DEAR EDITOR,

I read with interest Tony Gardiner's comparison of a ULEAC Higher Tier GCSE paper with a UCLES O Level paper (*Math. Gaz.* July 1998). I agree with most of the criticisms he made of the GCSE questions, yet there is one relevant factor to which he did not draw attention, and of which he may be unaware. Although the GCSE paper 5 he looked at is for the Higher Tier, aimed at candidates who should achieve grades A* to C, there is significant overlap in content with the Intermediate Tier paper 3, aimed at a highest grade of B. Several of the same questions appear on both papers, and these are usually the earlier questions on paper 5. Thus the first ten questions he sampled would not be representative, in all respects, of the paper as a whole.

In fact I don't think this invalidates his criticisms. After all, the questions do form a significant proportion of the paper. Why the Board chooses to lay out its papers this way, I am not sure. Presumably it helps them draw comparable grade boundaries for the different papers. Yet the consequence is that a significant proportion of the marks are assigned to work below the appropriate level for many of the candidates. The difference between an A and A* will depend on the answers to a very small part of the paper, such a candidate having to score very highly on all the mundane work which comes first, with little margin for error.

Yours sincerely,

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DEAR EDITOR,

Solution 3 of Problem 1997.4 given by Noriko Omiya and Florian Herzig on page 476 of the November 1997 of this *Gazette* falls a single step short of being a most elegant solution. Their approach could also have led to beautiful identities involving (that hold for) the angles A, B, C of an arbitrary triangle. For such A, B, C,

$$\cos A + \cos B + \cos C = \cos A + \cos B - \cos(A+B)$$

$$= 2\cos\frac{A+B}{2}\cos\frac{A-B}{2} - \left[2\cos^2\frac{A+B}{2} - 1\right]$$

$$= 1 + 2\cos\frac{A+B}{2}\left[\cos\frac{A-B}{2} - \cos\frac{A+B}{2}\right]$$

$$= 1 + 2\cos\frac{A+B}{2}\left[2\sin\frac{A}{2}\sin\frac{B}{2}\right]$$

$$= 1 + 4\sin\frac{C}{2}\sin\frac{A}{2}\sin\frac{B}{2}$$

because $(A+B+C)/2 = 90°$.

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Thus we have the elegant identity

$$\cos A + \cos B + \cos C = 1 + 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}. \quad (1)$$

In a similar way, one can obtain

$$\sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}. \quad (2)$$

From (1), one sees that the expression $\cos A + \cos B + \cos C$ is greater than 1 and that it attains its maximum when $A = B = C$. This follows from the fact that if $x > y$, then

$$\begin{aligned} 2 \sin x \sin y &= \cos (x - y) - \cos (x + y) \\ &< 1 - \cos (x + y) = 2 \sin^2 \frac{x + y}{2} \end{aligned}$$

Thus

$$1 < \cos A + \cos B + \cos C \leqslant 1 + 4 \sin^3 30° = 3/2. \quad (3)$$

Also, it is clear from (2) and from the fact that if $x > y$, then

$$\begin{aligned} 2 \cos x \cos y &= \cos (x - y) + \cos (x + y) \\ &< 1 + \cos (x + y) = 2 \cos^2 \frac{x + y}{2} \end{aligned}$$

that the expression $\sin A + \sin B + \sin C$ is greater than 0 and that it attains its maximum when $A = B = C$.

Thus

$$0 < \sin A + \sin B + \sin C \leqslant 4 \cos^3 30° = 3\sqrt{3}/2. \quad (4)$$

That the bounds in (3) and (4) are the best possible is quite trivial.

References

1. Problem 1997.4, *Math. Gaz.* **81** (1997) pp. 474-476.

Yours sincerely,

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