Note on Professor Allardice's Paper "On the Locus of the Foci of a System of Similar Conics through three Points." [*Proceedings*, Vol. XXVII.]

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The fundamental equation in this paper,

$$a_1^2 + b_1^2 + c_1^2 - 2b_1c_1\cos A - 2c_1a_1\cos B - 2a_1b_1\cos C = e^2I^2,$$

can be derived, easily and without using trilinears, from the following property of conics.

Let F be one focus of a conic circumscribed to the triangle ABC; draw AL perpendicular to the corresponding directrix from $A(x_1y_1)$, BM from $B(x_2y_2)$, CN from $C(x_3y_3)$. Then

$$\Sigma a^2 . FA^2 - 2\Sigma FB . FC . bc\cos A$$

$$= \Sigma a^2(FA - FB)(FA - FC) = e^2\Sigma a^2(AL - BM)(AL - CN)$$

$$= e^2\Sigma[(x_2 - x_3)^2 + (y_2 - y_3)^2](x_1 - x_2)(x_1 - x_3).$$

The expression $\Sigma(x_2 - x_3)^2(x_1 - x_2)(x_1 - x_3)$ clearly vanishes; and the expression $\Sigma(y_2 - y_3)^2(x_1 - x_2)(x_2 - x_3)$ is found to be the square of $\Sigma(x_1y_2 - x_2y_1)$. Thus $\Sigma a^2 . FA^2 - 2\Sigma FB . FC . bc\cos A = 4e^2\Delta^2$.

As $a/2\sin A = b/2\sin B = c/2\sin C = \Delta/I = R$, we get

$$\Sigma FA^2\sin^2 A - 2\Sigma FB\sin B . FC\sin C . \cos A = e^2I^2.$$

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Now $FA^2 \sin^2 A$ is the square of the line joining the feet of the perpendiculars from F on AB and AC, and is thus equal to $\beta^2 + \gamma^2 + 2\beta\gamma \cos A$, *i.e.* to a_1^2. Hence we deduce the result above referred to.

It would appear that the property was obtained simultaneously by Professor M. T. Naraniengar: see the solution of his Question 16835 in the *Educational Times*, October 1910.