Introduction

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Few articles can reasonably be described as epoch-making. Einstein's 'Zur Elektrodynamik bewegter Körper' (1905) is undoubtedly one such; Turing's 'On Computable Numbers, with an Application to the Entscheidungsproblem' (1936) is undoubtedly another. But standing equally tall among these ranks should surely be the article to which this volume – and so much else besides – owes its existence: Emmy Noether's 'Invariante Variationsprobleme' (1918). In that one article, Noether proved two theorems (and their converses), forging links between symmetries and conserved quantities which were to go on – whether by her intentions or not – to constitute the bedrock of modern theoretical physics.

But – perhaps surprisingly, perhaps not – the significance of an epoch-maker is not always recognised in the moment. Whether this is so in the case of Einstein is debatable; it is certainly true for Turing – and arguably even more so for Noether. Strikingly, the significance of what Noether proved in her 1918 article was not well appreciated until as late as the 1970s: only at that point were all of the theorems of the 1918 piece widely understood; and only at that point did they begin to be generalised and applied in substantially novel ways. Since then, progress has not stopped, and this volume – born out of an international 2018 centenary conference held at the London campus of the University of Notre Dame – represents the next episode in the same continuation. Bringing together historians, physicists, mathematicians, and philosophers, the volume constitutes the cutting edge of our understanding of (the application of) Noether's seminal work on variational problems, one hundred years on from her original article.

Why do we add 'the application of' in parentheses above? It is now well-known that Noether remarked little on the physical applications of her mathematical results; see, for example, the contributions of Kosmann-Schwarzbach and Rowe in this volume. In light of this, one should distinguish the creativity of Noether's methods – the creativity of her *technique* – from the creativity of their application – the creativity of the physical *representations* effected on the basis of her methods. In the post-1970s literature, the former are at least relatively well understood (albeit still not completely; see, for example, the contributions of Baez and Olver to this volume, which continue to add to such understanding); not so for the latter, in the case of which we are only beginning to explore a rich orchard of fruits.

The contributions to this volume pursue a number of distinct threads on both Noether's techniques and their applications to physics; these we will summarise here as succinctly as

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possible. We begin with Noether's history: both regarding specifically her work on variational problems, and more generally. In Chapter 1, **Yvette Kosmann-Schwarzbach** reviews this historical context of 'Invariante Variationsprobleme', from its prehistory, to its doldrums in the mid-twentieth century, to (as alluded to above) its revival in the post-1970s literature. Following on from this, in Chapter 2 **David Rowe** focuses on the interactions between Noether and Felix Klein in the years surrounding the appearance of her 'Invariante Variationsprobleme', and specifically on the role of differential invariants in Noether's two theorems. In Chapter 3, **Tomoko Kitagawa** focuses on another specific episode highlighted by Kosmann-Schwarzbach: namely, Noether's deliberations preceding her move to Bryn Mawr College.

Having presented this updated Noether history, we turn to the mathematics of her theorems, both generalisations and applications. In Chapter 4, **John Baez** illuminates the content of Noether's (first) theorem in the Hamiltonian context by pursuing a (Jordan–Lie) algebraic – rather than the traditional geometric – approach. In Chapter 5, **Kasia Rejzner** continues this study of Noetherian themes from an algebraic point of view (this time via homological algebras), by exploring the 'BV formalism' – an extension of the BRST prescription, in which auxiliary fields enjoying rigid symmetries are introduced, and in which the Noether charges associated with those symmetries are then quantised – from the point of view of perturbative algebraic quantum field theory.

Next, we turn to more philosophical questions regarding the explanatory arrow running from symmetries to conservation laws which is often (misleadingly, our authors would have it!) taken to be an important moral drawn from Noether's theorems. In Chapter 6, **Peter Olver** considers the significance of the fact that one can define infinitely many inequivalent Lagrangians invariant under a stipulated set of variational symmetries: should these Lagrangians be understood as encoding 'equivalent' physics – and if so, why? In Chapter 7, **Harvey R. Brown** questions the reasons for which, in light of the converse of Noether's first theorem, symmetries are often indeed considered to have this explanatory priority over conservation laws. In Chapter 8, **Mark Baker**, **Niels Linnemann**, and **Chris Smeenk** deploy the under-appreciated work of Bessel-Hagen in order to demonstrate how the converse of Noether's first theorem can be used to resolve ambiguities over what should be regarded as the 'physical' energy-momentum tensor in field theories.

The next three chapters address the significance of Noether's theorems in the context in which they were originally developed: the foundations of general relativity. In Chapter 9, **Sebastian de Haro** provides both a crystal-clear survey of the role of Noether's theorems in considerations of gravitational energy in general relativity, as well as a substantial novel contribution to recent philosophical discussions regarding the status of gravitational energy in that theory, including quasi-local notions. In Chapter 10, **James Read** continues these discussions, arguing that the pseudotensorial quantities obtained on application of Noether's theorems to general relativity are best interpreted physically from within the framework of 'perspectival realism'. Finally, in Chapter 11, **Laurent Freidel** and **Nicholas J. Teh** apply Noether's theorems in order to shed new light on three infamously vexed notions in the foundations of spacetime theories: (i) general covariance, (ii) the Principle of Relativity, and (iii) the status of conserved charges (including, again, gravitational energy).

Leading on from Teh and Freidel's contribution, the following pair of chapters address the question of the empirical significance of symmetry transformations in physics. In Chapter 12, **Henrique Gomes** brings to bear the symmetry (in)variance of Noether charges upon the *à la mode* question of whether and when gauge transformations (such as the familiar $U(1)$ transformations in electromagnetism) have 'direct empirical significance'. In Chapter 13, **David Wallace** develops a systematic account of when symmetry transformations in physics can relate empirically distinct states of affairs, again by considering subsystem-environment decompositions, and relates this to the content of Noether's theorems.

The final topic addressed by our authors is the gauge argument – i.e., the well-known (but infamously problematic!) principle by which new physical fields are introduced into one's physics in order to restore invariance under a certain set of transformations. In Chapter 14, **Henrique Gomes**, **Bryan Roberts**, and **Jeremy Butterfield** use Noether's second theorem to shed light on the gauge argument.

The above should suffice to convince the reader that the field of matters Noetherian is rich and blossoming. In a recent interview, Edward Witten defined beautiful mathematics as that which 'gives you more out than you put in'.[1] Given that, by their own admission, the contributions to this volume still only begin to scratch the surface of the potential stock of applications for Noether's theorems, the results of 'Invariante Variationsprobleme' surely constitute beautiful mathematics (and, via their applications, mathematical physics) in the highest degree. Insofar as physicists, mathematicians, and philosophers are united in their attraction to exquisite things, there is every reason to think that the epoch-making status of Noether's results will endure, and that 'Invariante Variationsprobleme' has secured its rightful place amongst the Immortals.

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[1] https://youtu.be/O3isFuQ2q2A. Accessed 8 July 2021.