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Zach Norwood, ***The Combinatorics and Absoluteness of Definable Sets of Real Numbers***, University of California, Los Angeles, USA, 2018. Supervised by Itay Neeman. MSC: 03E60, 03E57.

Abstract

This thesis divides naturally into two parts, each concerned with the extent to which the theory of $L(\mathbf{R})$ can be changed by forcing.

The first part focuses primarily on applying generic-absoluteness principles to how that definable sets of reals enjoy regularity properties. The work in Part I is joint with Itay Neeman and is adapted from our paper *Happy and mad families in* $L(\mathbf{R})$, JSL, 2018. The project was motivated by questions about *mad families*, maximal families of infinite subsets of ω of which any two have only finitely many members in common. We begin, in the spirit of Mathias, by establishing (Theorem 2.8) a strong Ramsey property for sets of reals in the Solovay model, giving a new proof of Törnquist's theorem that there are no infinite mad families in the Solovay model.

In Chapter 3 we stray from the main line of inquiry to briefly study a game-theoretic characterization of filters with the Baire Property.

Neeman and Zapletal showed, assuming roughly the existence of a proper class of Woodin cardinals, that the boldface theory of $L(\mathbf{R})$ cannot be changed by proper forcing. They call their result the Embedding Theorem, because they conclude that in fact there is an elementary embedding from the $L(\mathbf{R})$ of the ground model to that of the proper forcing extension. With a view toward analyzing mad families under AD^+ and in $L(\mathbf{R})$ under large-cardinal hypotheses, in Chapter 4 we establish *triangular* versions of the Embedding Theorem. These are enough for us to use Mathias's methods to show (Theorem 4.5) that there are no infinite mad families in $L(\mathbf{R})$ under large cardinals and (Theorem 4.9) that AD^+ implies that there are no infinite mad families. These are again corollaries of theorems about strong Ramsey properties under large-cardinal assumptions and AD^+, respectively. Our first theorem improves the large-cardinal assumption under which Todorcevic established the nonexistence of infinite mad families in $L(\mathbf{R})$. Part I concludes with Chapter 5, a short list of open questions.

In the second part of the thesis, we undertake a finer analysis of the Embedding Theorem and its consistency strength. Schindler found that the the Embedding Theorem is consistent relative to much weaker assumptions than the existence of Woodin cardinals. He defined *remarkable cardinals*, which can exist even in L, and showed that the Embedding Theorem is equiconsistent with the existence of a remarkable cardinal. His theorem resembles a theorem of Harrington–Shelah and Kunen from the 1980s: the absoluteness of the theory of $L(\mathbf{R})$ to ccc forcing extensions is equiconsistent with a weakly compact cardinal. Joint with Itay Neeman, we improve Schindler's theorem by showing that absoluteness for σ-closed $*$ ccc posets—instead of the larger class of proper posets—implies the remarkability of $\aleph_1^V$ in L. This requires a fundamental change in the proof, since Schindler's lower-bound argument uses Jensen's reshaping forcing, which, though proper, need not be σ-closed $*$ ccc in that context. Our proof bears more resemblance to that of Harrington–Shelah than to Schindler's.

The proof of Theorem 6.2 splits naturally into two arguments. In Chapter 7 we extend the Harrington–Shelah method of coding reals into a specializing function to allow for trees with uncountable levels that may not belong to L. This culminates in Theorem 7.4, which asserts that if there are $X \subseteq \omega_1$ and a tree $T \subseteq \omega_1$ of height ω_1 such that X is codable along T (see Definition 7.3), then $L(\mathbf{R})$-absoluteness for ccc posets must fail.

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We complete the argument in Chapter 8, where we show that if in any σ-closed extension of V there is no $X \subseteq \omega_1$ codable along a tree T, then $\aleph_1^V$ must be remarkable in L.

In Chapter 9 we review Schindler's proof of generic absoluteness from a remarkable cardinal to show that the argument gives a level-by-level upper bound: a strongly λ^+-remarkable cardinal is enough to get $L(\mathbf{R})$-absoluteness for λ-linked proper posets.

Chapter 10 is devoted to partially reversing the level-by-level upper bound of Chapter 9. Adapting the methods of Neeman, *Hierarchies of forcing axioms II*, we are able to show that $L(\mathbf{R})$-absoluteness for $|\mathbf{R}| \cdot |\lambda|$-linked posets implies that the interval $[\aleph_1^V, \lambda]$ is Σ_1^2-remarkable in L.

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Sandro Márcio da Silva Preto, ***Semantics modulo satisfiability with applications: function representation, probabilities and game theory***, Institute of Mathematics and Statistics, University of São Paulo, São Paulo, Brazil, 2021. Supervised by Marcelo Finger. MSC: 03B70, 68T27, 60A86, 91A40. Keywords: Coherence of constraints, formal methods, function representation, Łukasiewicz infinitely-valued logic, Nash equilibrium, neural networks, non-classical probabilities, piecewise linear functions, probabilistic constraints, probabilistic satisfiability, propositional logics, rational McNaughton functions, uncertain games, valuation semantics.

Abstract

In the context of propositional logics, we apply semantics modulo satisfiability—a restricted semantics which comprehends only valuations that satisfy some specific set of formulas—with the aim to efficiently solve some computational tasks. Three possible such applications are developed.

We begin by studying the possibility of implicitly representing rational McNaughton functions in Łukasiewicz Infinitely-valued Logic through semantics modulo satisfiability. We theoretically investigate some approaches to such representation concept, called representation modulo satisfiability, and describe a polynomial algorithm that builds representations in the newly introduced system. An implementation of the algorithm, test results and ways to randomly generate rational McNaughton functions for testing are presented. Moreover, we propose an application of such representations to the formal verification of properties of neural networks by means of the reasoning framework of Łukasiewicz Infinitely-valued Logic.

Then, we move to the investigation of the satisfiability of joint probabilistic assignments to formulas of Łukasiewicz Infinitely-valued Logic, which is known to be an NP-complete problem. We provide an exact decision algorithm derived from the combination of linear algebraic methods with semantics modulo satisfiability. Also, we provide an implementation for such algorithm for which the phenomenon of phase transition is empirically detected.

Lastly, we study the game theory situation of observable games, which are games that are known to reach a Nash equilibrium, however, an external observer does not know what is the exact profile of actions that occur in a specific instance; thus, such observer assigns subjective probabilities to players actions. We study the decision problem of determining if a set of these probabilistic constraints is coherent by reducing it to the problems of satisfiability of probabilistic assignments to logical formulas both in Classical Propositional Logic and Łukasiewicz Infinitely-valued Logic depending on whether only pure equilibria or also mixed equilibria are allowed. Such reductions rely upon the properties of semantics modulo satisfiability. We provide complexity and algorithmic discussion for the coherence problem and, also, for the problem of computing maximal and minimal probabilistic constraints on actions that preserves coherence.