T^2$ ["/year]	0

(1) M is the mean anomaly of Mars.

(2) Λ is the angle between Γ_t and the Mars perigee

$$\dot{h}_E = \dot{\pi} \cot I \sin (h - \Pi) = 0\overset{''}{.}22924/\text{year} \tag{11}$$

$$\dot{I}_E = -\dot{\pi} \cos (h - \Pi) = 0\overset{''}{.}43810/\text{year} \tag{12}$$

3.3. EFFECTS DUE TO PHOBOS, DEIMOS, THE EARTH AND JUPITER

Phobos and Deimos, the two satellites of Mars, are bodies of very small dimension, respectively 11.1 and 6.2 km for the mean radius. But, they are close to Mars, for their distances correspond approximately to 2.76 and 6.91 radii of Mars. So, we want to estimate their effects on the precession of Mars. On the other hand the Earth and Jupiter have a mass big enough to influence the martian rotation. We have carried out calculations as for the Sun. For Phobos and Deimos, we use the series ESAPHO and ESADE (Chapront-Touzé, 1990). For the planets Mars, Jupiter and the Earth, we use VSOP87 in rectangular coordinates. Results of these calculations are presented in Table 2.

4. Conclusion

As a conclusion, we can remark that the martian precession-nutation terms greater than 10^{-3} arcseconds come from the Sun's influence and, for two

TABLE 2. Phobos, Deimos, Jupiter and Earth influence on the variables h and I. Periods are in days, nutations in arcseconds and the precession in arcseconds per year.

Phobos' influence				Deimos' influence			
argument	period	Δh(sin)	ΔI(cos)	argument	period	Δh (sin)	ΔI(cos)
$N_{Pho}{}^{(1)}$	−825.64	−0.01206	0	$N_{Dei}{}^{(1)}$	−19998	−0.00439	0
$N_{Pho} - \dot{h}^{(2)}$	−825.65	0	−0.00358	$N_{Dei} - \dot{h}$	−20004	0	−0.00133
$N_{Pho} - 2\dot{h}$	−825.66	0	−0.00089	$N_{Dei} - 2\dot{h}$	−20011	0	−0.00031
$N_{Pho} + \dot{h}$	−825.63	0	−0.00062	$N_{Dei} + \dot{h}$	−19992	0	−0.00021
precession	.	−0.00028	0	precession	.	−0.00025	0
Direct Jupiter's influence				Direct Earth's influence			
argument	period	Δh (sin)	ΔI(cos)	argument	period	Δh (sin)	ΔI(cos)
$2\lambda_{Ju} - 2\dot{h}$	2166.1	−0.00019	−0.00009	$\lambda_{Ea} - 4\lambda_{Ma} + 2\dot{h}$	−2881.7	0.00008	−0.00004
				$\lambda_{Ea} - 2\lambda_{Ma}$	−5760.1	0.00014	0
precession	.	−0.00022	0	precession	.	−0.00008	0.

(1) N_{Pho}, N_{Dei} are the nodes of Phobos and Deimos on the equatorial plane of Mars.

(2) $\dot{h} \approx$-7.″5$/year$ is the constant of Mars precession.

nutation terms, from Phobos and Deimos. Indirect effects due to the planets are higher than direct effects, but all these effects are smaller than $5 \cdot 10^{-4}$ arcseconds. At last it is important to insist on the present lack of precision of the term ($Mr^2/C \simeq 2.727$), higher than 5%. Nowadays, observational techniques cannot reduce this lack of precison. So, this term is determined only in a theoretical way and this problem limits our investigation.

References

Bretagnon, P.: 1982, "Théorie du mouvement de l'ensemble des planétes. Solution VSOP82*", *Astron. Astrophys.* **114**, 278.

Capitaine, N., Guinot, B., and Souchay, J.: 1986, "A non-rotating origin on the instantaneous equator: Definition, properties and use", *Celest. Mech.* **39**, 283.

Chapront-Touzé, M.: 1990, "Orbits of the Martian satellites from ESAPHO and ESADE theories", *Astron. Astrophys.* **240**, 159.

Guinot, B.: 1979, "Basic problems in the kinematics of the rotation of the Earth", in: *Time and the Earth's Rotation 7-8* (D.D. McCarthy, J.D. Pilkington, eds), Reidel, Dordrecht.

Hilton, J.L. : 1991, "The motion of Mars' pole. 1. Rigid body precession and nutation", *Astron. J.* **102**, 1510.

Kinoshita, H.: 1972, "First order perturbations of the two finite body problem", *Publ. Astron. Soc. Japan* **24**, 423.

Kinoshita, H.: 1977, "Theory of the rotation of the rigid Earth", *Celest. Mech.* **26**, 296.

Kinoshita, H. and Souchay, J.: 1990, "The theory of the nutation for the rigid Earth model at the second order", *Celest. Mech.* **48**, 187.

https://doi.org/10.1017/S0252921100046753 Published online by Cambridge University Press