Macroeconomic Dynamics (2023), **27**, pp. 2056–2085
doi:10.1017/S1365100522000591

ARTICLE

Income taxation and job creation

Johanna Röhrs

University of Regensburg, Regensburg, Germany
Email: johanna.roehrs@ur.de

Abstract
This paper augments the DMP model with large firms and intrafirm wage bargaining by an endogenous decision to become an entrepreneur that is based on heterogeneous entrepreneurial abilities. If workers' wage bargaining power is not too large and the match efficiency is not too low, the decentralized market equilibrium features an inefficiently high number of entrepreneurs, because they appropriate large parts of the surplus from matches. A realistic calibration with empirically plausible parameters shows this case to be the relevant one. Consequently, introducing a tax on the profits of entrepreneurs restores the constrained first-best allocation by affecting occupational choices. It drives rather unproductive entrepreneurs out of the market since the marginal entrepreneur is affected and not the average one. Thus, the negative effects on job creation are small.

Keywords: Optimal taxation; imperfect labor markets; job creation; entrepreneurship

1. Introduction

When talking about the taxation of entrepreneurs or job creators, a common fear is that higher taxation might lead to job cuts or less business creation. Moreover, if entrepreneurs are very innovative, taxation might lead to less innovation and is harmful for growth. If the trickle-down effect matters, it is beneficial for the whole economy to not tax entrepreneurs heavily, since they create jobs and help reduce unemployment. The question that arises is whether entrepreneurs' profits suitably reflect their economic contribution or whether these profits might be too high because they can acquire a too large share of output. If the latter is the case, taxation of entrepreneurial profits is justified even from an efficiency perspective. To satisfyingly answer the question whether higher tax rates for entrepreneurs can be welfare-enhancing, it is important to make realistic assumptions about the labor market. We therefore use the labor matching model developed by Diamond, Mortensen, and Pissarides (DMP Model)[1] augmented with heterogeneous agents and endogenous occupational choices. Moreover, by assuming that wage bargaining takes place according to Stole and Zwiebel (1996), the paper also extends the literature on intrafirm wage bargaining in matching models [Cahuc and Wasmer (2001), Cahuc et al. (2008)] by adding heterogeneous entrepreneurial abilities and taxation. Individuals are endowed with an entrepreneurial ability and can decide to become a regular worker or an entrepreneur. In equilibrium, every worker in regular employment earns a wage independent of her entrepreneurial talent, whereas entrepreneurs with higher ability employ more workers and earn higher profits.

In the first section, we describe the model setup, the firms' optimal hiring decisions, and vacancy posting and the occupational choice that each individual must face. Next, the social planner's maximization problem is outlined and the first-best allocation is derived. We obtain an equation stating how the planner sets the optimal threshold ability. Individuals with lower entrepreneurial ability become workers and individuals with an ability above the threshold become entrepreneurs. We then compare the outcome in the market with the social planner's solution and show that the market equilibrium is in general not efficient. Hereby and in the following sections, we focus on steady states to make the problem more tractable. Inefficiencies in the market can arise for the following two reasons: First, entrepreneurs post too many vacancies if their private return of a match exceeds the social return and if they can suppress wages by hiring more workers. The inefficiencies caused if the Hosios condition does not hold have been discussed extensively in the literature[2]. But even when the Hosios condition holds, entrepreneurs may post too many vacancies since overhiring decreases the wage that is paid to the employees like in Stole and Zwiebel (1996). Second, there are too many entrepreneurs if they can acquire a significant fraction of the surplus of a match. The occupational choice of individuals creates a new inefficiency that has not been treated by the literature so far. Even if the number of jobs created by entrepreneurs is efficient, the decision to become an entrepreneur can still be inefficient. Whether too many or too few individuals decide to become an entrepreneur depends on the relative value of becoming a regular worker or an entrepreneur and is ultimately a quantitative question. If entrepreneurs are able to acquire a too large share of the surplus of a match and workers thus receive inefficiently low wages, it is more profitable for individuals with low entrepreneurial abilities to become an entrepreneur. Thus, if entrepreneurs and regular workers are drawn from the same pool of individuals, occupational choices might be inefficient and exert externalities. By becoming an entrepreneur, individuals do not take into account that it becomes more difficult for the other, potentially more able, entrepreneurs to fill their vacancies.

Therefore, when introducing taxation, it is important to consider these two margins of market failure simultaneously. A first tax instrument has to be designed to make hiring efficient. Therefore, vacancy posting needs to be taxed such that overhiring workers become more expensive and firms post the first-best number of vacancies. A second tax rate or subsidy on entrepreneurial incomes should affect the decision to become an entrepreneur without distorting the vacancy posting choice of existing firms. If the worker's wage bargaining power is not too large, too many individuals decide to become an entrepreneur and we show that there is a tax on the profits of entrepreneurs that allows to restore the first-best with fewer entrepreneurs than in the market equilibrium. If instead the worker's wage bargaining power is very large or the matching efficiency is low, too few individuals open up a firm, and entrepreneurial incomes must be subsidized to restore the first-best. We calculate the tax rate (or subsidy) on entrepreneurial profits by comparing the equilibrium condition for the optimal number of entrepreneurs in a social planner setting with the condition in the decentralized market equilibrium. Whether entrepreneurial incomes must be taxed or subsidized thus depends on the wage bargaining power and match efficiency. Under a realistic calibration, the laissez-faire market equilibrium features too many entrepreneurs since they can acquire large parts of the surplus of a match. The taxation of profits from job creation is efficiency enhancing since it makes being an entrepreneur less attractive and reduces the number of individuals that decide to become an entrepreneur. Moreover, it crowds out only the least productive firms whereas entrepreneurs with high entrepreneurial ability are not affected in their decision to open up a firm. Efficiency increases if those entrepreneurs who are unproductive relative to others become workers. Furthermore, we demonstrate that the introduction of income taxation does not distort the vacancy posting and hiring decisions of the remaining more productive entrepreneurs.

To sum up, we find efficiency reasons that support a higher taxation of incomes from job creation, at least under a realistic parametrization, whereas most of the literature argues that entrepreneurship should be subsidized. When thinking about the taxation of entrepreneurial incomes, the argument is that it is also important to consider that rather low-productivity entrepreneurs might start a firm if they have the possibility to extract large parts of the surplus of a match. This leads to an inefficiently large number of "not so able" entrepreneurs who compete with more productive firms for workers. The analysis in this paper can therefore also be understood as a word of caution not to exaggerate the support of entrepreneurship since this could have adverse effects on welfare. The numerical simulation of the model shows that for realistic parameter values the tax rate on entrepreneurial income is around 30%. The result of having too many entrepreneurs in the decentralized market reverses if the worker's wage bargaining power is larger than 0.2 meaning that entrepreneurs can acquire less than 80% of the surplus of a match.[3]

This paper contributes to the literature that analyses taxation in search models. In a model setup similar to mine, with search and scarce entrepreneurial talent, Boone and Bovenberg (2002) analyze under which circumstances workers or entrepreneurs can reap surpluses and derive how different labor supply and labor demand elasticities influence the optimal tax system. They find that the labor market tightness should not be distorted by taxation. Hungerbühler et al. (2006) derive optimal tax rates in a matching model with directed search and workers with different skill levels but without endogenous occupational choices. They find positive marginal tax rates even at the top of the income distribution and larger marginal taxes compared to a Mirrlees setting. Lehmann et al. (2011) use the same model as Hungerbühler et al. (2006) but introduce individuals with heterogeneous values of nonmarket activities and study the influence of participation decisions on optimal redistributive taxation. They find a progressive tax schedule if the participation elasticities decrease with increasing skill levels. Furthermore, the calculated marginal tax rates are higher than in a framework with competitive labor markets without frictions. An overview of further literature on income taxation within the search and matching model framework can be found in Boadway and Tremblay (2013).

Moreover, the paper contributes to the literature dealing with the effects of taxation on innovation and entrepreneurship. There are several papers that find sizeable negative effects of high top tax rates on innovation activity and the location choice of innovators [see, e.g. Akcigit et al. (2016, 2022), Akcigit and Stantcheva (2020)]. Another strand considers the role of entrepreneurs in the top income distribution since entrepreneurs are highly represented in the group of top income earners. These papers include entrepreneurial activity in general equilibrium models to replicate the empirically observed income and wealth distributions with high concentrations at the top and analyze optimal taxation within these frameworks [see, e.g. Quadrini (2000), Cagetti and De Nardi (2006), Brueggemann (2021)]. In the mentioned papers, borrowing constraints limit the number of entrepreneurs whereas labor markets are complete in the sense that wages equal marginal products. The contribution of my paper is the focus on an endogenous occupational choice in imperfect labor markets where wages deviate from marginal products, which yields the possibility of rent-seeking and therefore an inefficient allocation of resources.

There is some evidence in the literature that parts of the incomes of top earners are caused by rent-seeking behavior. Bivens and Mishel (2013), for example, argue that the increase in incomes of the top 1 percent since the 1980s is largely caused by the creation or redistribution of rents. In their opinion, very high incomes are not just efficient marginal returns to specific skills or high ability, as is claimed, for example, by Kaplan and Rauh (2013). If this holds true, higher taxes on high-income earners might be justified and potentially welfare-enhancing since they reduce the returns to rent-seeking. Rothschild and Scheuer (2016) calculate Pareto optimal income taxes when agents can work either in a traditional sector, where private and social returns coincide, or in a rent-seeking sector which exercises externalities. In contrast to the above-mentioned papers, we do not need to make the assumption that individuals behave in some way of rent-seeking. The appropriation of surpluses is included in the model through the way how firms and workers match and bargain about wages. Therefore, it provides some micro-foundation for rent-seeking.

The remainder of the paper is organized as follows. Section 2 describes the setup of the theoretical model, Section 3 examines the social planner's allocation, and Section 4 analyses the market equilibrium's inefficiency. Taxation and its effects on the efficient allocation are described in Section 5, and the results of a numerical simulation can be found in Section 6. Finally, the last section provides a brief conclusion.

https://doi.org/10.1017/S1365100522000591 Published online by Cambridge University Press

2. Model setup

We consider a closed economy with a continuum of individuals who live forever. Their mass is normalized to one. Every individual has an entrepreneurial ability that is denoted by a. The cumulative distribution function of this ability is given by $\Phi(a)$ and the density by $\phi(a)$.

Individuals can decide at the beginning of each period whether they want to become entrepreneurs or workers. If they decide to become a worker, they can either be employed or unemployed. An employed worker earns the wage w_t, which is independent of the worker's talent for job creation, and an unemployed worker receives home production z.

If an individual instead decides to become an entrepreneur, she starts a firm and must decide how many workers l_t to hire and how many vacancies v_t to open in every period. The cost of posting a vacancy γ is constant over time. As an outcome, there is a cutoff level $\bar{a}_t$ for which all individuals with talent $a \geq \bar{a}_t$ become entrepreneurs and the less talented ones become workers. An entrepreneur with talent a posts $v_t(a)$ vacancies and hires $l_t(a)$ workers. The marginal entrepreneur with entrepreneurial talent $\bar{a}_t$ then opens $v_t(\bar{a}_t)$ vacancies and employs $l_t(\bar{a}_t)$ workers. The number of all vacancies in the economy in period t therefore is $V_t = \int_{\bar{a}_t}^{\infty} v_t(a)d\Phi(a)$.

In aggregate, there are $\Phi(\bar{a}_t)$ workers who can be divided into employed workers L_t and unemployed workers N_t. $1 - \Phi(\bar{a}_t)$ then gives the number of entrepreneurs because the number of individuals in the economy is normalized to one. There is a resource constraint on the supply of labor. The fraction engaged in creating jobs plus the fraction engaged as employees must be smaller or equal to one in each period : $1 - \Phi(\bar{a}_t) + L_t \leq 1$. This can also be written as $L_t \leq \Phi(\bar{a}_t)$, which means that labor demand must be smaller or equal to labor supply. Consequently, the aggregate number of employed workers in period t can be written as $L_t = \int_{\bar{a}_t}^{\infty} l_t(a)d\Phi(a) = \Phi(\bar{a}_t) - N_t$ and the unemployed are $N_t = \Phi(\bar{a}_t) - L_t$.

How firms and workers come together and form a match is described by the matching technology $m(N_t, V_t)$. It gives the number of aggregate contacts between the mass of vacancies and unemployed workers. The matching function is assumed to be homogeneous of degree one.

The tightness of the labor market is denoted with $\theta_t = \frac{V_t}{N_t}$. The probability to fill an open vacancy per unit of time can thus be written as $\frac{m(N_t,V_t)}{V_t} = q(\theta_t)$ with $q'(\theta_t) < 0$. If the labor market tightness increases, it gets more difficult for entrepreneurs to fill their vacancies. Moreover, the probability of finding a new job is $\frac{m(N_t,V_t)}{N_t} = \theta_t q(\theta_t)$. It rises if the labor market gets less tight for the unemployed, so $\frac{d[\theta_t q(\theta_t)]}{d\theta_t} > 0$.

When workers and firms are matched, production y_t takes place according to a production function $y_t = af(l_t)$ with $f'(l_t) > 0$ and $f''(l_t) < 0$. A match between a worker and a firm does not have to last forever. There is an exogenous job destruction rate s that is assumed to be constant over time. Once a match is destroyed, the worker becomes unemployed, and the entrepreneur must post vacancies to hire a new worker. The dynamics of unemployment are therefore described as follows: $N_{t+1} = (1 - \theta_t q(\theta_t))N_t + s(\Phi(\bar{a}_t) - N_t)$. In every period, there are $\theta_t q(\theta_t)$ unemployed people who find a job and leave the unemployment pool, and there are s employees who lose their job and join the unemployed.

2.1 The regular worker

It is assumed that there is no storage technology and that individuals are risk neutral. An employee receives the wage w_t and an unemployed worker engages in home production z. The rate of time preference is denoted by β.

One can now set up the value equations for the different types of individuals. The value equation for an employed worker is the following:

$$W_t^e = w_t + \beta \left[sW_{t+1}^n + (1-s)W_{t+1}^e \right].$$

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It depends on the current wage and the future value of being a worker. The employed worker becomes unemployed with probability and stays employed with probability $1-s$.

The value of being an unemployed worker can be written as

$$W_t^n = z + \beta\left[\theta_t q(\theta_t) W_{t+1}^e + (1-\theta_t q(\theta_t)) W_{t+1}^n\right].$$

The unemployed worker engages in home production in the current period and knows that with probability $\theta_t q(\theta_t)$ she will be matched with a firm and will become employed in the next period. Otherwise, she stays in unemployment.

2.2 The firm

If an individual decides to become an entrepreneur, her value equation depends on the firm's profit, which is maximized by choosing the optimal number of workers and vacancies. After having posted the vacancies and being matched with workers, wage bargaining takes place. Since multiple workers bargain with a firm, the wage-setting is more complex than in the standard DMP model. If the production function exhibits decreasing returns in labor inputs, entrepreneurs can exploit the diminishing returns to manipulate wages as is shown by Stole and Zwiebel (1996). In their model, wage-setting is an ongoing process within the firm because contracts are non-binding and workers can quit the firm at any time. Therefore, it is optimal for firms to overhire workers and to decrease the marginal product of labor to lower the wages of the incumbent workers and acquire larger rents.

The firm's maximization problem for given $w_t(a,l_t)$ and $q(\theta_t)$ is the following:

$$W_t^f(a,l_t) = \max_{l_{t+1},v_t}\left\{af(l_t) - w_t(a,l_t)l_t - \gamma v_t + \beta W_{t+1}^f(a,l_{t+1})\right\}$$

$$\text{s.t. } l_{t+1} = (1-s)l_t + q(\theta_t)v_t.$$

Solving the maximization problem by using a Lagrangian function,[4] one obtains the job creation condition:

$$\frac{\gamma}{q(\theta_t)} = \beta\left[af'(l_{t+1}) - w_{t+1}(a,l_{t+1}) - \frac{\partial w_{t+1}(a,l_{t+1})}{\partial l_{t+1}}l_{t+1} + (1-s)\frac{\gamma}{q(\theta_{t+1})}\right]. \quad (1)$$

It states that the expected costs of hiring a worker have to be equal to the value generated by having an additional worker. A hired worker increases the firm's production by the marginal product of labor multiplied by the entrepreneur's ability minus the wage that is paid to him or her. The derivative of the wage with respect to labor multiplied by labor input is subtracted and reflects the benefits of overhiring. With probability $1-s$, the worker stays at the firm in the next period and the continuation value must be added. The job creation condition can be rearranged to

$$\gamma\left[\frac{1}{\beta q(\theta_t)} - \frac{(1-s)}{q(\theta_{t+1})}\right] = af'(l_{t+1}) - \frac{\partial w_{t+1}(a,l_{t+1})}{\partial l_{t+1}}l_{t+1} - w_{t+1}(a,l_{t+1}).$$

The term on the left-hand side does not depend on any of the choice variables of the firm. Therefore, the right-hand side must be equal across all firms in equilibrium and the following equation must hold:

$$af'(l_{t+1}(a)) - \frac{\partial w_{t+1}(a)}{\partial l_{t+1}}l_{t+1}(a) - w_{t+1}(a) = \bar{a}f'(l_{t+1}(\bar{a})) - \frac{\partial w_{t+1}(\bar{a})}{\partial l_{t+1}}l_{t+1}(\bar{a}) - w_{t+1}(\bar{a}).$$

The instantaneous marginal value of employing an additional worker is constant across firms because the average costs of employing an additional worker are the same for all entrepreneurs. It implies that the wage paid by all firms is the same in equilibrium and does not vary with entrepreneurial ability. There is no wage dispersion across firms.[5] If one rearranges the job creation condition and defines the surplus of having an additional worker as P_t, so that $P_{t+1} = \frac{\gamma}{\beta q(\theta_t)}$

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holds, one obtains

$$P_t = af'(l_t) - w_t(a, l_t) - \frac{\partial w_t(a, l_t)}{\partial l_l} l_t + \beta(1-s)P_{t+1}.$$

P_t is the value of having a match for the firm or the value of an occupied job. If there are no differences in wages and hiring costs across firms, more able entrepreneurs hire more workers so that the marginal product of labor is the same in each firm.

After having posted their vacancies according to the job creation condition, firms and workers are matched randomly according to the matching technology and they bargain about wages. Any wage-setting process is consistent with the model as long as the wage is not too low, so that the present value of being unemployed is not higher than the present value of being employed, or too high which makes the match unprofitable for the entrepreneur. It is assumed that wages are determined by the generalized Nash bargaining solution. Workers and entrepreneurs bargain about the surplus of the match. The worker's surplus of being employed consists of the difference between the value equations of being employed and unemployed. The entrepreneur bargains with the marginal worker and thus compares the value of having an additional worker P_t to the option of not employing that worker, which is zero. The wage is therefore determined as

$$w_t(a, l_t) = \arg\max(W_t^e - W_t^n)^{\xi} P_t^{1-\xi},$$

with the worker's bargaining power $\xi \in [0, 1]$. The wage must thus satisfy the first-order condition

$$\xi P_t = (1-\xi)(W_t^e - W_t^n). \tag{2}$$

As a result, the solution to the Nash wage bargaining[6] is the wage curve

$$w_t(a, l_t) = \xi \left[af'(l_t) - \frac{\partial w_t(a, l_t)}{\partial l_t} l_t + \gamma\theta_t \right] + (1-\xi)z. \tag{3}$$

The wage consists of the fraction ξ of the worker's marginal product, the derivative of the wage with respect to labor input, and the hiring costs plus a fraction $1-\xi$ of home production. Workers are therefore rewarded for the saving of vacancy posting costs since the firm does not have to pay it anymore after a match is formed.[7] The wage curve also includes the effect that the hiring of an additional worker exerts on wages.[8] For given employment, wages are higher in the DMP model with intrafirm wage bargaining than in the standard model because hiring an additional worker has a higher value for the firm since it decreases wages for the already employed workers within the firm. Nevertheless, since increasing employment reduces the wage bill, firms post more vacancies compared to a standard DMP model without intrafirm wage bargaining. The marginal product of workers declines with increased hiring, which in turn lowers the wages paid to them.

2.3 Occupational choice

The marginal entrepreneur is indifferent between being a worker or becoming an entrepreneur. The profit of the marginal entrepreneur with talent $\bar{a}_t$ has to equal her outside option of being a worker in every period: $W_t^f(l_t, \bar{a}_t) = W_t^e$.[9] Plugging in for the respective value equations leads to the indifference equation

$$\begin{aligned}\bar{a}_t f(l_t(\bar{a}_t)) - w_t l_t(\bar{a}_t) - \gamma v_t(\bar{a}_t) + \beta W_{t+1}^f(\bar{a}_t, l_{t+1}(\bar{a}_t)) \\ = w_t + \beta \left[s W_{t+1}^n + (1-s) W_{t+1}^e \right].\end{aligned} \tag{4}$$

The profits of the marginal entrepreneur plus the future value of being an entrepreneur have to be equal to the wage this very individual would earn as an employed worker plus the future value of being a worker, which comes with some uncertainty because she can lose her job with probability s.

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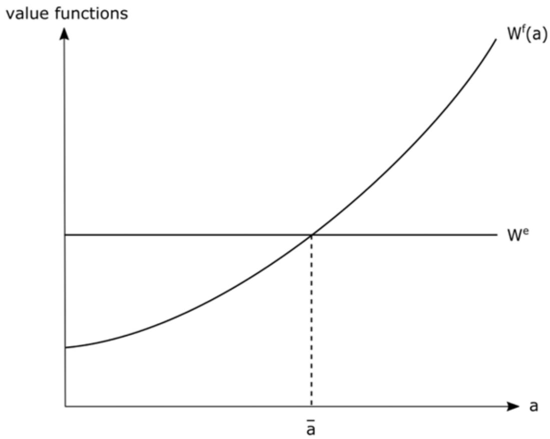

Figure 1. Decision to become an entrepreneur.

Figure 1 shows the value functions for regular workers and for entrepreneurs in dependence on talent a. If an individual has a talent above a certain threshold $\bar{a}$, it is optimal for her to become an entrepreneur because the value function exceeds the value function of being a regular employed worker. Therefore, all individuals with an entrepreneurial talent below $\bar{a}$ prefer to become workers whereas the others open up their own firm.

If a is bounded above, it must be made sure that at least one individual decides to become an entrepreneur and employs workers. Therefore, the upper bound must be sufficiently large so that it is more profitable for at least one individual to be an entrepreneur instead of being a regular worker. The upper bound $\hat{a}$ therefore must fulfill the following equation:

$$\hat{a}f(l_t(\hat{a})) - w_t l_t(\hat{a}) - \gamma v_t(\hat{a}) + \beta W^f_{t+1}(\hat{a}, l_{t+1}(\hat{a}))$$
$$\geq w_t + \beta \left[s W^n_{t+1} + (1-s) W^e_{t+1} \right].$$

If the equation holds, there will be at least one entrepreneur who opens up a firm.

2.4 Market equilibrium

After having described the model setup and the individuals' optimization problems, we can define the market equilibrium.

Definition 1. *$W^f_t(a, l_t)$, W^e_t, W^n_t, $v_t(a)$, θ_t, and $\bar{a}_t$ define a market equilibrium if the following conditions hold for all t:*

- *W^e_t, W^n_t, and $W^f_t(a, l_t)$ fulfill the value equations stated above and satisfy (2),*
- *optimal vacancy posting $v_t(a)$ takes place according to the job creation condition (1),*
- *the threshold $\bar{a}_t$ is set in line with the indifference equation (4),*
- *the labor market tightness θ_t is given with $\frac{V_t}{N_t}$.*

The next section introduces the social planner's optimization problem to characterize the efficient allocation as a benchmark. The market outcome described in the former sections can then be compared to the social optimal allocation.

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3. The social planner

The social planner wants to maximize the aggregate sum of the utilities of employees, unemployed workers, and entrepreneurs. The utility functions depend only on consumption and are assumed to be linear. Therefore, they are of the form $u(c_t^i) = c_t^i$ for $i = e, f, n$. An individual consumes c_t^f if she becomes an entrepreneur and starts a firm. Employed workers consume c_t^e and unemployed workers c_t^n. In aggregate, there are $\Phi(\bar{a}_t) - N_t$ employed workers, N_t unemployed workers, and $1 - \Phi(\bar{a}_t)$ entrepreneurs. The social planner is constrained by a resource constraint, a labor supply constraint, and the law of motion for unemployment. His maximization problem therefore reads

$$\max_{c_t^e, c_t^n, c_t^f, V_t, N_{t+1}, \bar{a}_t, l_t(a)} \sum_{t=0}^{\infty} \beta^t \left[(\Phi(\bar{a}_t) - N_t) u(c_t^e) + N_t u(c_t^n) + \int_{\bar{a}_t}^{\infty} u(c_t^f(a)) d\Phi(a) \right]$$

$$\text{s.t. } (\Phi(\bar{a}_t) - N_t) c_t^e + N_t c_t^n + \int_{\bar{a}_t}^{\infty} c_t^f(a) d\Phi(a) + \gamma V_t = \int_{\bar{a}_t}^{\infty} a f(l_t(a)) d\Phi(a) + N_t z,$$

$$N_{t+1} = N_t - m(N_t, V_t) + s(\Phi(\bar{a}_t) - N_t),$$

$$\Phi(\bar{a}_t) - N_t = \int_{\bar{a}_t}^{\infty} l_t(a) d\Phi(a).$$

The matching function is assumed to be Cobb-Douglas:

$$m(N_t, V_t) = N_t^{\alpha} V_t^{1-\alpha}.$$

Since the social planner still faces the matching function and cannot directly match workers with firms without vacancy posting, the outcome of his maximization problem is a constrained first-best outcome. Combining the first-order conditions from the maximization problem and simplifying them,[10] one obtains two central equations that describe the social planner's optimal choice. First, it must hold that

$$\frac{\gamma}{\beta q(\theta_t)} = (1 - \alpha) \left[a f'(l_{t+1}(a)) - z \right] - \alpha \gamma \theta_{t+1} + \beta (1 - s) \frac{\gamma}{\beta q(\theta_{t+1})}, \tag{5}$$

and, second,

$$\bar{a}_t f(l_t(\bar{a}_t)) + \frac{s\gamma}{(1 - \alpha) q(\theta_t)} - \bar{a}_t f'(l_t(\bar{a}_t))(1 + l_t(\bar{a}_t)) = 0, \tag{6}$$

must be fulfilled.

Equation (5) states that today's discounted costs of having an additional employed worker must equal the social benefit of having that additional worker in the firm in the next period plus her future value.

Equation (6) instead describes the optimal choice of $\bar{a}_t$, the threshold above which every individual becomes an entrepreneur. Rearranging (6) to

$$\bar{a}_t f \left[l_t(\bar{a}_t) \right] - \bar{a}_t f' \left[l_t(\bar{a}_t) \right] l_t(\bar{a}_t) = \bar{a}_t f' \left[l_t(\bar{a}_t) \right] - \frac{s\gamma}{(1 - \alpha) q(\theta_t)},$$

one can see that the threshold must be set such that the value added to GDP that is attributable to the marginal entrepreneur is equal to her contribution if she had been an employed worker. The left-hand side describes the production in the marginal entrepreneur's firm minus the part of production that is assignable to the workers working at that firm. The entrepreneur provides only her technology $\bar{a}$, but she does not provide any labor herself. The contribution as a worker on the right-hand side of the equation consists of production attributable to that worker reduced by the hiring costs that must be paid in case the worker loses the job. In contrast, an additional entrepreneur means that there is one additional firm in the economy that opens vacancies,

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employs workers, and contributes to aggregate production. Nevertheless, that entrepreneur is not an employed worker anymore and is not available for production within a firm. Moreover, moving an individual from being a worker to being an entrepreneur affects the tightness of the labor market. It gets harder for the firms to fill vacancies, since there is increased competition for fewer workers.

In the following, we will focus on steady states. In a steady state, (5) can be reformulated to

$$\frac{\gamma}{q(\theta)} = \frac{\beta(1-\alpha)}{1-\beta(1-s)+\alpha\beta\theta q(\theta)}\left[af'(l)-z\right]. \tag{7}$$

Using this equation, the optimal marginal product of labor multiplied with total factor productivity can be calculated as

$$af'(l(a)) = z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{\left[1-\beta+\alpha\beta\theta q(\theta)\right]}{(1-\alpha)} + \frac{s\gamma}{(1-\alpha)q(\theta)}. \tag{8}$$

Since $af'(l(a))$ in steady state depends only on exogenously given parameters and θ, which is the same for all entrepreneurs, the marginal products of labor multiplied with total factor productivities are constant across firms. Thus, wages are constant across firms as well. The steady-state version of (6) is

$$\bar{a}f(l(\bar{a})) + \frac{s\gamma}{(1-\alpha)q(\theta)} - \bar{a}f'(l(\bar{a}))(1+l(\bar{a})) = 0. \tag{9}$$

Equations (7) and (9) describe the equilibrium resulting from the social planner's optimization problem.

In the next section, we describe the steady-state market equilibrium and analyze its efficiency properties.

4. Efficiency in market equilibrium

The market equilibrium in steady state is characterized by the job creation condition, the wage curve, the Beveridge curve, the indifference equation, and the value equations for employees, unemployed workers, and entrepreneurs.

The first-order condition for hiring in steady state is

$$\frac{\gamma}{q(\theta)} = \frac{\beta(1-\xi)}{1-\beta(1-s)+\xi\beta\theta q(\theta)}\left[af'(l) - \frac{\partial w}{\partial l} l - z\right]. \tag{10}$$

The costs of employing an additional worker have to be equal to the returns of employing that additional worker. The wage curve in steady state resulting from Nash wage bargaining after a match is formed is given with

$$w = \xi\left[af'(l) - \frac{\partial w}{\partial l} l + \gamma\theta\right] + (1-\xi)z.$$

Plugging (10) into the wage equation, the steady-state wage reads

$$w = z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{\xi}{(1-\xi)}\left[1-\beta(1-s)+\beta\theta q(\theta)\right], \tag{11}$$

and depends only on exogenously given parameters and the endogenously determined labor market tightness in equilibrium. The Beveridge curve states that in steady state the aggregate flows into unemployment must equal the aggregate flows out of unemployment:

$$s\int_{\bar{a}}^{\infty} l(a)d\Phi(a) = \theta q(\theta)\left[\Phi(\bar{a}) - \int_{\bar{a}}^{\infty} l(a)d\Phi(a)\right].$$

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Therefore, unemployment in steady state can be derived as $N=\frac{s\Phi(\bar{a})}{s+\theta q(\theta)}$.

The indifference equation (4) in steady state takes the form

$$\begin{aligned}\bar{a}fl(\bar{a})-\gamma v(\bar{a})-wl(\bar{a})&=w+\beta s(W_n-W_e)\\&=(1-\beta)W_e.\end{aligned}$$

The marginal entrepreneur is indifferent between being an entrepreneur and receiving the immediate profits of the firm and being an employed worker, who earns the wage w but might get unemployed in the future with probability s. Using the steady-state wage, the value equation for being an employee is

$$(1-\beta)W_e=z+\frac{\gamma}{\beta q(\theta)}\cdot\frac{\xi}{(1-\xi)}\left[1-\beta+\beta\theta q(\theta)\right], \tag{12}$$

and the value equation for being an unemployed worker is

$$(1-\beta)W_n=z+\frac{\xi}{1-\xi}\gamma\theta. \tag{13}$$

By inserting (12) into the indifference equation in steady state, the marginal entrepreneur's profit in the market equilibrium is calculated:[11]

$$\begin{aligned}\bar{a}f(l(\bar{a}))-wl(\bar{a})-\gamma v(\bar{a})&=z+\frac{\gamma}{\beta q(\theta)}\cdot\frac{\xi}{(1-\xi)}\left[1-\beta+\beta\theta q(\theta)\right]\\&=w-\beta s\frac{\xi}{(1-\xi)}\frac{\gamma\theta}{\beta\theta q(\theta)}.\end{aligned}$$

In the market equilibrium, the marginal entrepreneur's profit is equal to the wage she would earn as an employed worker minus the discounted difference between the value of being employed and being unemployed[12] multiplied by the probability of losing the job s. The marginal entrepreneur's profits are lower than an employed worker's instantaneous income, but as an employed worker, there is always the risk of becoming unemployed, in which case the worker would earn less than the marginal entrepreneur. The following proposition describes the steady-state market equilibrium.

Proposition 1. *The steady-state market equilibrium is characterized by w, θ, N, and $\bar{a}$ that fulfill*

- *the wage curve:* $w=\xi\left[af'(l)-\frac{\partial w}{\partial l}l+\gamma\theta\right]+(1-\xi)z$
- *the job creation condition:* $\frac{\gamma}{q(\theta)}=\frac{\beta(1-\xi)}{1-\beta(1-s)+\xi\beta\theta q(\theta)}\left[af'(l)-\frac{\partial w}{\partial l}l-z\right]$
- *the Beveridge curve:* $N=\frac{s\Phi(\bar{a})}{s+\theta q(\theta)}$
- *the indifference equation:* $\bar{a}f(l(\bar{a}))-wl(\bar{a})-\gamma v(\bar{a})=z+\frac{\gamma}{\beta q(\theta)}\cdot\frac{\xi}{(1-\xi)}\left[1-\beta+\beta\theta q(\theta)\right]$

The wage curve and the job creation condition have a unique intersection in a (θ, w)-space and thus pin down the unique equilibrium for wages and labor market tightness, as can be seen in Figure 2. The wage increases linearly in tightness, whereas the job creation condition is convex and decreases in θ. If wages are higher, firms create fewer jobs, and hence, there are fewer vacancies per worker. The equilibrium for vacancies and unemployment is determined by the job creation condition and the Beveridge curve. In a (u, v)-space, the job creation condition is upward sloping and uniquely intersects with the Beveridge curve, which is convex and downward sloping in u. As the number of posted vacancies rises, unemployment declines because it is easier to be matched with a firm. The unique equilibrium threshold $\bar{a}$ is determined by the indifference equation. For a further discussion, see Section 4.2, which describes the optimal threshold $\bar{a}$.

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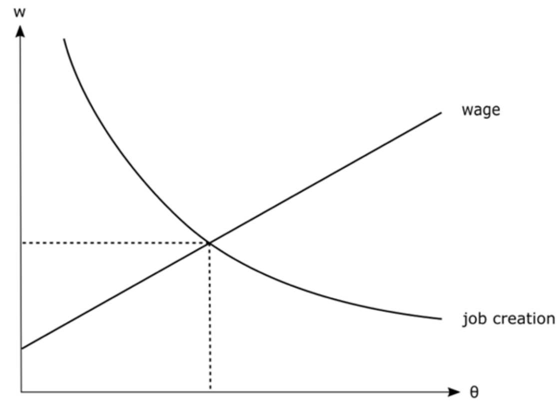

Figure 2. Equilibrium wages and labor market tightness.

4.1 Efficient hiring

The indifference equation together with equation (10) describes the equilibrium in the decentralized market. In the following, we compare the conditions describing the steady-state market equilibrium to the conditions for an optimal allocation in the social planner's setting.

First, if one compares (10) and (7), it is obvious that the Hosios condition $\xi = \alpha$ is not sufficient for efficient job creation if we assume that the threshold $\bar{a}$ is the same in the market as in the efficient case. The Hosios condition[13] states that the market solution in the DMP model is efficient if the private returns of a match ξ are equal to the social returns α. If, for example, $\alpha > \xi$, entrepreneurs will create too many vacancies compared to the efficient situation because their returns of a match would exceed the socially optimal returns. Thus, equilibrium unemployment is too low. The entrepreneurs in decentralized markets do not consider that their creation of jobs poses a negative externality on other entrepreneurs, since it gets harder for them to fill their own vacancies. In the described model, job creation is inefficient even if the Hosios condition holds because entrepreneurs hire too many workers since this suppresses wages. Nevertheless, by taxing or subsidizing vacancy creation, it can be assured that job creation is efficient.

Second, we can determine whether the allocation of individuals into workers and entrepreneurs who start a firm is efficient. This is done in the next sections.

4.2 The optimal threshold $\bar{a}$

In this section, we will first describe how the optimal number of entrepreneurs is set by the social planner and then compare it to the amount of entrepreneurs in the market equilibrium.

Equation (8) and (9) pin down the constrained first-best equilibrium. Combining both equations, one obtains

$$\frac{\bar{a}f(l(\bar{a}))}{(1+l(\bar{a}))} - \frac{l(\bar{a})}{(1+l(\bar{a}))} \cdot \frac{s\gamma}{(1-\alpha)q(\theta)} = z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{[1-\beta+\alpha\beta\theta q(\theta)]}{(1-\alpha)}. \tag{14}$$

The left-hand side of this condition can be interpreted as the value of the marginal firm relative to employment, whereas the right-hand side is the value of a worker. If an individual decides to be an entrepreneur, he or she cannot be a worker anymore. The marginal firm contributes the production $\bar{a}f(l(\bar{a}))$ to total output. This production value is distributed to all workers plus the entrepreneur. But in the case that workers lose their jobs, vacancies must be posted, which is costly.

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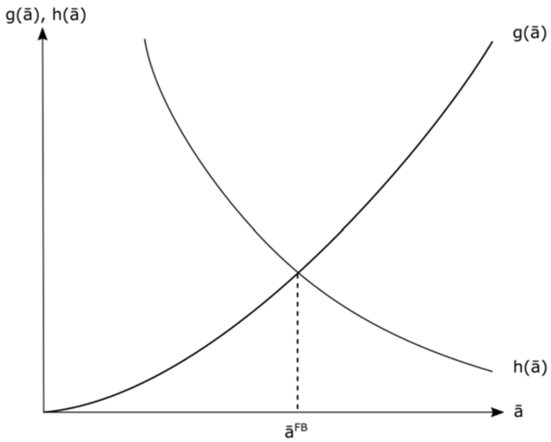

Figure 3. Optimal threshold $\bar{a}$.

Vacancy posting costs are sunk costs and therefore lower overall welfare. As a worker instead, the individual would at least produce z and would have an influence on the labor market tightness, which potentially saves vacancy posting costs. If more individuals decide to become workers, the labor market tightness decreases, which makes it easier for entrepreneurs to fill their vacancies. Therefore, they do not have to post so many vacancies in excess only to make sure that some workers are hired and save costs. Moreover, if more individuals become workers, there are fewer entrepreneurs per se, which lowers the competition among them for workers. In equilibrium, the value of the marginal firm must be equal to the value of a worker, which determines the optimal number of entrepreneurs. If we define the left-hand side of equation (14) as function $g(\bar{a})$ and the right-hand side as function $h(\bar{a})$, the intersection of these two functions pins down the unique threshold $\bar{a}$.

First, we focus on a partial equilibrium analysis. For given l, $g(\bar{a})$ increases in $\bar{a}$, whereas $h(\bar{a})$ decreases. This is depicted in Figure 3, where the value of the marginal firm is an increasing function of $\bar{a}$, and the value of a worker falls in $\bar{a}$. As $\bar{a}$ goes up, the production of the marginal firm increases as well. A higher $\bar{a}$ means that there is one fewer entrepreneur but one more worker available in the economy. Therefore, the hiring costs decrease ceteris paribus, since it gets easier for the remaining firms to fill their vacancies. Put differently, labor supply increases if the former marginal entrepreneur decides to become a worker. The market tightness declines since there are more available workers and fewer firms looking for a worker. Hiring costs decrease. With lower hiring costs and unchanged policy functions, labor demand rises and a new equilibrium is established.

In general equilibrium, if $\bar{a}$ grows by a small amount, equation (14) shows that θ must increase since the left-hand side is clearly increasing in $\bar{a}$ and the right-hand side can only go up if θ gets larger. The tightness of the labor market is therefore increasing in $\bar{a}$. A higher $\bar{a}$ means that there are fewer rather unproductive firms. Therefore, it is easier for the remaining more productive firms to fill their vacancies, which increases overall production. Wages rise, but not as much as productivity itself. At higher productivity, the profit from job creation is higher because the wages do not fully absorb the surge in productivity. Therefore, entrepreneurs post more vacancies, which increases labor market tightness. To sum up, the optimal allocation of individuals into workers and entrepreneurs must ensure that the value of the marginal firm for the economy equals the value of that firm's founder being a worker.

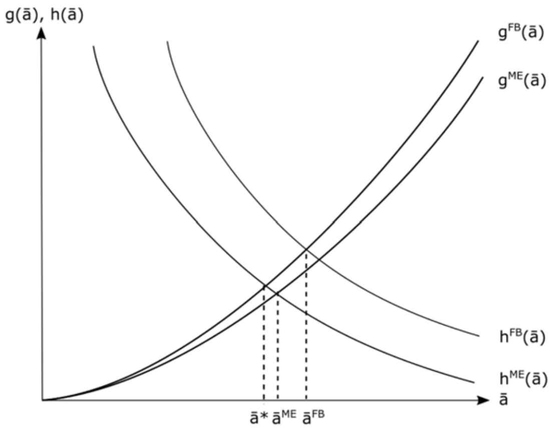

Figure 4. Threshold $\bar{a}$ in market equilibrium compared to first-best $\bar{a}$.

4.3 The threshold in market equilibrium

Now, we determine how the threshold $\bar{a}$ is set in the decentralized market. To compare the market equilibrium with the social planner's solution, it is convenient to reformulate the indifference equation[14] to

$$\frac{\bar{a}f(l(\bar{a}))}{1+l(\bar{a})} - \frac{l(\bar{a})}{(1+l(\bar{a}))} \cdot \frac{s\gamma}{(1-\xi)q(\theta)} = z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{\left[1-\beta+\xi\beta\theta q(\theta)\right]}{(1-\xi)} - \frac{(1-\beta)}{\beta} \cdot \frac{\gamma}{q(\theta)}. \quad (15)$$

The term on the left side is the value of the marginal entrepreneur, which is denoted as $g^{ME}(\bar{a})$ in the following, and the right side defines the value of a worker in market equilibrium, denoted as $h^{ME}(\bar{a})$. If intrafirm wage bargaining would not take place, the equation above differs only from (14) with respect to the term $\frac{(1-\beta)}{\beta}\frac{\gamma}{q(\theta)}$, which is subtracted on the right side, and with respect to ξ. The left-hand sides would be equal to each other if $\alpha = \xi$ and the threshold abilities in the first-best and the market equilibrium align. In the following, it is assumed that $\alpha = \xi$ for simplicity. Without intrafirm wage bargaining, the entrepreneurial threshold ability in the market equilibrium $\bar{a}^{ME}$ thus would be smaller than $\bar{a}^{FB}$, which denotes the threshold ability in the social planner's allocation. With intrafirm wage bargaining, which leads to inefficient job creation even if the Hosios condition holds, the left-hand side of the above equation would differ from the left-hand side of (14). By comparing the labor demand in the market equilibrium given with (10) to the first-best labor demand (8), it is obvious that, for a given $\bar{a}$, the labor demand in the decentralized market is smaller. The threshold $\bar{a}^{ME}$ and θ^{ME} are thus smaller or larger in the market equilibrium than in the constrained first-best depending on the specification of parameter values.

Proposition 2. *If the Hosios condition holds, $l(\bar{a}) \geq 0$, and ξ is not too large, then*

$$\bar{a}^{ME} < \bar{a}^{FB},$$

which means that there are more entrepreneurs in the market equilibrium compared to the social planner's allocation.

Although Stole and Zwiebel (1996) describe the over-creation of jobs at the intensive margin, the channel here is rather the over-creation of jobs at the extensive margin. Figure 4 shows that the threshold $\bar{a}$ in market equilibrium can be smaller than in the social planning problem. There are too many firms compared to the efficient situation, since the value of being a worker is too low in the market equilibrium, which makes being an entrepreneur more profitable. Because of

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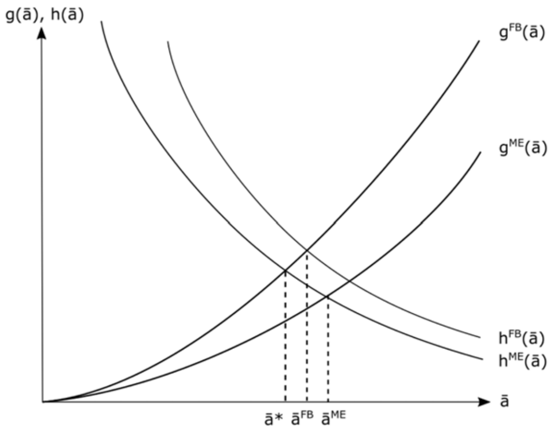

Figure 5. Threshold $\bar{a}$ in market equilibrium compared to first-best $\bar{a}$.

the wage bargaining process, workers in the decentralized market receive a smaller part of the surplus of a match than what is allocated to them by the social planner. This is reflected by $h^{ME}(\bar{a})$ lying below $h^{FB}(\bar{a})$. Put differently, entrepreneurs can acquire an inefficiently large part of the surplus. For a given $\bar{a}$, $g^{FB}(\bar{a})$ is larger than $g^{ME}(\bar{a})$. The reason is that entrepreneurs overhire workers to decrease wages. This in turn increases the competition for workers among firms and thus labor market tightness, which makes being an entrepreneur less attractive. Depending on the relative effect sizes, we can have too few or too many entrepreneurs. Nevertheless, the equilibrium illustrated in Figure 4 features too many entrepreneurs compared to the first-best.

Thus, the acquisition of large shares of the surplus of a match outweighs the negative effects of overhiring on being an entrepreneur, and overall, it is too attractive to start a firm. In the shown case, entrepreneurs with lower ability do not consider the negative effect that they have on the marginal product of more efficient firms. For an individual with a low a, it might be optimal to become an entrepreneur but it is inefficient for the whole economy since that entrepreneur competes for workers with other firms. Within these other firms with more talented entrepreneurs, an employed worker would contribute more to overall production. Entrepreneurs in the decentralized market are able to acquire a larger part of the worker's marginal product through wage bargaining and an entrepreneur with a given ability for job creation makes higher profits in the market than in the social planner's setting. It is therefore profitable for an individual with a low a to be an entrepreneur in the market equilibrium, whereas instead the social planner assigns her to become a worker. Without intrafirm bargaining, $g^{ME}(\bar{a})$ would always be equal to $g^{FB}(\bar{a})$ because, if the Hosios condition holds, hiring is efficient. The threshold ability in the decentralized market would be equal to $\bar{a}^*$ and thus further away from the optimal value because overhiring does not occur and the competition among firms for workers is less severe, which induces entrepreneurs with even lower abilities to start a firm.

The larger ξ, the closer the threshold $\bar{a}^{ME}$ gets to the efficient one. If the worker's bargaining power increases, the entrepreneur cannot acquire that much of the worker's marginal product and the saved vacancy posting costs. If ξ rises further, $h^{ME}(\bar{a})$ moves upwards because θ decreases. With intrafirm wage bargaining, the slope of $g^{ME}(\bar{a})$ changes, because as $\alpha = \xi$ increases, entrepreneurs inefficiently post too many vacancies. The intersection of $g^{ME}(\bar{a})$ and $h^{ME}(\bar{a})$ can thus lie on the right side of the first-best intersection point depicted in Figure 5. In that case, the former results would be reversed, and there would be too few entrepreneurs and too many workers. Being an entrepreneur in that scenario would be too unattractive compared to the constrained first-best. Without intrafirm wage bargaining, the result of having too many entrepreneurs in the market equilibrium would prevail. The threshold ability $\bar{a}^*$ would be closer to

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the first-best threshold, but still smaller. Without intrafirm wage bargaining, $h^{ME}(\bar{a})$ moves closer to $h^{FB}(\bar{a})$ as ξ goes up, because being a worker becomes more attractive. Nevertheless, $g^{ME}(\bar{a})$ is the same as $g^{FB}(\bar{a})$. Thus, the counteracting effect of overhiring on the attractiveness of being an entrepreneur does not exist and $\bar{a}^*$ still lies on the left side of $\bar{a}^{FB}$.

To summarize, an efficient market equilibrium does not exist. Comparing (7), (10), (14), and (15), it is not possible that the threshold $\bar{a}$ in the social planner's problem equals the one in the market equilibrium and job creation is efficient at the same time.[15] If job creation is efficient, the market equilibrium is still inefficient since too many or too few individuals decide to become entrepreneurs. If, on the other hand, the threshold $\bar{a}$ is the same in the market as in the social planner's allocation, the number of entrepreneurs in the market is efficient, but these entrepreneurs inefficiently post too many or too few vacancies. Therefore, the market equilibrium is in general inefficient and taxation might be useful to restore efficiency.

5. Taxation

This section analyses how taxation can be used to restore the constrained first-best allocation. Since we are confronted with two channels for inefficiencies, we need two tax instrument to correct for them. Entrepreneurs' incomes are taxed with marginal tax rate τ_t^f and vacancy posting is either taxed or subsidized with marginal tax rate τ_t^v.[16]

The firm's optimization problem thus becomes

$$W_t^f(a, l_t) = \max_{l_{t+1}, v_t} \left\{ (1-\tau_t^f)\left[af(l_t) - w_t(a, l_t)l_t - (1+\tau_t^v)\gamma v_t\right] + \beta W_{t+1}^f(a, l_{t+1}) \right\}$$

$$\text{s.t. } l_{t+1} = (1-s)l_t + q(\theta_t)v_t.$$

Wages and vacancy posting costs can be deducted from taxed gross profits. Setting up the Lagrange function, deriving the first-order conditions, and using the envelope theorem, the condition for optimal job creation is

$$\frac{(1-\tau_t^f)(1+\tau_t^v)\gamma}{\beta q(\theta_t)} = (1-\tau_{t+1}^f)\left[af'(l_{t+1}) - w_{t+1} - \frac{\partial w_{t+1}}{\partial l_{t+1}}l_{t+1}\right] + (1-s)\frac{\beta(1-\tau_{t+1}^f)(1+\tau_{t+1}^v)\gamma}{\beta q(\theta_{t+1})}.$$

Assuming that the tax rate on entrepreneurial incomes is fixed over time, τ^f disappears from the above equation. Therefore, income taxation does not distort the vacancy posting behavior of firms. Defining $P_{t+1} := \frac{(1+\tau_t^v)\gamma}{\beta q(\theta_t)}$, the condition can be written as

$$P_t = af'(l_t) - w_t - \frac{\partial w_t}{\partial l_t}l_t + \beta(1-s)P_{t+1}.$$

The workers' value equations are not affected by the introduction of taxation except that both the employed and unemployed workers receive the lump sum transfer in each period. Using the value equations and the surplus of a match for the entrepreneur, the Nash wage bargaining result[17] is

$$w_t = \xi\left[af'(l_t) - \frac{\partial w_t}{\partial l_t}l_t + (1+\tau_t^v)\gamma\theta_t\right] + (1-\xi)z,$$

which differs from the wage curve in the former sections. The average vacancy posting costs are now multiplied by $(1+\tau_t^v)$. If the tax on vacancy posting increases, the wage increases as well because a hired worker saves the firm higher vacancy posting costs. In the following, we

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again focus on steady states to calculate the marginal tax rates that recreate the social planner's allocation. Moreover, for simplicity, we assume that $\xi = \alpha$ holds.

First, the focus lies on the taxation or subsidization of vacancy posting. The first-order condition for job creation in steady state becomes

$$\frac{(1+\tau^{v})\gamma}{q(\theta)} = \frac{\beta(1-\xi)}{1-\beta(1-s)+\xi\beta\theta q(\theta)}\left[af'(l) - \frac{\partial w}{\partial l}l - z\right],$$

if we plug in for the wage. Comparing this first-order condition to (7), the tax rate τ^{v} can be calculated that leads to efficient job creation. It is thus given with

$$\tau^{v} = \frac{-\frac{\partial w}{\partial l}l}{af'(l)-z},$$

and ensures that entrepreneurs efficiently post vacancies. Since the derivative of the wage with respect to labor input is negative, the above tax rate is always positive if z is not too large. Now, we can analyze the entrepreneurial decision under efficient hiring by setting τ^{v} as above and $\tau^{f} = 0$. The indifference equation becomes

$$\frac{\bar{a}f(l(\bar{a}))}{(1+l(\bar{a}))} - \frac{l(\bar{a})}{(1+l(\bar{a}))}\cdot\frac{s\gamma}{(1-\xi)q(\theta)} = z + \frac{\gamma}{\beta q(\theta)}\frac{\left[1-\beta+\alpha\theta q(\theta)\right]}{(1-\alpha)} - \frac{\gamma}{\beta q(\theta)}\left\{(1-\beta)(1+\tau^{v}) - \frac{\tau^{v}}{(1-\alpha)}\left[1-\beta\left(1-\alpha\theta q(\theta)-\frac{sl(\bar{a})}{1+l(\bar{a})}\right)\right]\right\}.$$

If we compare this equation to the first-best indifference equation, we can calculate the threshold τ^{v} below which there are still too many entrepreneurs in the decentralized market equilibrium. If the term in curly brackets is larger than one, too many individuals decide to start a firm, even if job creation itself is efficient. This corresponds to[18]

$$\tau^{v} < \frac{(1-\alpha)(1-\beta)}{\alpha\left[1-\beta+\beta\theta q(\theta)\right]+\frac{\beta sl(\bar{a})}{1+l(\bar{a})}}.$$

Whenever the tax rate that restores efficient hiring is smaller than the fraction on the right, entrepreneurial profits have to be taxed to ensure an efficient number of firms and workers.

After having derived τ^{v}, we concentrate on the taxation of entrepreneurial incomes under efficient hiring, that is, it is assumed that τ^{v} is set as described above.[19] The indifference equation for the marginal entrepreneur becomes

$$(1-\tau^{f})\left[\bar{a}f(l(\bar{a})) - wl(\bar{a}) - (1+\tau^{v})\gamma v(\bar{a})\right] = z + (1+\tau^{v})\frac{\xi}{(1-\xi)}\frac{\gamma}{\beta q(\theta)}\left[1-\beta+\beta\theta q(\theta)\right].$$

It yields the net income of the marginal entrepreneur depending only on exogenously given parameters and labor market tightness. Substituting further for the wage on the left-hand side of the equation, it can be rearranged to

$$\frac{\bar{a}f(l(\bar{a}))}{(1+l(\bar{a}))} - \frac{l(\bar{a})}{(1+l(\bar{a}))}\cdot\frac{s\gamma}{(1-\xi)q(\theta)} = \frac{\frac{1}{1-\tau^{f}}+l(\bar{a})}{(1+l(\bar{a}))}\left\{\frac{(1+\tau^{v})\gamma}{\beta q(\theta)}\frac{\left[1-\beta+\xi\beta\theta q(\theta)\right]}{(1-\xi)} - \frac{(1-\beta)}{\beta}\frac{(1+\tau^{v})\gamma}{q(\theta)} + z\right\} + \tau^{v}\frac{l(\bar{a})}{(1+l(\bar{a}))}\frac{s\gamma}{(1-\xi)q(\theta)}, \tag{16}$$

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which is easily comparable to (14) which gives us the efficient number of entrepreneurs. The τ^f that restores the first-best allocation therefore is

$$\tau^f = \frac{A(1+l) - B\tau^v(1+l) - \beta\gamma\tau^v sl}{Al + B(1-\tau^v l) - \beta\gamma\tau^v sl + (1-\alpha)\beta q(\theta)z}$$

with

$$A = (1-\alpha)(1-\beta)(1+\tau^v)\gamma \quad \text{and} \quad B = \left[1-\beta+\alpha\beta\theta q(\theta)\right]\gamma.$$

The tax rate acts as a Pigouvian tax and reconstitutes the first-best. If ξ is not too large, there are too many entrepreneurs in the market equilibrium without taxation. A tax that makes being an entrepreneur less attractive relative to being a worker is therefore efficiency enhancing. Profit taxation makes it unprofitable for rather unproductive entrepreneurs to stay in that occupation. For them, it is now better to become a worker, and the number of entrepreneurs declines. If there are fewer entrepreneurs and more workers, the labor market tightness decreases and it gets easier for the remaining firms with better technologies to fill their vacant positions.

The above tax rate rises in $l(\bar{a})$, so it increases in the number of workers who are hired by the marginal entrepreneur. It also surges in θ if home production is sufficiently high. The tighter the labor market is, the higher the tax will be, since it induces more entrepreneurs to become workers, which in turn relaxes the labor market. Moreover, the tax rate decreases in α or ξ. A higher ξ means that workers have higher bargaining power in the wage negotiations. Entrepreneurs can therefore only acquire a smaller part of the worker's marginal product, and the tax rate on the firms' profits decreases. The tax rate can thus become negative if the worker's bargaining power is high. In that case, being a worker is too attractive and there is an inefficiently low number of entrepreneurs in the decentralized market equilibrium. Being an entrepreneur thus must be subsidized.

Proposition 3. *The constrained first-best allocation can be achieved with two distinct marginal tax rates on vacancy posting and entrepreneurial income:*

$$\tau^v = \frac{-\frac{\partial w}{\partial l} l}{af'(l) - z},$$

and

$$\tau^f = \frac{A(1+l) - B\tau^v(1+l) - \beta\gamma\tau^v sl}{Al + B(1-\tau^v l) - \beta\gamma\tau^v sl + (1-\alpha)\beta q(\theta)z},$$

with

$$A = (1-\alpha)(1-\beta)(1+\tau^v)\gamma \quad \textit{and} \quad B = \left[1-\beta+\alpha\beta\theta q(\theta)\right]\gamma.$$

The first tax rate ensures that vacancy posting is efficient, whereas the second one ensures that the occupational choice to become an entrepreneur is efficient.

To sum up, we calculate tax rates on vacancy posting and on entrepreneurs' profits that restore the constrained first-best allocation. The taxation of entrepreneurial profits can be justified by pure efficiency arguments. It corrects for the private decision of too many individuals to become an entrepreneur, which causes a loss in welfare because of inefficiently high vacancy posting costs that are caused by a tight labor market. Moreover, a second tax policy can correct for inefficient vacancy posting by subsidizing or taxing the named and directly targeting the originator of the positive or negative externality.

Table 1. Parameter values used for the numerical simulation

Parameter	Value	
α	0.5	Petrongolo and Pissarides (2001)
β	0.9879	Annual discount factor of 0.95
γ	2.5	
η	0.7	Short-term labor share
ξ	0.1	Card et al. (2018)
μ_a	8.5	
σ_a^2	0.15	
s	0.1	Shimer (2005)
z	1	

6. Numerical simulation

In this section, we use a numerical simulation of the described model in steady state to briefly describe the magnitude and impact of the tax rates calculated above. The unit of time is considered to be a quarter of the year. The production function used for the numerical simulation takes the form $f(l) = l^\eta$. For the parameters, we draw on values that are usually used in the literature. A summary of the parameter values is provided in Table 1. Petrongolo and Pissarides (2001) analyze estimates for the exponent of a Cobb-Douglas matching function. Relying on their survey of estimates, which mostly lie between 0.5 and 0.7, the exponent is set to $\alpha = 0.5$. As Card et al. (2018) estimate the worker's bargaining power to be in a range of 5% to 15%, we set $\xi = 0.1$. Entrepreneurial ability is distributed according to a log-normal distribution $\ln \mathcal{N}(\mu_a, \sigma_a^2)$ with $\mu_a = 8.5$ and $\sigma_a^2 = 0.15$. The monthly separation rate is estimated by Shimer (2005) to be on average equal to 0.034, which leads us to a quarterly probability of losing the job of 10%. Therefore, $s = 0.1$. We use $\beta = 0.9879$, consistent with an annual discount factor of 0.95. Home production z is normalized to 1. We set $\gamma = 2.5$ to receive an unemployment rate of 5.32% in the market equilibrium, which is approximately in line with the observed long-term unemployment rate in the USA. The tax revenue is assumed to be redistributed as a lump sum transfer to every individual. The tax rates that maximize welfare are thus $\tau^v = 400\%$ and $\tau^f = 28.8\%$. Figure 6 depicts overall welfare under the described parametrization for different values of τ^f. The marginal tax rate τ^v is set to correct for the overhiring behavior of firms. If $\tau^f = 0\%$, welfare increases from 0 to approximately 0.6. This rise is purely caused by correcting vacancy posting through τ^v. Increasing the marginal tax rate on entrepreneurial incomes to $\tau^f = 28.8\%$ increases welfare to 1.8, the maximum. Figure 7 depicts the marginal tax rate τ^f depending on μ_a while holding everything else constant. For a very low μ_a, the tax rate is negative. It falls as μ_a increases, peaks at around 30% and then declines.

Figure 8 shows the tax rates for different values of α and ξ if the respective other parameter is fixed. Panels a and b depict the marginal tax rates on entrepreneurial incomes and vacancy posting if $\alpha = 0.5$ and ξ varies. For small values of ξ, which are empirically relevant as Card et al. (2018) argue, entrepreneurial income is taxed because otherwise there would be too many entrepreneurs in the market equilibrium. If ξ is low, they can acquire a large share of the surplus of a match. The tax on entrepreneurs decreases in ξ as a larger wage bargaining power makes being an entrepreneur less attractive. On the other hand, the tax rate on vacancy posting falls as ξ increases. If ξ is small and matching is quite efficient with an $\alpha = 0.5$, overhiring workers comes with a small cost for the firms. They engage in a lot of overhiring, and thus, τ^v must be large to correct for that. In contrast, Panels c and d illustrate τ^f and τ^v depending on α if ξ is fixed at 0.1. The matching elasticity and τ^f are positively correlated. If the matching efficiency is high and

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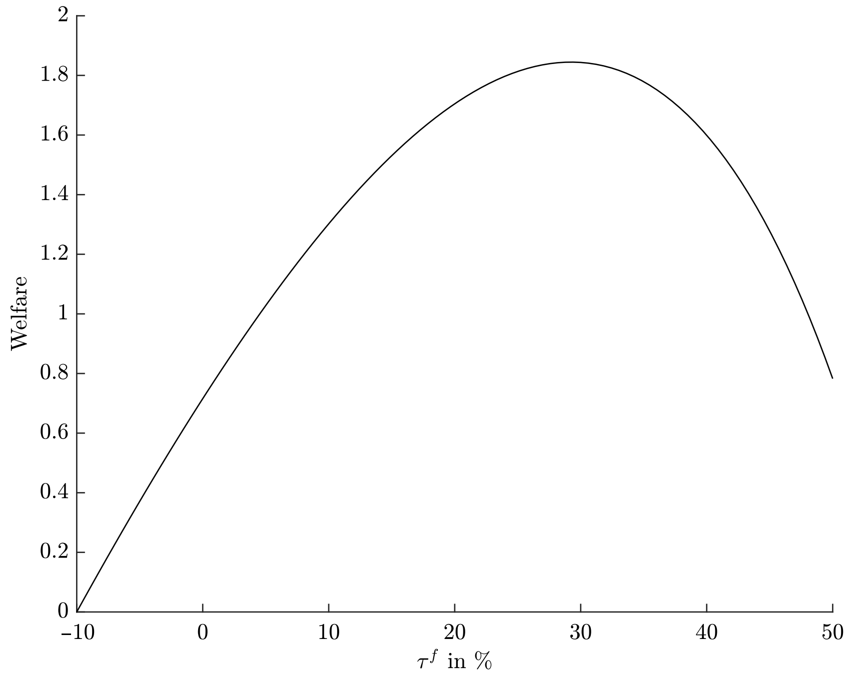

Figure 6. Welfare for different τ^f.

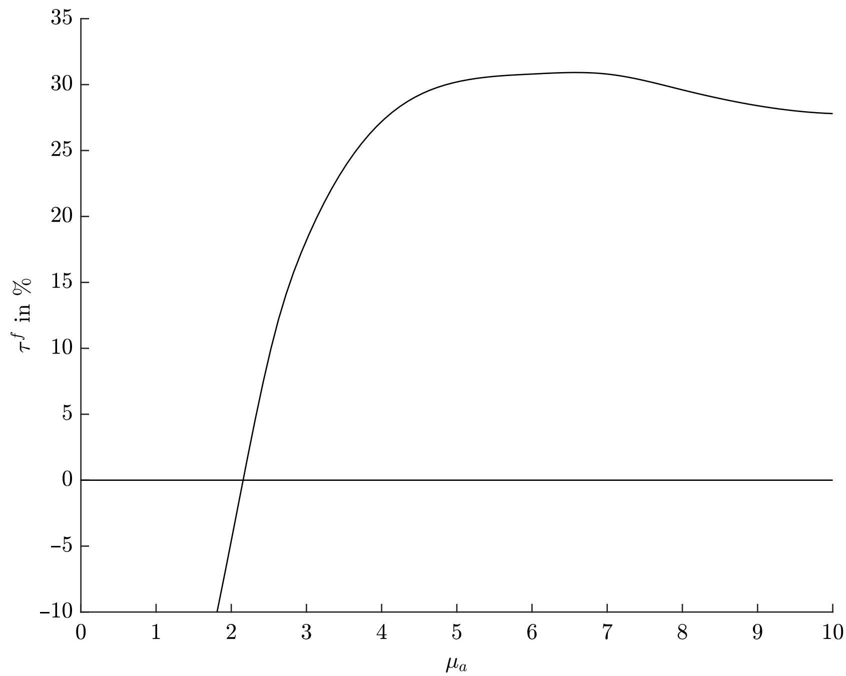

Figure 7. τ^f for varying levels of μ_a.

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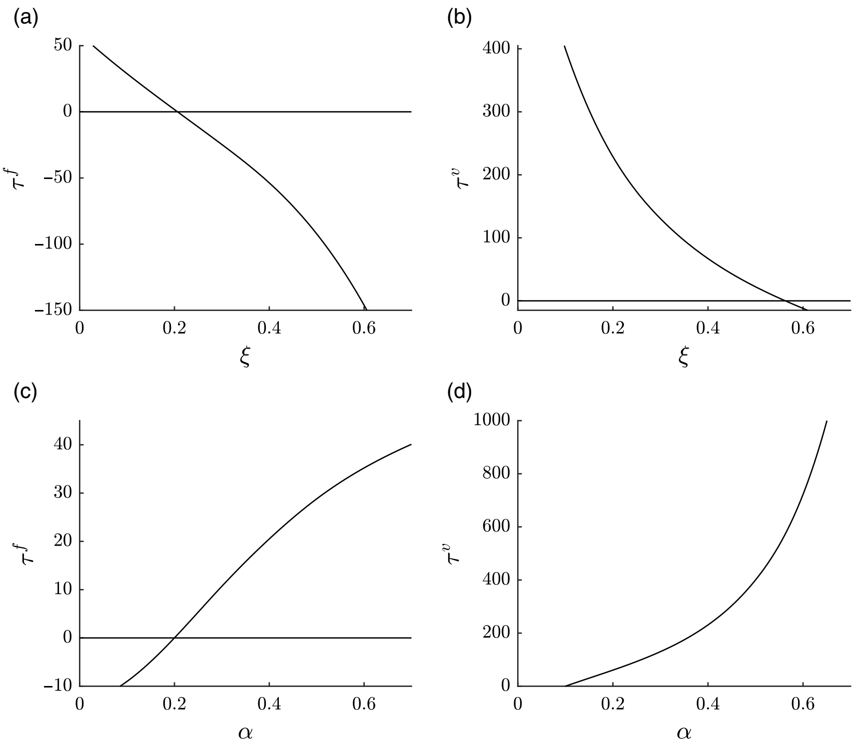

Figure 8. Tax rates for varying ξ and α.

the wage bargaining power is low, being an entrepreneur is very attractive because wages are low and vacancies are filled in a short amount of time. Thus, as matching becomes more efficient and vacancy posting costs decrease, the tax rate on entrepreneurial income must increase to lower the attractiveness of being an entrepreneur. The marginal tax rate τ^ν rises in α. A large α means that matching is quite efficient, which makes overhiring more attractive for firms. To counteract the increasing overhiring behavior of firms, the tax on vacancy posting must become larger. Moreover, Panels b and d point out that τ^ν takes more extreme values the larger the gap between α and ξ is. Nevertheless, if α is equal to ξ, the tax on vacancy posting is still positive because of the overhiring behavior of firms.

Overall, if α and ξ both increase proportionally, the marginal tax rate on entrepreneurial incomes decreases, whereas the tax on vacancy posting goes up. For example, if $\alpha = \xi = 0.2$, the tax rates from the numerical simulation are $\tau^f = -15.6\%$ and $\tau^\nu = 8.0\%$, with the other parameters being as reported in Table 1. For $\alpha = \xi = 0.7$, they are $\tau^f = -54.7\%$ and $\tau^\nu = 27.9\%$. Hence, the effect of the worker's higher wage bargaining power outweighs the effect of an increase in match efficiency with regard to the attractiveness of being an entrepreneur. As α and ξ are equal and rise, being an entrepreneur becomes less attractive and must be taxed with a low rate or even subsidized. On the other hand, τ^ν increases in α and ξ being equal. The higher matching efficiency induces firms to engage massively in overhiring which cannot be outweighed by higher wages caused by a higher ξ.

7. Conclusion

In this paper, we have shown that a market equilibrium can feature an inefficiently high number of entrepreneurs. Since entrepreneurs can acquire a large part of the surplus from a match with

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a worker, it is optimal for an individual to become an entrepreneur in the decentralized market, whereas the social planner would assign that individual to become a regular worker. If an individual with a mediocre entrepreneurial talent decides to become an entrepreneur, she competes with more productive firms for the available workers who would contribute more to production in a more productive firm with a better entrepreneur. This overall effect on total production and labor market tightness is not considered in individual utility maximization. Moreover, inefficiencies also arise because of intrafirm wage bargaining as described in Stole and Zwiebel (1996). Entrepreneurs post too many vacancies so that they can overhire workers to decrease the wages paid to them.

Having outlined the above problem, we calculate two Pigouvian tax rates. The first tax rate ensures that vacancy posting, and thus hiring, is efficient because firms tend to overhire workers to decrease wages. The tax on entrepreneurial profits corrects for the externalities caused by a too large number of entrepreneurs and increases the costs of engaging in job creation. It restores the first-best allocation without distorting labor demand and the vacancy posting choice of individual firms. Without intrafirm wage bargaining, the market equilibrium would always feature too many entrepreneurs compared to the first-best. With intrafirm bargaining, the result depends on the worker's bargaining power and match efficiency. If matching efficiency is high, overhiring is severe and the negative effect of overhiring on the attractiveness to open up a firm outweighs the positive effect of the entrepreneur's ability to acquire parts of the surplus of a match.

Since the analysis in this paper is limited on a steady-state analysis, future work should concentrate on characterizing complete policy functions and transition paths that occur from tax reforms and should analyze the effects of a taxation of workers' incomes as well. Then, it must be possible to investigate the progressiveness of a complete tax schedule. Moreover, the model can be extended with various features influencing the taxation of entrepreneurs. One can argue that entrepreneurs might face a higher risk when opening up a firm than regular workers. Additionally, potential entrepreneurs might be exposed to financial frictions and are restricted by collateral constraints. All these mentioned expansions rather speak in favor of lowering taxes on incomes from job creation. Therefore, it might be interesting to study the tradeoffs of these different channels influencing the optimal taxation of entrepreneurial incomes. A further aspect for future work is the introduction of on-the-job search into the described model. On-the-job search might lead to wage dispersion with more able entrepreneurs paying higher wages. This would make being an entrepreneur less attractive for individuals with lower abilities and thus counteracts the effects this paper describes at least to some extent. The welfare effects of on-the-job search are ambiguous and thus left for future work.

Notes

1 See for example Diamond (1982), Pissarides (1985), Mortensen and Pissarides (1994), and Pissarides (2000). Yashiv (2007) gives an overview on how the labor search and matching model is growingly used in macroeconomics.

2 See as example Hosios (1990) and exemplary Mortensen and Pissarides (2002).

3 Card et al. (2018) review the literature on rent-sharing elasticities and argue that the wage-productivity elasticity lies in the range of 0.05 to 0.15. Even if some studies report slightly higher elasticities, a wage bargaining power larger than 0.2 is empirically quite implausible.

4 For the derivation of the job creation condition, see Appendix A.1.

5 See Appendix A.2.

6 For derivation, see Appendix A.2.

7 The average hiring costs for unemployed workers are $\gamma\theta_t = \frac{\gamma V_t}{N_t}$.

8 This result is in line with Cahuc et al. (2008) who show that the CRS model in a matching framework converges to the Pissarides solution. If instead the production function is of Cobb-Douglas form and without CRS, the marginal product in the wage equation is multiplied by a fraction that includes the worker's bargaining power and the returns to scale parameter.

9 Here, we assume that the entrepreneur directly becomes an employed worker if the threshold shifts.

10 Detailed derivations can be found in Appendix A.3.

11 For the derivation of the steady-state wage, the value equations, and the entrepreneur's profits, see Appendix A.4.

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12 Using (12) and (13), one can calculate $W_e - W_n = \frac{\xi}{(1-\xi)} \frac{\gamma\theta}{\beta\theta q(\theta)}$.
13 See Hosios (1990).
14 The derivation can be found in Appendix A.4.1.
15 The market equilibrium is of course efficient when $\xi = \alpha = 1$. This is the trivial case when we have efficient matching, for example, when every unemployed worker directly finds a job. Then, there are no labor market frictions and no unemployment.
16 The entrepreneur's taxable income is the net profit from the firm. We abstract from corporate taxation. Moreover, we assume that the tax revenue is redistributed via a lump sum transfer to each individual. Since a lump sum transfer does not alter the results, it does not appear in the respective equations for simplicity.
17 We assume that workers and entrepreneurs bargain about gross wages.
18 See Appendix A.5.1 for a derivation of the result.
19 For a more detailed derivation of the following results, see Appendix A.5.

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A. Appendix

A.1 The firm's maximization problem

The Lagrangian function from the maximization problem of the firm has the form

$$\mathcal{L} = af(l_t) - w_t(a, l_t)l_t - \gamma v_t + \beta W^f_{t+1}(a, l_{t+1}) + \lambda\left[(1-s)l_t + q(\theta_t)v_t - l_{t+1}\right].$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial v_t} = -\gamma + \lambda q(\theta_t) = 0,$$

$$\frac{\partial \mathcal{L}}{\partial l_{t+1}} = \beta \frac{\partial W^f_{t+1}(a, l_{t+1})}{\partial l_{t+1}} - \lambda = 0.$$

Combining the first-order conditions, one receives

$$\frac{\gamma}{q(\theta_t)} = \beta \frac{\partial W^f_{t+1}(a, l_{t+1})}{\partial l_{t+1}}.$$

Finally, using the Envelope condition

$$\frac{\partial W^f_t(a, l_t)}{\partial l_t} = af'(l_t) - w_t - \frac{\partial w_t(a, l_t)}{\partial l_t} l_t + \lambda(1-s),$$

one gets to the job creation condition.

A.2 Nash wage bargaining

The wage in a Nash wage bargaining process solves $w_t = \arg\max(W^e_t - W^n_t)^{\xi} P_t^{1-\xi}$.

The FOC is

$$\xi P_t \frac{\partial(W^e_t - W^n_t)}{\partial w_t} + (1-\xi)(W^e_t - W^n_t)\frac{\partial P_t}{\partial w_t} = 0.$$

Calculating the derivatives and rearranging, the FOC becomes

$$\xi P_t = (1-\xi)(W^e_t - W^n_t). \tag{A.1}$$

The surplus of a match for the firm is

$$P_t = af'(l_t) - w_t - \frac{\partial w_t(a, l_t)}{\partial l_t} l_t + \beta(1-s)P_{t+1}.$$

and for the worker, it is

$$W^e_t - W^n_t = w_t - z + \beta(1 - s - \theta_t q(\theta_t))(W^e_{t+1} - W^n_{t+1}).$$

Using (A.1), we can write the surplus of a match for the worker also as

$$W^e_{t+1} - W^n_{t+1} = \frac{\xi}{1-\xi} P_{t+1}.$$

If we plug in the surplus of a match for the entrepreneur and the worker, the FOC from wage bargaining becomes

$$\frac{\xi}{1-\xi}\left[af'(l_t)-\frac{\partial w_t(a,l_t)}{\partial l_t}l_t-w_t(a,l_t)+\beta(1-s)P_{t+1}\right]$$
$$=w_t(a,l_t)-z+\beta\left[1-s-\theta_t q(\theta_t)\right]\frac{\xi}{1-\xi}P_{t+1}$$
$$\Leftrightarrow\frac{\xi}{1-\xi}\left[af'(l_t)-\frac{\partial w_t(a,l_t)}{\partial l_t}l_t-w_t(a,l_t)\right]=w_t(a,l_t)-z-\beta\theta_t q(\theta_t)\frac{\xi}{1-\xi}P_{t+1}.$$

Using $P_{t+1}=\frac{\gamma}{\beta q(\theta_t)}$ from the job creation condition, it becomes

$$\Leftrightarrow\frac{\xi}{1-\xi}\left[af'(l_t)-\frac{\partial w_t(a,l_t)}{\partial l_t}l_t-w_t(a,l_t)\right]=w_t(a,l_t)-z-\theta_t\frac{\xi}{1-\xi}\gamma.$$

The left-hand side needs to be constant across all firms. So has to be the right-hand side, which implies that $w_t(a,l_t)$ is the same for all firms. Rearranging terms, the wage curve is derived:

$$w_t=\xi\left[af'(l_t)-\frac{\partial w_t(a,l_t)}{\partial l_t}l_t+\gamma\theta_t\right]+(1-\xi)z.$$

A.3 Social planner's maximization problem

The Lagrange function of the social planner's maximization problem is

$$\begin{aligned}\mathcal{L}=\sum_{t=0}^{\infty}\beta^t\Bigg\{&(\Phi(\bar{a}_t)-N_t)u(c_t^e)+N_tu(c_t^n)+\int_{\bar{a}_t}^{\infty}u(c_t^f(a))d\Phi(a)\\&+\lambda_t\left[\int_{\bar{a}_t}^{\infty}af(l_t(a))d\Phi(a)+N_tz-(\Phi(\bar{a}_t)-N_t)c_t^e-N_tc_t^n-\int_{\bar{a}_t}^{\infty}c_t^f(a)d\Phi(a)-\gamma V_t\right]\\&+\mu_t\left[N_{t+1}-N_t-s(\Phi(\bar{a}_t)-N_t)+m(N_t,V_t)\right]\\&+\nu_t\left[\Phi(\bar{a}_t)-N_t-\int_{\bar{a}_t}^{\infty}l_t(a)d\Phi(a)\right]\Bigg\},\end{aligned}$$

with λ_t being the Lagrange multiplier on the resource constraint, μ_t being the multiplier for the law of motion of unemployment and ν_t as multiplier on the labor supply constraint.

The FOCs are:

$$\frac{\partial\mathcal{L}}{\partial c_t^e}=1-\lambda_t=0,$$
$$\frac{\partial\mathcal{L}}{\partial c_t^n}=1-\lambda_t=0,$$
$$\frac{\partial\mathcal{L}}{\partial c_t^f(a)}=1-\lambda_t=0,$$
$$\frac{\partial\mathcal{L}}{\partial V_t}=-\lambda_t\gamma+\mu_t\frac{\partial m(N_t,V_t)}{\partial V_t}=0,$$
$$\frac{\partial\mathcal{L}}{\partial l_t(a)}=\lambda_t af'(l_t(a))-\nu_t=0,$$

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$$\frac{\partial \mathcal{L}}{\partial N_{t+1}} = \mu_t + \beta\left[u(c_{t+1}^n) - u(c_{t+1}^e)\right] + \beta\lambda_{t+1}\left[z + c_{t+1}^e - c_{t+1}^n\right] - \beta\mu_{t+1}\left[1 - s - \frac{\partial m(N_{t+1}, V_{t+1})}{\partial N_{t+1}}\right] - \beta\nu_{t+1} = 0,$$

$$\frac{\partial \mathcal{L}}{\partial \bar{a}_t} = \phi(\bar{a}_t)u(c_t^e) - \phi(\bar{a}_t)u(c_t^f(\bar{a}_t)) + \lambda_t\left[-\phi(\bar{a}_t)\bar{a}_t f(l_t(\bar{a}_t)) - \phi(\bar{a}_t)c_t^e + \phi(\bar{a}_t)c_t^f(\bar{a}_t) + \phi(\bar{a}_t)\gamma v_t(\bar{a}_t)\right] + \mu_t\left[-\phi(\bar{a}_t)\frac{\partial m(N_t, V_t)}{\partial V_t}v_t(\bar{a}_t) - \phi(\bar{a}_t)s\right] + \nu_t\left[\phi(\bar{a}_t) + \phi(\bar{a}_t)l_t(\bar{a}_t)\right] = 0.$$

The matching function is assumed to be of Cobb-Douglas form: $m(N_t, V_t) = N_t^{\alpha} V_t^{1-\alpha}$. The derivatives of the matching function with respect to N_t and V_t therefore are

$$\frac{\partial m(N_t, V_t)}{\partial V_t} = (1-\alpha)N_t^{\alpha}V_t^{-\alpha} = (1-\alpha)\left(\frac{m(N_t, V_t)}{V_t}\right) = (1-\alpha)q(\theta_t),$$

and

$$\frac{\partial m(N_t, V_t)}{\partial N_t} = \alpha N_t^{\alpha-1}V_t^{1-\alpha} = \alpha\left(\frac{m(N_t, V_t)}{N_t}\right) = \alpha\theta_t q(\theta_t).$$

The elasticity of the Cobb-Douglas matching function with respect to N_t is

$$\frac{\partial m(N_t, V_t)}{\partial N_t}\frac{N_t}{m(N_t, V_t)} = \alpha.$$

The utility function is assumed to be linear; therefore, $u(c_t^i) = c_t^i$ for $i = e, j, n$.

In the following, the Lagrange multipliers are eliminated from the derivatives of the Lagrangian with respect to N_t and $\bar{a}$.

From $\frac{\partial \mathcal{L}}{\partial l_t(a)}$, it is clear that the marginal product of labor has to be the same for each individual, no matter how high a is. Therefore, instead of $af'(l_t(a))$ one can always use $\bar{a}_t f'(l_t(\bar{a}_t))$.

Derivative with respect to N_t: First, $\frac{\partial \mathcal{L}}{\partial N_{t+1}}$ is rearranged to

$$\mu_t = \beta\left[u(c_{t+1}^e) - u(c_{t+1}^n)\right] + \beta\lambda_{t+1}\left[c_{t+1}^n - c_{t+1}^e - z\right] + \beta\mu_{t+1}\left[1 - s - \frac{\partial m(N_t, V_t)}{\partial N_t}\right] + \beta\nu_{t+1}.$$

Plugging in for μ_t, μ_{t+1}, λ_t, λ_{t+1}, and ν_{t+1}, one obtains

$$\frac{\gamma}{\beta q(\theta_t)} = (1-\alpha)\left[af'(l_{t+1}(a)) - z\right] - \alpha\gamma\theta_{t+1} + \beta(1-s)\frac{\gamma}{\beta q(\theta_{t+1})} \tag{A.2}$$

Derivative with respect to $\bar{a}_t$: Divide the derivative $\frac{\partial \mathcal{L}}{\partial \bar{a}_t}$ by $\phi(\bar{a})$ and replace λ_t, μ_t and ν_t. One obtains

$$\bar{a}_t f(l_t(\bar{a}_t)) + \frac{s\gamma}{(1-\alpha)q(\theta_t)} - \bar{a}_t f'(l_t(\bar{a}_t))(1 + l_t(\bar{a}_t)) = 0. \tag{A.3}$$

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A.4 Efficiency in market equilibrium

In the market equilibrium, we have

$$w = \xi \left[af'(l) - \frac{\partial w}{\partial l} l + \gamma\theta \right] + (1-\xi)z$$

$$= z + \xi \left[af'(l) - \frac{\partial w}{\partial l} l - z \right] + \xi\gamma\theta.$$

The first-order condition for hiring is

$$\frac{\gamma}{q(\theta)} = \frac{\beta(1-\xi)}{1-\beta(1-s)+\xi\beta\theta q(\theta)} \left[af'(l) - \frac{\partial w}{\partial l} l - z \right],$$

which we can rewrite as

$$af'(l) - \frac{\partial w}{\partial l} l - z = \frac{\gamma}{\beta q(\theta)} \cdot \frac{\left[1-\beta(1-s)+\xi\beta\theta q(\theta)\right]}{(1-\xi)}.$$

Hence, the wage equation reads

$$w = z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{\xi}{(1-\xi)} \left[1-\beta(1-s)+\xi\beta\theta q(\theta)\right] + \xi\gamma\theta$$

$$= z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{\xi}{(1-\xi)} \left[1-\beta(1-s)+\xi\beta\theta q(\theta)+(1-\xi)\beta\theta q(\theta)\right]$$

$$= z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{\xi}{(1-\xi)} \left[1-\beta(1-s)+\beta\theta q(\theta)\right]. \tag{A.4}$$

The value equations for an employed and an unemployed worker in steady state are

$$W_e = w + \beta\left[sW_n + (1-s)W_e\right], \tag{A.5}$$

and

$$W_n = z + \beta\left[\theta q(\theta) W_e + (1-\theta q(\theta)) W_n\right]. \tag{A.6}$$

Rearranging W_n gives

$$W_n = \frac{z + \beta\theta q(\theta) W_e}{1-\beta+\beta\theta q(\theta)}.$$

Inserting this into (A.5), one can solve for W_e:

$$W_e = w + \beta s \left(\frac{z + \beta\theta q(\theta) W_e}{1-\beta+\beta\theta q(\theta)} \right) + \beta(1-s) W_e$$

$$\Leftrightarrow \quad (1-\beta) W_e = w - \frac{\beta s (w-z)}{1-\beta(1-s)+\beta\theta q(\theta)}.$$

Analogously, rearranging (A.5) gives

$$W_e = \frac{w + \beta s W_n}{1-\beta(1-s)}. \tag{A.7}$$

By using (A.7), W_e can be eliminated from equation (A.6) and one obtains

$$(1-\beta) W_n = z + \frac{\beta\theta q(\theta)(w-z)}{1-\beta(1-s)+\beta\theta q(\theta)}.$$

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For w, one can insert (A.4) into both reformulated value equations and receives

$$(1-\beta)W_e = z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{\xi}{(1-\xi)}\left[1-\beta+\beta\theta q(\theta)\right], \tag{A.8}$$

and

$$(1-\beta)W_n = z + \frac{\xi}{1-\xi}\gamma\theta. \tag{A.9}$$

A.4.1 Reformulating the indifference equation. The indifference equation for the entrepreneur is

$$\bar{a}(f(l(\bar{a}))) - \gamma v(\bar{a}) - wl(\bar{a}) = z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{\xi}{(1-\xi)}\left[1-\beta+\beta\theta q(\theta)\right].$$

For the firm's steady state, we get

$$v(\bar{a}) = \frac{sl(\bar{a})}{q(\theta)}.$$

Plugging this, as well as (A.4), into the above indifference equation, we get

$$\begin{aligned}
&\bar{a}(f(l(\bar{a}))) - \frac{s\gamma l(\bar{a})}{q(\theta)} - \left[z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{\xi}{(1-\xi)}\left[1-\beta(1-s)+\beta\theta q(\theta)\right]\right] l(\bar{a}) \\
&\qquad = z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{\xi}{(1-\xi)}\left[1-\beta+\beta\theta q(\theta)\right] \\
\Leftrightarrow\quad & \bar{a}(f(l(\bar{a}))) - \frac{s\gamma l(\bar{a})}{q(\theta)} - \frac{s\gamma l(\bar{a})}{q(\theta)} \cdot \frac{\xi}{(1-\xi)} = \left[z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{\xi}{(1-\xi)}\left[1-\beta+\beta\theta q(\theta)\right]\right](1+l(\bar{a})) \\
\Leftrightarrow\quad & \frac{\bar{a}f(l(\bar{a}))}{(1+l(\bar{a}))} - \frac{l(\bar{a})}{(1+l(\bar{a}))} \cdot \frac{s\gamma}{(1-\xi)q(\theta)} = z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{\xi}{(1-\xi)}\left[1-\beta+\beta\theta q(\theta)\right] \\
\Leftrightarrow\quad & \frac{\bar{a}f(l(\bar{a}))}{(1+l(\bar{a}))} - \frac{l(\bar{a})}{(1+l(\bar{a}))} \cdot \frac{s\gamma}{(1-\xi)q(\theta)} = z + \frac{\gamma}{\beta q(\theta)} \frac{\left[1-\beta+\xi\beta\theta q(\theta)\right]}{(1-\xi)} - \frac{(1-\beta)}{\beta} \cdot \frac{\gamma}{q(\theta)}
\end{aligned}$$

A.5 Tax rates

The maximization problem of the entrepreneur is

$$\max_{v_t, l_{t+1}} \; W_t^f(a, l_t) = (1-\tau^f)\left[af(l_t) - w_t l_t - (1+\tau^v)\gamma v_t\right] + \beta W_{t+1}^f(a, l_{t+1})$$

$$\text{s.t.} \quad l_{t+1} = (1-s)l_t + q(\theta_t)v_t.$$

The related Lagrangian function is

$$\begin{aligned}
\mathcal{L} = {} & (1-\tau^f)\left[af(l_t) - w_t l_t - (1+\tau^v)\gamma v_t\right] + \beta W_{t+1}^f(a, l_{t+1}) \\
& + \lambda_t\left[(1-s)l_t + q(\theta_t)v_t - l_{t+1}\right].
\end{aligned}$$

The FOC thus are

$$\frac{\partial \mathcal{L}}{\partial v_t} = -(1-\tau^f)(1-\tau^v)\gamma + \lambda_t q(\theta_t) = 0,$$

$$\frac{\partial \mathcal{L}}{\partial l_{t+1}} = \beta \frac{\partial W_{t+1}^f(a, l_{t+1})}{\partial l_{t+1}} - \lambda_t = 0,$$

which can be combined to

$$\frac{(1-\tau^f)(1-\tau^v)\gamma}{q(\theta_t)} = \beta \frac{\partial W^f_{t+1}(a, l_{t+1})}{\partial l_{t+1}}.$$

Using the Envelope condition

$$\frac{\partial W^f_t(a, l_t)}{\partial l_t} = (1-\tau^f)\left[af'(l_t) - w_t - \frac{\partial w_t}{\partial l_t} l_t\right] + \lambda_t(1-s),$$

we derive the job creation condition

$$\begin{aligned}\frac{(1-\tau^f)(1-\tau^v)\gamma}{\beta q(\theta_t)} &= (1-\tau^f)\left[af'(l_{t+1}) - w_{t+1} - \frac{\partial w_{t+1}}{\partial l_{t+1}} l_{t+1}\right] - \lambda_{t+1}(1-s)\\ &= (1-\tau^f)\left[af'(l_{t+1}) - w_{t+1} - \frac{\partial w_{t+1}}{\partial l_{t+1}} l_{t+1}\right]\\ &\quad - \beta(1-s)\frac{(1-\tau^f)(1-\tau^v)\gamma}{\beta q(\theta_{t+1})}.\end{aligned}$$

Dividing by $(1-\tau^f)$, one receives

$$\frac{(1-\tau^v)\gamma}{\beta q(\theta_t)} = af'(l_{t+1}) - w_{t+1} - \frac{\partial w_{t+1}}{\partial l_{t+1}} l_{t+1} - \beta(1-s)\frac{(1-\tau^v)\gamma}{\beta q(\theta_{t+1})}.$$

Nash wage bargaining. For Nash wage bargaining, I define $P_{t+1} := \frac{(1-\tau^v)\gamma}{\beta q(\theta_t)}$. The wage thus solves

$$w_t = \arg\max \left(W^e_t - W^n_t\right)^{\xi} P_t^{1-\xi},$$

which results in the FOC

$$\xi P_t = (1-\xi)\left[W^e_t - W^n_t\right].$$

As

$$P_t = af'(l_t) - w_t - \frac{\partial w_t}{\partial l_t} l_t + \beta(1-s)P_{t+1},$$

and

$$\begin{aligned}W^e_t - W^n_t &= w_t - z + \beta\left[1-s-\theta_t q(\theta_t)\right]\left(W^e_{t+1} - W^n_{t+1}\right)\\ &= w_t - z + \beta\left[1-s-\theta_t q(\theta_t)\right]\frac{\xi}{(1-\xi)} P_{t+1},\end{aligned}$$

the FOC from wage bargaining becomes

$$\xi\left[af'(l_t) - w_t - \frac{\partial w_t}{\partial l_t} l_t + \beta(1-s)P_{t+1}\right] = (1-\xi)\left[w_t - z + \beta\left[1-s-\theta_t q(\theta_t)\right]\frac{\xi}{(1-\xi)} P_{t+1}\right].$$

If we plug in $P_{t+1} := \frac{(1-\tau^v)\gamma}{\beta q(\theta_t)}$ and solve for the wage, it becomes

$$w_t = \xi\left[af'(l_t) - \frac{\partial w_t}{\partial l_t} l_t + (1-\tau^v)\gamma\theta_t\right] + (1-\xi)z.$$

Deriving τ^v. The following derivations are done in steady state. The wage in steady state can be written as

$$w = z + \xi \left[af'(l) - \frac{\partial w}{\partial l} l - z \right] + (1 + \tau^v)\xi\gamma\theta.$$

By plugging in the wage, the first-order condition for job creation is:

$$\frac{(1+\tau^v)\gamma}{q(\theta)} = \frac{\beta(1-\xi)}{1-\beta(1-s)+\xi\beta\theta q(\theta)} \left[af'(l) - \frac{\partial w}{\partial l} l - z \right]. \tag{A.10}$$

We then compare this equation to the first-best job creation condition to calculate τ^v under the assumption that $\xi = \alpha$. The tax rate τ^v thus solves

$$\frac{\beta(1-\alpha)}{1-\beta(1-s)+\alpha\beta\theta q(\theta)} \left[af'(l) - z \right] = \frac{\beta(1-\alpha)}{\left[1-\beta(1-s)+\alpha\beta\theta q(\theta)\right](1+\tau^v)} \left[af'(l) - \frac{\partial w}{\partial l} l - z \right].$$

Solving for τ^v, one obtains

$$\tau^v = \frac{-\frac{\partial w}{\partial l} l}{af'(l) - z}.$$

Deriving τ^f. Rewriting (A.10) as

$$af'(l) - \frac{\partial w}{\partial l} l - z = \frac{(1-\tau^v)\gamma}{\beta q(\theta)} \frac{\left[1-\beta(1-s)+\xi\beta\theta q(\theta)\right]}{(1-\xi)},$$

and plugging it into the steady-state wage equation, the wage becomes

$$w = z + \frac{(1+\tau^v)\gamma}{\beta q(\theta)} \cdot \frac{\xi}{(1-\xi)} \left[1-\beta(1-s)+\beta\theta q(\theta)\right].$$

The indifference equation for the entrepreneur is

$$(1-\tau^f)\left[\bar{a}f(l) - wl - (1+\tau^v)\gamma v\right] = w - \beta s \frac{(w-z)}{\left[1-\beta(1-s)+\beta\theta q(\theta)\right]}$$

$$= z + (1+\tau^v)\frac{\xi}{(1-\xi)} \frac{\gamma}{\beta q(\theta)} \left[1-\beta+\beta\theta q(\theta)\right].$$

Plugging in for the wage and $v(\bar{a})$ and rearranging, we receive

$$\frac{\bar{a}f(l(\bar{a}))}{(1+l(\bar{a}))} - \frac{l(\bar{a})}{(1+l(\bar{a}))} \cdot \frac{s\gamma}{(1-\xi)q(\theta)}$$

$$= \frac{\frac{1}{(1-\tau^f)} + l(\bar{a})}{1+l(\bar{a})} \cdot \left\{ \frac{(1+\tau^v)\gamma}{\beta q(\theta)} \frac{\left[1-\beta+\xi\beta\theta q(\theta)\right]}{(1-\xi)} - \frac{(1-\beta)}{\beta} \frac{(1+\tau^v)\gamma}{q(\theta)} + z \right\}$$

$$+ \tau^v \frac{l(\bar{a})}{(1+l(\bar{a}))} \frac{s\gamma}{(1-\xi)q(\theta)}.$$

If we set this equation equal to the first-best indifference equation and solve for τ^f, we receive

$$\tau^f = \frac{(1-\alpha)(1-\beta)(1+\tau^v)\gamma(1+l) - \left[1-\beta+\alpha\beta\theta q(\theta)\right]\gamma\tau^v(1+l) - \tau^v\beta\gamma sl}{(1-\alpha)(1-\beta)(1+\tau^v)\gamma l + \left[1-\beta+\alpha\beta\theta q(\theta)\right]\gamma(1-\tau^v l) + (1-\alpha)\beta q(\theta)z - \tau^v\beta s\gamma l}$$

https://doi.org/10.1017/S1365100522000591 Published online by Cambridge University Press

or

$$\tau^f = \frac{A(1+l) - B\tau^v(1+l) - \beta\gamma\tau^v sl}{Al + B(1+\tau^v l) - \beta\gamma\tau^v sl + (1-\alpha)\beta q(\theta)z}$$

with

$$A = (1-\alpha)(1-\beta)(1-\tau^v)\gamma \quad \text{and} \quad B = \left[1-\beta+\alpha\beta\theta q(\theta)\right]\gamma.$$

A.5.1 Entrepreneurial decision under efficient hiring. Set τ^v at first-best and $\tau^f = 0$ and use $\alpha = \xi$. The indifference equation becomes

$$\begin{aligned} &\frac{\bar{a}f(l(\bar{a}))}{(1+l(\bar{a}))} - \frac{l(\bar{a})}{(1+l(\bar{a}))}\cdot\frac{s\gamma}{(1-\xi)q(\theta)} - \tau^v\frac{l(\bar{a})}{(1+l(\bar{a}))}\cdot\frac{s\gamma}{(1-\xi)q(\theta)} \\ &= z + \frac{\alpha}{(1-\alpha)}\frac{\gamma}{\beta q(\theta)}\left[1-\beta+\beta\theta q(\theta)\right] + \tau^v\frac{\alpha}{(1-\alpha)}\frac{\gamma}{\beta q(\theta)}\left[1-\beta+\beta\theta q(\theta)\right] \\ &= z + \frac{\gamma}{\beta q(\theta)}\frac{\left[1-\beta+\alpha\theta q(\theta)\right]}{(1-\alpha)} - \frac{(1-\beta)}{\beta}\frac{\gamma}{q(\theta)} + \tau^v\frac{\alpha}{(1-\alpha)}\frac{\gamma}{\beta q(\theta)}\left[1-\beta+\beta\theta q(\theta)\right]. \end{aligned}$$

Rearranging further, it can be written as

$$\begin{aligned} \frac{\bar{a}f(l(\bar{a}))}{(1+l(\bar{a}))} - \frac{l(\bar{a})}{(1+l(\bar{a}))}\cdot\frac{s\gamma}{(1-\xi)q(\theta)} &= z + \frac{\gamma}{\beta q(\theta)}\frac{\left[1-\beta+\alpha\theta q(\theta)\right]}{(1-\alpha)} \\ &\quad - \frac{\gamma}{\beta q(\theta)}\left\{(1-\beta)(1+\tau^v) - \frac{\tau^v}{(1-\alpha)}\left[1-\beta\left(1-\alpha\theta q(\theta) - \frac{sl(\bar{a})}{1+l(\bar{a})}\right)\right]\right\}. \end{aligned}$$

We then compare this indifference equation to the first-best one. There are too many entrepreneurs if the term in curly brackets is larger than one:

$$\begin{aligned} &(1-\beta)(1+\tau^v) - \frac{\tau^v}{(1-\alpha)}\left[1-\beta\left(1-\alpha\theta q(\theta) - \frac{sl(\bar{a})}{1+l(\bar{a})}\right)\right] > 0 \\ \Leftrightarrow\ &\frac{1+\tau^v}{\tau^v} > \frac{1-\beta\left[1-\alpha\theta q(\theta) - \frac{sl(\bar{a})}{1+l(\bar{a})}\right]}{(1-\alpha)(1-\beta)} \\ \Leftrightarrow\ &\tau^v < \frac{(1-\alpha)(1-\beta)}{\alpha\left[1-\beta+\beta\theta q(\theta)\right] + \frac{\beta sl(\bar{a})}{1+l(\bar{a})}}. \end{aligned}$$

Cite this article: Röhrs J (2023). "Income taxation and job creation." *Macroeconomic Dynamics* **27**, 2056–2085. https://doi.org/10.1017/S1365100522000591