APPLICATIONS TO A THEORY OF BONUS SYSTEMS

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This paper deals with bonus systems used in Denmark, Finland, Norway, Sweden, Switzerland and West Germany. These systems are studied by methods given by Mr. Loimaranta [1]). The bonus rules of Denmark and Sweden have been modified because they contradict to one of the assumptions of the theory.

It is assumed that the number of claims in a year follows the Poisson distribution with a mean λ. Further it is assumed that the value of λ is independent of time.
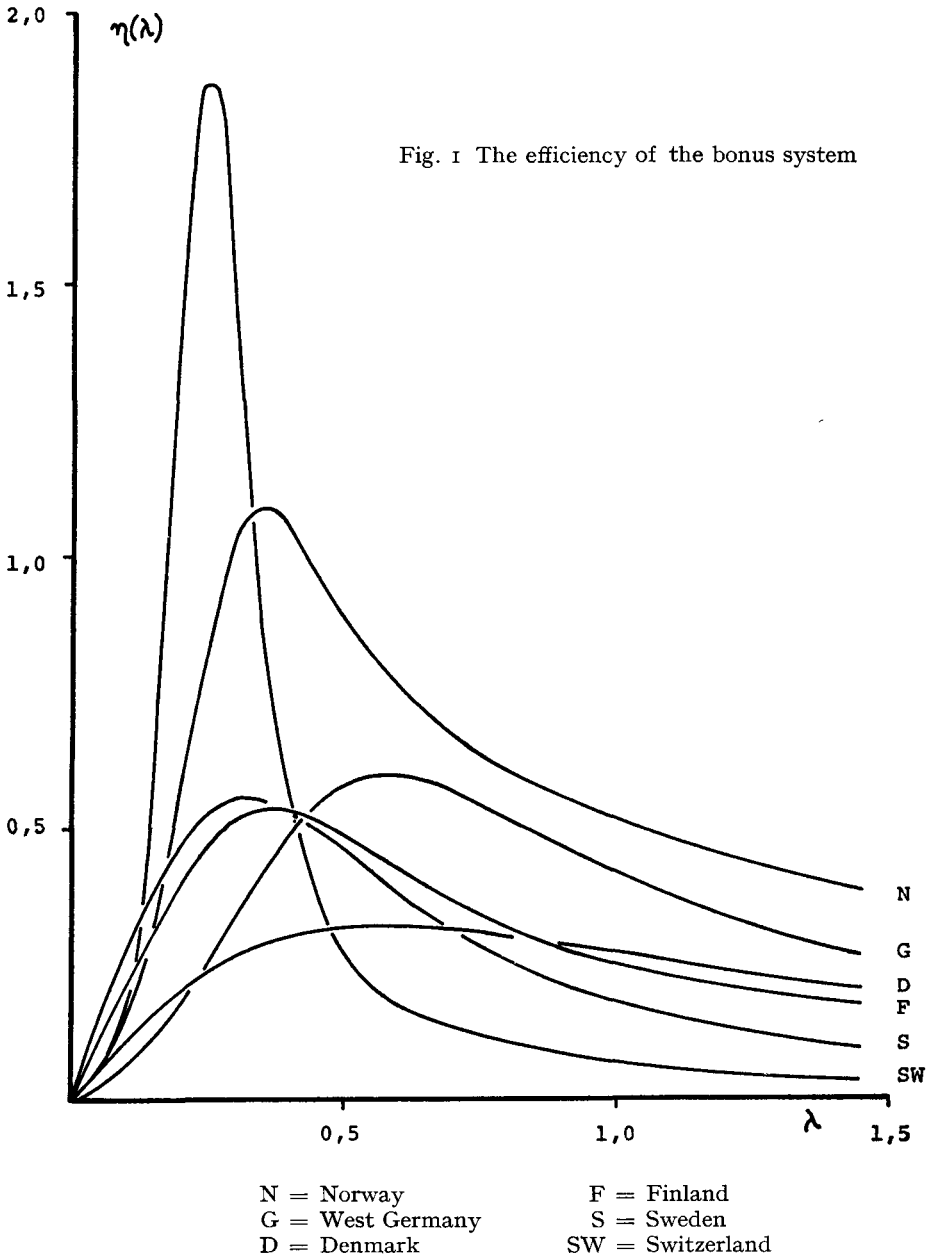
In each case bonus rules are given in form of transformations T_k defined in ref. 1., i.e. $T_k(i) = j$ when a policy moves from class i to class j after k claims. The class where a new policy starts from is called the initial class. Bonus scales, the vectors B, are normed so that the premium of the initial class is 100.
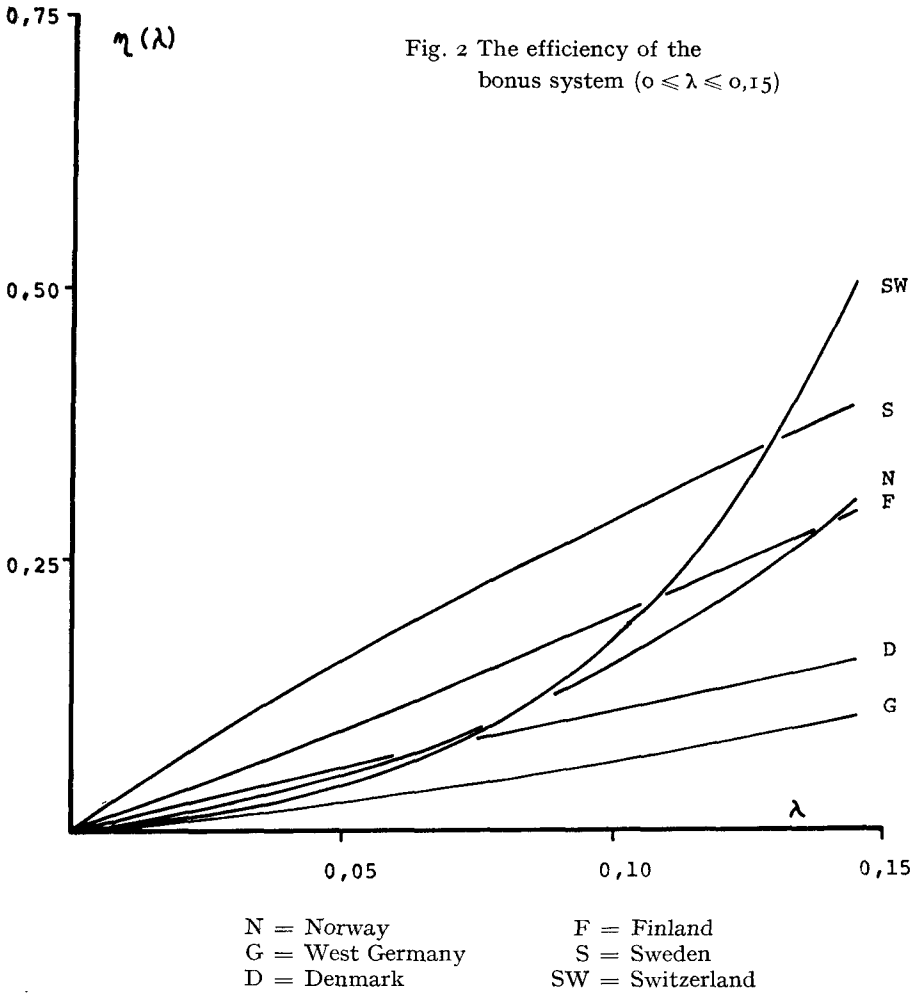
For each bonus system the efficiency η of the system and the discrimination power d of the bonus rules as a function of the mean claim frequency λ have been calculated. The graphs of these functions are presented in the figures 2 to 4 respectively. In the following pages, different bonus rules are described in detail and some simple analysis based on the curves on pp. 211-214 has been made. The calculations were performed quite recently and any deeper analysis of the results has not been possible because of lack of time.

1. *Denmark*

In order to be able to apply the theory of Markov chains to a bonus system we must require among others that the transition to a certain class depends only on the number of claims occurred during last period ignoring possible former claims. The Danish system has originally four bonus classes labeled from 0 to 3.

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[1]) K. Loimaranta: Some Asymptotic Properties of Bonus Systems Astin Colloquium 1970, Randers.

Fig. 1 The efficiency of the bonus system

N = Norway   F = Finland
G = West Germany   S = Sweden
D = Denmark   SW = Switzerland

Fig. 2 The efficiency of the
bonus system ($0 \leqslant \lambda \leqslant 0,15$)

N = Norway   F = Finland
G = West Germany   S = Sweden
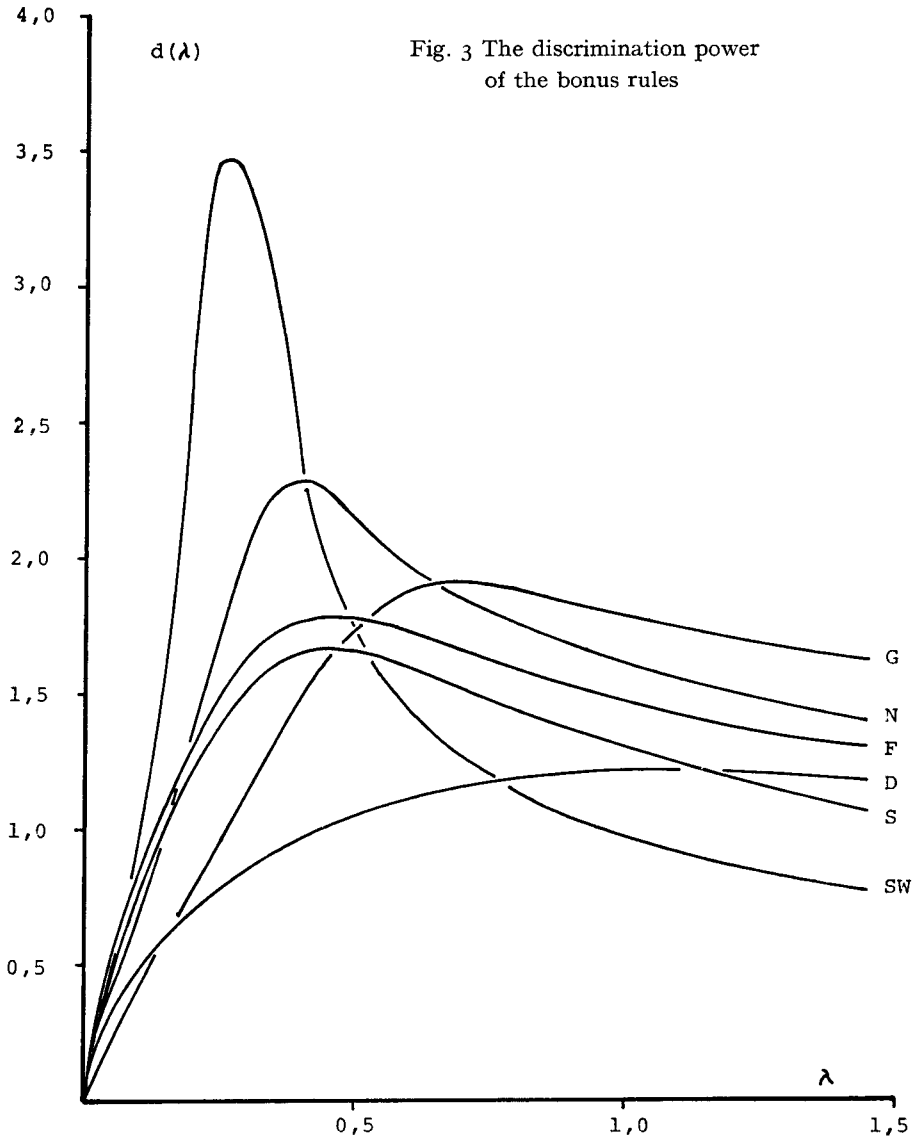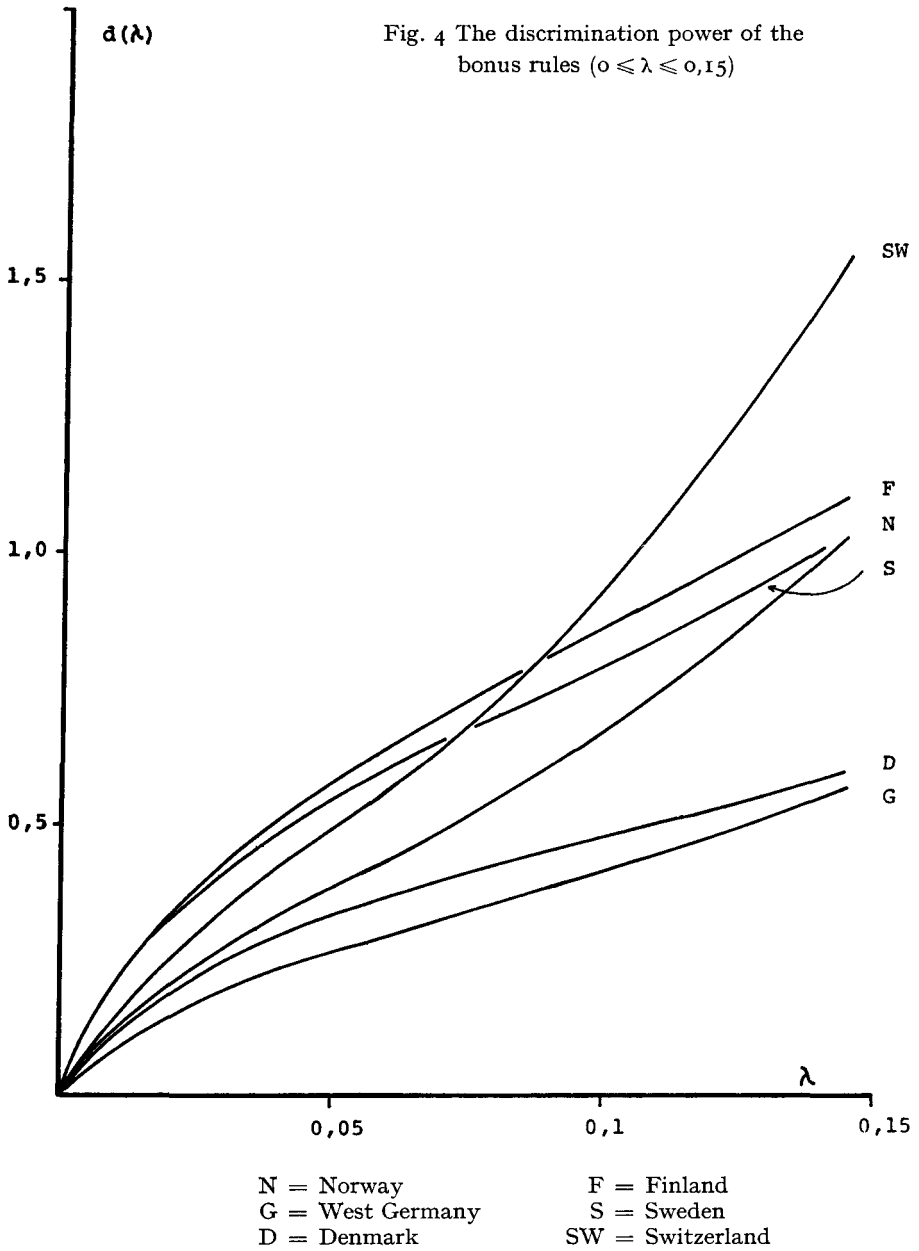D = Denmark   SW = Switzerland

d(λ)

Fig. 3 The discrimination power
of the bonus rules

N = Norway   F = Finland
G = West Germany   S = Sweden
D = Denmark   SW = Switzerland

Fig. 4 The discrimination power of the bonus rules $(0 \leqslant \lambda \leqslant 0,15)$

N = Norway   F = Finland
G = West Germany   S = Sweden
D = Denmark   SW = Switzerland

The bonus rules do not completely satisfy the above requirement: after two claims during one year *or* one claim during both of two years the new class will be 0. In order to avoid this difficulty we add to the labels another digit, which is 0 or 1 depending on whether there was a claim year before or not. The transformations T_k defining the bonus rules, and the premium vector B will then be as follows: (the new policies are placed in class 11)

class	T_0	T_1	$T_k(k \geqslant 2)$	B
00	11	00	00	133.33..
10	21	00	00	100
11	21	10	00	100
21	31	10	00	75
31	31	10	00	56.25

From the efficiency curves on page 212 we can see that the efficiency $\eta(\lambda)$ of the system for common values of λ is rather small but this is not so unusual. Only the efficiencies of Finnish and Swedish systems attain larger values when $\lambda < 0.05$. The function $\eta(\lambda)$ attains its maximum $\eta = 0.32$ when $\lambda = 0.55$. The discrimination power $d(\lambda)$ of the bonus rules (pp. 213-214) also seems to be relatively small for all values of λ, obviously the small number of classes is a restrictive factor.

2. Finland

The Finnish bonus system satisfies all the requirements stated in ref. 1. There are 13 bonus classes labeled from 0 to 12; new policies are placed in the class 2. The bonus rules and the premium vector are given in the table below:

class	T_0	T_1	T_2	T_3	T_4	$T_k(k \geqslant 5)$	B
0	2	0	0	0	0	0	140
1	2	0	0	0	0	0	120
2	3	1	0	0	0	0	100
3	4	2	1	0	0	0	100
4	5	2	1	0	0	0	80
5	6	3	2	1	0	0	70
6	7	4	2	1	0	0	60
7	8	5	3	2	1	0	50
8	9	5	3	2	1	0	50
9	10	5	3	2	1	0	50
10	11	5	3	2	1	0	50
11	12	5	3	2	1	0	50
12	12	6	4	2	1	0	40

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When $\lambda < 0.1$, the efficiency η for this system has a value higher than for most other systems under study. One of the reasons for this is that the ratio between the premiums of the first and last classes has the great value 3.5 (while for example in Danish system the ratio is 2.4). The maximum value of $\eta(\lambda)$ is attained when $\lambda = 0.35$. The discrimination power d of the Finnish bonus rules for small values of λ, has higher value than for other systems. One reason for this is that the occurrence of a single claim in higher bonus classes $(i \geq 6)$ causes the return to the class 5 or 6.

3. Norway

Also the bonus system of Norway can be treated without any modifications by means of the theory under consideration. There are 17 bonus classes (labeled from 0 to 16) and the initial class is the class 10. The premium vector B and the bonus rules are given in the table below:

class	T_0	T_1	T_2	T_3	T_4	T_5	T_6	T_7	$T_k(k \geqslant 8)$	B
0	8	0	0	0	0	0	0	0	0	300
1	8	0	0	0	0	0	0	0	0	280
2	8	0	0	0	0	0	0	0	0	260
3	8	1	0	0	0	0	0	0	0	240
4	8	2	0	0	0	0	0	0	0	220
5	8	3	1	0	0	0	0	0	0	200
6	8	4	2	0	0	0	0	0	0	180
7	8	5	3	1	0	0	0	0	0	160
8	9	6	4	2	0	0	0	0	0	140
9	10	7	5	3	1	0	0	0	0	120
10	11	8	6	4	2	0	0	0	0	100
11	12	9	7	5	3	1	0	0	0	90
12	13	10	8	6	4	2	0	0	0	80
13	14	11	9	7	5	3	1	0	0	70
14	15	12	10	8	6	4	2	0	0	60
15	16	13	11	9	7	5	3	1	0	50
16	16	14	12	10	8	6	4	2	0	40

From the graph corresponding to the Norwegian system (page 212) we can see that for the moderate and great values of $(\lambda > 0.2)$ the efficiency $\eta(\lambda)$ attain very high values. Only in the over-efficient system of Switzerland the function $\eta(\lambda)$ gets still higher values. The maximum value $\eta = 1.09$ is attained at the point $\lambda = 0.37$. Some reasons for this are easily founded. The bonus scale is very steep,

the ratio between the premiums in classes o and 16 is 7.5. Another reason for high values of $\eta(\lambda)$ with great values of λ is the quick return from malus classes: After one claimfree period a policy in classes 0-7 moves straight to the class 8. This is also reason for the high values of the discrimination power (see fig. 3). For small values of the claim frequency the system seems not to be very efficient and also $d(\lambda)$ attains values smaller than the corresponding values of Finnish, Swedish or Swiss systems.

4. *Sweden*

As in the Danish system the assumption (ii) in ref. 1. fails to be true. The transition to a certain class may depend on even six years old claims. However we can modify the Swedish bonus system by the same way as the Danish one. Originally there are seven bonus classes labeled from o to 6. In addition to this label we take a second digit and define this to be n if there have been no claims during last n years. So the number of possible classes will be 21, and the different classes and the transition rules are following: (the initial class is the class oo)

class	T_0	T_1	T_2	$T_k(k \geqslant 3)$	B
00	11	00	00	00	100
10	21	00	00	00	80
11	22	00	00	00	80
20	31	00	00	00	70
21	32	00	00	00	70
22	33	00	00	00	70
30	41	10	00	00	60
31	42	10	00	00	60
32	43	10	00	00	60
33	44	10	00	00	60
40	51	20	00	00	50
41	52	20	00	00	50
42	53	20	00	00	50
43	54	20	00	00	50
44	55	20	00	00	50
51	52	30	10	00	40
52	53	30	10	00	40
53	54	30	10	00	40
54	55	30	10	00	40
55	66	30	10	00	40
66	66	40	20	00	25

The graphs of $\eta(\lambda)$ and $d(\lambda)$ are very similar to the graphs of the Finnish system. However, for $\lambda > 0.5$ the efficiency of the Swedish system is rather low because there are no malus classes, and the transition to the highest bonus class is relatively slow. For smaller values of the claim frequency ($\lambda < 0,1$) the efficiency has the highest value of all systems under study, mainly because the maximum bonus is great, 75% of the premium of the first class. The discrimination power is also relatively high for common values λ.

5. *Switzerland*

In this system there are 22 bonus classes. The new policies are placed in the class 13. The bonus rules and the premium vector B are:

class	T_0	T_1	T_2	T_3	T_4	T_5	T_6	$T_k(k \geqslant 7)$	B
0	1	0	0	0	0	0	0	0	270
1	2	0	0	0	0	0	0	0	270
2	3	0	0	0	0	0	0	0	230
3	4	0	0	0	0	0	0	0	230
4	5	1	0	0	0	0	0	0	200
5	6	2	0	0	0	0	0	0	200
6	7	3	0	0	0	0	0	0	170
7	8	4	1	0	0	0	0	0	170
8	9	5	2	0	0	0	0	0	140
9	10	6	3	0	0	0	0	0	140
10	11	7	4	1	0	0	0	0	120
11	12	8	5	2	0	0	0	0	120
12	13	9	6	3	0	0	0	0	100
13	14	10	7	4	1	0	0	0	100
14	15	11	8	5	2	0	0	0	80
15	16	12	9	6	3	0	0	0	80
16	17	13	10	7	4	1	0	0	70
17	18	14	11	8	5	2	0	0	70
18	19	15	12	9	6	3	0	0	60
19	20	16	13	10	7	4	1	0	60
20	21	17	14	11	8	5	2	0	50
21	21	18	15	12	9	6	3	0	50

The efficiency $\eta(\lambda)$ attains the very high maximum $\eta = 1.87$ at $\lambda = 0.24$. Also the discrimination power has its maximum (3.5) about at the same point. One reason for high values of η is the steepness of the bonus scale. In addition, according to the bonus rules a policy moves three classes down if one claim occurs and one

class up after a claimfree year. If a claim occurs about every thirth or fourth year ($\lambda \approx 0.25\text{-}0.3$), the efficiency must attain a high value. The low values of η and d for $\lambda > 0.5$ are obviously due to the slow transition to the highest bonus class.

6. *West Germany*

A bonus system used in West Germany is presented here: In the system there are 8 bonus classes and the initial class is the class 3. The components of B-vector are the premiums of the "tariff group M". The bonus rules and the premium vector are:

class	T_0	T_1	T_2	T_3	$T_k(k \geqslant 4)$	B
0	3	0	0	0	0	150
1	3	0	0	0	0	130
2	3	1	0	0	0 ·	110
3	5	2	1	0	0	100
4	5	2	1	0	0	95
5	6	4	2	0	0	90
6	7	5	4	1	0	70
7	7	6	5	2	0	50

The efficiency and the discrimination power of this system are relatively great for $\lambda > 0.5$. Similar features were found in the Norwegian system and a reason could be of the same kind: the quick return from malus classes 0-2 to the class 3. For small values of the claim frequency λ the values of η and d are lower than the respective values of any other system under consideration.