FLUID TECHNIQUES AND EVOLUTION OF ANISOTROPY

RAINER SPURZEM
Institut für Astronomie und Astrophysik der Universität, Olshausenstraße 40, D-24098 Kiel
e-mail: supas028@astrophysik.uni-kiel.d400.de

Abstract.

Fluid dynamical techniques to model the dynamical evolution of star clusters, and their successors, gaseous models using an equation of heat conductivity to model relaxation effects, including anisotropy, are presented. The historical merits of such models are reviewed as well as the current status of their credibility, based on quantitative comparisons with other methods, like orbit-averaged Fokker-Planck solutions and direct N-body simulations.

1. Introduction with some historical remarks

Fluid or gasdynamical models of star clusters have successfully been used since many years. As Sugimoto (1985) expressed it ten years ago in the last IAU symposium on the dynamics of star clusters "we can understand physics of self-gravitating systems in terms of gaseous models in so far that their global nature and effects of self-gravity are concerned. ... They include gravothermal collapse/expansion ... and post-collapse evolution with gravothermal oscillation." In this review I want to discuss the progress, which has been made in the understanding of the relevance of fluid or gas dynamical models during the past ten years and convince the reader that there are prospects to create realistic models of stellar systems, including effects of particle-particle interactions, on the basis of such models.

I want to use this occasion first to stress some of the historical merits of the models, which have served for many important discoveries in the past. Thereafter one of them, the anisotropic gaseous model is presented in some more detail, its comparison with direct N-body simulations and direct solutions of the orbit-averaged Fokker-Planck equation is discussed, and finally first results on the path towards real models of star clusters are presented.

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The term fluid dynamical model was used by Larson in his seminal series of papers on star clusters (Larson 1969, 1970a, b). He derived dynamical equations from the fundamental kinetic Boltzmann equation, including a Fokker-Planck collisional term based on the derivation of Rosenbluth, McDonald & Judd (1958). He assumed that some higher order moments of the velocity distribution function can be derived as in the case of a Maxwell-Boltzmann distribution. Using a series expansion of the distribution function in Legendre polynomials up to second order he could derive the local collisional anisotropy decay timescale (note, however, that his derivation was not fully self-consistent, since he used an isotropized background distribution function). In a direct comparison between Larson's models, direct N-body simulations, and Monte-Carlo models of star clusters Aarseth, Hènon & Wielen (1974) showed, however, that Larson's models deviated from the other models in late core collapse.

The physics of global instabilities of self-gravitating gas spheres in the linear approximation has been studied by Hachisu & Sugimoto (1978), who showed that the gravothermal catastrophe, detected by Antonov (1962) and Lynden-Bell & Wood (1968) could be understood as a global instability against the redistribution of heat in a self-gravitating isothermal gas cloud. It was the effect of the negative specific heat in the core of self-gravitating systems, which caused the runaway. In a similar approach Hachisu (1979) studied the so-called "gravo-gyro" catastrophe, caused by a negative specific momentum of inertia in self-gravitating systems. However, until recently there has not been paid much attention to models of rotating star clusters (but see C. Einsel & R. Spurzem, and P.Y. Longaretti, this volume). Spurzem (1991) showed that thermodynamic arguments (maximizing an entropy functional) could also be used to understand the linear response of anisotropy to a redistribution of heat.

Hachisu et al. (1978) first utilized the gas dynamical equations (which can be seen as isotropic version of the moment equations of Larson) with an equation of heat transfer and various scalings of the heat conductivity $\Lambda \propto \rho^{\alpha}T^{\beta}$ to model a star cluster. This is a phenomenological closure of the moment equations, instead of a specialization of the distribution function as in Larson's case. Lynden-Bell & Eggleton (1980) found that $\Lambda \propto \rho T^{-1/2}$ is the physical case in which the conductivity scales with the standard stellar dynamical two-body relaxation rate. With such a model fair agreement (e.g. of the self-similar solution for gravothermal collapse) could be reached with the at that time recent competitive models based on the numerical solution of the 1D orbit-averaged Fokker-Planck equation (Cohn 1980). But it was much less clear, to what extent these models were really appropriate for real star clusters like globulars.

Although the process of core collapse can be understood without inclusion of anisotropy, the prospect of modelling real star clusters, which exhibit anisotropy (e.g. Lupton et al. 1987), would require anisotropic models. Here anisotropy is understood as a difference between the radial and tangential velocity dispersions ("temperatures") in a spherically symmetric system. Unfortunately the very efficient scheme of Cohn (1980) to numerically solve the 1D orbit-averaged Fokker-Planck equation could not easily be generalized to the 2D case. So until very recently (Takahashi 1995, K. Takahashi, this volume) anisotropic Fokker-Planck models were not available (with the exception of an early attempt by Cohn (1979), who did not continue this work because of problems with numerical accuracy). Remarkably early, however, Stodółkiewicz (1982, 1986) developed a Monte-Carlo method based on Hènon's work, which was able to numerically simulate large N

star clusters including anisotropy. Such a model has been revisited recently by M. Giersz (this volume).

The moment equations, as they were used e.g. by Larson (1970a) include anisotropy; however, it is less obvious how to generalize the closure equation of heat transport in an anisotropic case. So-called one-flux and two-flux closures have been examined, (Bettwieser & Spurzem 1986). In a more systematic study (Louis & Spurzem 1991) it could be shown, that at least the self-similar solutions in both cases were very similar, and even close to the solutions of a higher order moment model of Louis (1990).

Soon the question what happens to globular clusters after core collapse was raised. After Hènon (1975) and Stodółkiewicz (1982) first extended their Monte-Carlo models to the post-collapse phase Inagaki & Lynden-Bell (1983) showed that there is a self-similar post-collapse solution with a central pointlike energy source by using a gaseous sphere model. Bettwieser & Sugimoto (1984), and Heggie (1984) put into their gas sphere models a distributed heating term tailored to describe the energy generation in the core due to formation and hardening of three-body binaries. Here a post-collapse model with a non-singular core was reached, in the case of the first two papers large amplitude gravothermal oscillations were found. Until such oscillations were also detected in the solutions of the Fokker-Planck equation (Cohn, Hut & Wise 1989) it was widely believed that they are an artifact of the gaseous model or, even worse, of particular codes to solve the model equations. Goodman (1987) proved that post-collapse oscillations can be understood as an instability of a self-similar solution. Both gaseous and Fokker-Planck models exhibit a rich dynamical behaviour of their oscillating solutions, with period doublings and possibly chaotic behaviour, similar to the non-linear dynamics in the case of the Rössler- and Lorentz attractors (see e.g. Jackson 1990), which originate from heat conduction problems as well (Breeden et al. 1994, Spurzem 1994).

The question whether large amplitude gravothermal oscillations occur in real, discrete N-body systems made the need clear for quantitative, detailed comparisons between direct simulations and models based on the Fokker-Planck equation (gaseous models as well as models based on orbit-averaging, henceforth denoted as statistical models). It has long been argued (cf. e.g. Inagaki 1986) that stochastic fluctuations at core bounce (which always occurs at a very small core particle number, provided three-body binary formation dominates and there were no primordial binaries) destroy the characteristic temperature inversion, which creates the steady gravothermal expansion in the statistical models. To start with the most recent result, J. Makino (this volume) has shown that such a temperature inversion indeed occurs in a real N-body system of 32k particles near core bounce and thus should trigger a gravothermal reexpansion.

But there are more questions than that of gravothermal oscillations. The validity of the Fokker-Planck approximation (uncorrelated small angle two-body encounters dominate the evolution) and the possibility to model energy transport by two-body encounters in a nearly collisionless (mean free path long compared to systems dimensions) stellar system by a phenomenological heat transport equation, which can be strictly derived only in the case of a collisional Boltzmann gas, are open theoretical questions. Surprisingly there have not been many quantitative comparative studies between direct N-body and other models (despite of considerable development of software and hardware. except of the pioneering study of Aarseth, Hènon & Wielen (1974). Recently, however, Giersz & Heggie (1994a,

b) and Giersz & Spurzem (1994, henceforth GS) have shown that for the equal point mass case there is fair agreement between all models for $N \leq 2000$, provided the stochastic N-body fluctuations are overcome by ensemble averaging of several statistically independent N-body simulations.

2. The anisotropic gaseous model

To be definite and clear for the reader unfamiliar with gaseous models I would like to give in the following a short, but complete description of the variables and equations used for a standard model.

Let the dependent variables be the mass M_r contained in a sphere of radius r, the local mass density ρ, radial and tangential pressure p_r, p_t, bulk mass transport velocity u, and transport velocities v_r, v_t of the radial and tangential energy, respectively. As auxiliary quantities the radial and tangential 1-D velocity dispersions $\sigma_r^2 = p_r/\rho$, $\sigma_t^2 = p_t/\rho$, the average velocity dispersion $\sigma^2 = (\sigma_r^2 + 2\sigma_t^2)/3$, the anisotropy $A = 2 - 2\sigma_t^2/\sigma_r^2$ and the relaxation time

$$T = \frac{9}{16\sqrt{\pi}} \frac{\sigma^3}{G^2 m \rho \log(\gamma N)} \tag{1}$$

in the definition of Larson (1970a) are used, where N is the total particle number of the star cluster, m the individual stellar mass and γ a numerical constant whose value will be discussed below. The equations are

$$\frac{\partial M_r}{\partial r} = 4\pi r^2 \rho \ \ ; \ \ \frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho u r^2) = 0 \tag{2}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{G M_r}{r^2} + \frac{1}{\rho}\frac{\partial p_r}{\partial r} + 2\frac{p_r - p_t}{\rho r} = 0 \tag{3}$$

$$\frac{\partial p_r}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(p_r u r^2) + 2p_r\frac{\partial u}{\partial r} + \frac{3}{r^2}\frac{\partial}{\partial r}(p_r(v_r - u)r^2)$$
$$- 4\frac{p_t(v_t - u)}{r} = -\frac{2}{3}\frac{p_r - p_t}{\lambda_A T_A} + \left(\frac{\delta p_r}{\delta t}\right)_{\rm bin3} \tag{4}$$

$$\frac{\partial p_t}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(p_t u r^2) + 2\frac{p_t u}{r} + \frac{1}{r^2}\frac{\partial}{\partial r}(p_t(v_t - u)r^2)$$
$$+ 2\frac{p_t(v_t - u)}{r} = \frac{1}{3}\frac{p_r - p_t}{\lambda_A T_A} + \left(\frac{\delta p_t}{\delta t}\right)_{\rm bin3} \tag{5}$$

$$v_r - u + \frac{\lambda}{4\pi G \rho T}\frac{\partial \sigma^2}{\partial r} = 0 \ \ ; \ \ v_r = v_t \tag{6}$$

The net transport velocities for radial and tangential energy $(v_r - u)$ and $(v_t - u)$ can be derived from the energy fluxes F_r and F_t (which are identified with the third order moments of the velocity distribution) by dividing out a convenient multiple of the relevant pressure ($2p_t$ for $(v_t - u)$, $3p_r$ for $(v_r - u)$). The reader interested in more details about this and the connection of the variables to moments of the stellar velocity distribution is referred to Louis & Spurzem (1991).

The numerical constants λ_A, λ and γ occurring in Eqs. (4) to (6) are related to the timescales of collisional anisotropy decay and heat transport, and to the Coulomb logarithm, respectively. λ is related to the standard C constant in isotropic gaseous models (see e.g. Heggie 1984) by $\lambda = 2.7\sqrt{\pi}C$. T_A is the anisotropy decay timescale for an anisotropic local velocity distribution function; for a generalization of Larson's (1970a) distribution function (series of Legendre polynomials) including anisotropy it is $T_A = 10\,T/9$ (Louis & Spurzem 1991). λ_A is discussed in Sect. 4. Additional terms due to the average heating by formation and hardening of three body binaries (see e.g. Cohn 1985) are

$$\left(\frac{\delta p_r}{\delta t}\right)_{\rm bin3} = \frac{2}{3} C_b \frac{\rho^3}{m^2\sigma^2}\left(\frac{Gm}{\sigma}\right)^5 \quad ; \quad \left(\frac{\delta p_t}{\delta t}\right)_{\rm bin3} = \left(\frac{\delta p_r}{\delta t}\right)_{\rm bin3} . \qquad (7)$$

This is an isotropic energy input. It is shown in GS and Giersz and Heggie (1994a, b) that for particle numbers between $N = 1000$ and $N = 10.000$ the best agreement between direct N-body calculations, direct solutions of the orbit-averaged Fokker-Planck equation and this anisotropic gaseous models is achieved for one set of parameters, namely $\lambda = 0.4977$ (i.e. $C = 0.104$), $\gamma = 0.11$, $\lambda_A = 0.1$, and $C_b = 90$. The latter value used to be the standard value derived from theoretical arguments, based on a numerical factor of $\tilde{C} = 0.9$ in the formula for the formation rate of three-body binaries (Hut 1985). Recently, Goodman & Hut (1993) argue, that $\tilde{C} = 0.75$ is a better value, but still within some uncertainty. The results of comparisons with N-body simulations show that $C_b = 90$ is a fairly reasonable value, but within the uncertainty $C_b = 75$ (which would ensue with the new formation rate) cannot be ruled out. Note that the value of γ found empirically is somewhat smaller than e.g. Spitzer's (1987) standard value ($\gamma = 0.4$).

As for multi-component models the simplest approach is to take dynamical equations like those above (including the closure equation) for each component separately, then coupling them by gravity (via Euler's equation) only and self-consistent collisional terms for the decay of anisotropy and the exchange of energy. The results for the collisional terms have been reported in the Appendix of Spurzem & Takahashi (1995, henceforth ST). Therein we also argue, that such a model is in much better agreement with direct Fokker-Planck solutions than previously argued (Bettwieser & Inagaki 1985). The main reason is a much more complicated additional coupling between the components within the conductivity equation adopted by Bettwieser & Inagaki (1985) as compared with the new model (Spurzem 1992).

3. Comparisons

Figs. 1 to 3 are taken from GS and visualize the quality of agreement between N-body an standard gaseous model for an equal mass system in pre- and post-collapse (low N, averaged N-body simulation). It also illustrates the influence of a possible variation of C_b. Note that the agreement is non-trivial, because the main free parameter in the gaseous model (λ has been fixed already by comparison with the orbit-averaged Fokker-Planck model.

Fig. 4 shows a similar comparison, but now for an *individual* $N = 10000$-body simulation, compared with the standard anisotropic gaseous model ($C_b = 90$). The

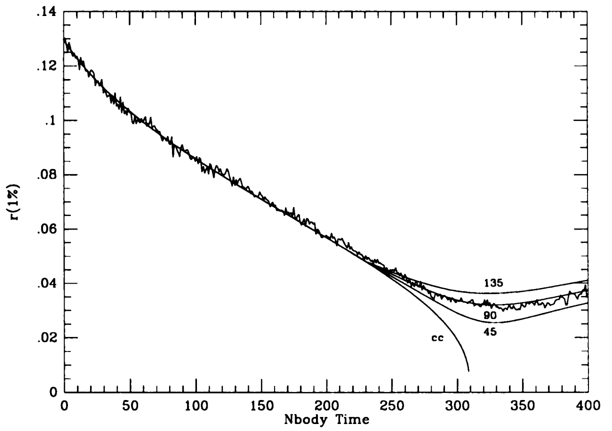

Figure 1. Evolution of the 1% Lagrangian radius in an averaged $N = 1000$ N-body model in comparison to the standard anisotropic gaseous model for varying strengths of the binary energy generation (curves labelled by C_b-value, see main text). The curve labelled *cc* is the one for $C_b = 0$, i.e. pure core collapse.

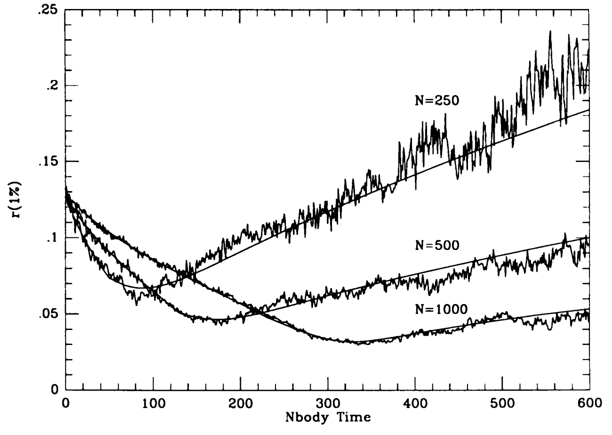

Figure 2. Evolution of the 1% Lagrangian radius in averaged N-body models for $N = 250, 500, 1000$ (here taken $C_b = 55$, $C_b = 70$, $C_b = 90$, respectively). For the reasons why to take smaller C_b for small N see discussion in GS.

good agreement in pre-collapse again underlines that the Fokker-Planck approximation is valid in this evolutionary phase, however, we now note a significant discrepancy in collapse times and oscillations of the N-body central density which do not show up in the gaseous model. The discrepancy in collapse time is seen as a result of poor statistics. One can estimate an expected spread in collapse times for an $N = 10000$ system of about 130 time units, so the actual collapse time is just 1.4 σ apart from the average (Spurzem & Aarseth 1996). How about the oscillations of the N-body model? Are they gravothermal? The author has spent considerable effort in looking at these data for inversions of the temperature gradients, or its signature in the cumulative distribution function to find any trace of this necessary feature of gravothermal oscillations, as it was already clearly outlined in the original paper by Bettwieser & Sugimoto (1984). Since they could *not* be found the oscillations are interpreted as binary-driven, generated by stochastic binary

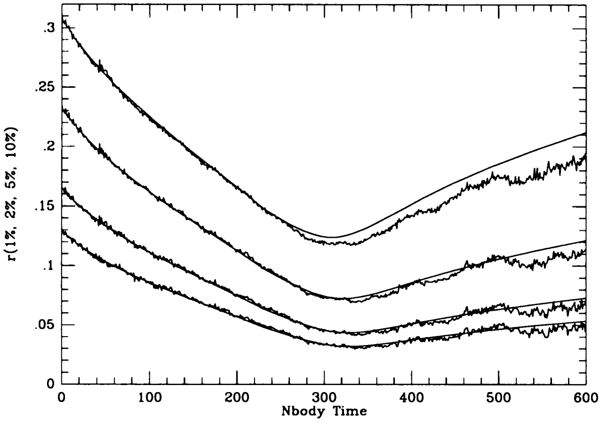

Figure 3. Evolution of the 1 to 10% Lagrangian radius in an averaged $N = 1000$ N-body model in comparison to the standard anisotropic gaseous model ($C_b = 90$).

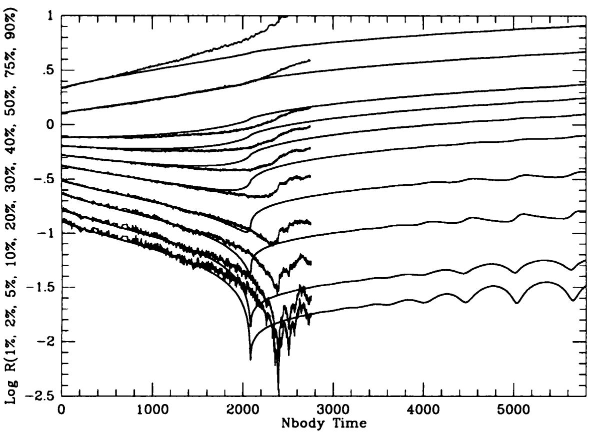

Figure 4. Evolution of the 1 to 10% Lagrangian radius in a single $N = 10000$ N-body model in comparison to the standard anisotropic gaseous model ($C_b = 90$).

encounter events, which do not persist for long times *after* the binary activity has ceased (see also the critical assessment of what real gravothermal oscillations are by S.L.W. McMillan, this volume). Such interpretation is supported by observations in the N-body simulation that at the beginning of an expansion phase there is a strong binary scattering event and that the expansion ceases after an active binary has been lost by escape or becomes inactive (e.g. by ejection into the halo) (Spurzem & Aarseth 1996).

It is interesting to note that Takahashi (1995, and this volume) finds in his new 2D Fokker-Planck models a collapse time for an $N = 10000$ model, which agrees much better with the here presented N-body data. He claims that the collapse time is considerably longer in the anisotropic case, which coincides with a result published by Louis (1990), based on a fifth-order moment model. But presently it is difficult to judge about this conjecture from the viewpoint of N-body simulations, because the variations of collapse times in $N = 10000$ N-body simulations is of the same order as the postulated difference between isotropic and anisotropic

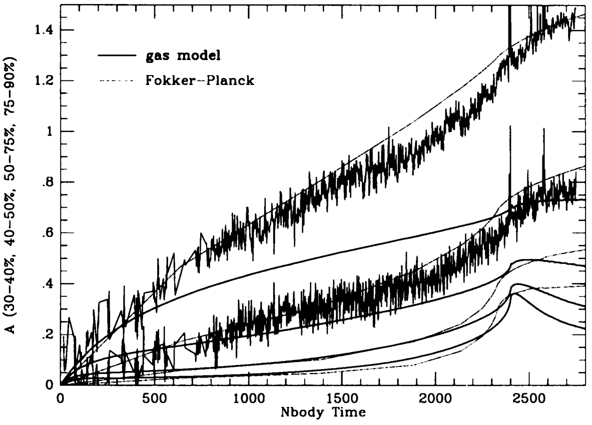

Figure 5. Anisotropy averaged between the indicated Lagrangian radii for direct $N = 10000$ body simulation (fluctuating curves, only for two outermost zones) and for comparison 2D Fokker-Planck model by Takahashi (1995) and standard anisotropic gaseous model. Times were rescaled such that core bounce occurs at the same point of the abscissa for all models in order to allow a better comparison.

models. So we have to wait for better statistics and more N-body models with larger N. Since the scatter in collapse times in relation to the collapse time itself becomes smaller with increasing N a few larger N simulations will give much more significant results on the average collapse time.

4. The anisotropy

It turns out that a canonical value of λ_A in the gaseous model equations, as it would turn out for some standard anisotropic distribution function, yields much larger anisotropy than in the N-body models. Reasonable agreement inside the the half-mass radius can be achieved for $\lambda_A = 0.1$ for all cases of N, single and two-mass models (see GS and ST). Such a result is theoretically not well understood, could be related, however, to the fact, that the local approximation used in the gaseous model becomes obsolete in the halo regions, where anisotropy prevails. Stars on radial orbits suffer encounters during there passage through the core, where there is a much higher density, thus the collisional decay of anisotropy ought to be shorter than by a local estimate in the halo, which is consistent with the above findings. Since the orbit average of the direct Fokker-Planck solution includes for a given orbit contributions to the diffusion coefficients from different radii such a discrepancy should not occur in 2D orbit-averaged Fokker-Planck models. Indeed first results by Takahashi (1995, this volume) point this out. In Fig. 5 we show the anisotropy for the outer mass shells ($N = 10000$) in comparison between his new 2D Fokker-Planck results, standard anisotropic gaseous model and the direct N-body simulations. The 2D Fokker-Planck model agrees in the outermost shells much better with the N-body system. For the intermediate shells (Lagrangian radii containing 30 to 50 % of total mass) fluctuations of the measured N-body anisotropy were too large to allow for a reasonable plot.

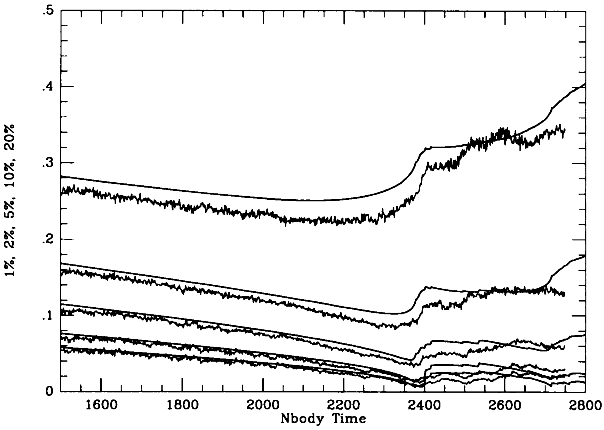

Figure 6. Lagrangian radii containing 1 to 20 % of the total mass for an anisotropic gaseous model with stochastic binaries compared to the direct $N = 10000$ N-body simulation. To compare the characteristic features of the post-collapse evolution the data have been scaled in time such that both models have core bounce at the same point of the abscissa, as in Fig. 5.

5. Outlook and Conclusion

It has been shown that the anisotropic gaseous model of star clusters matches for a wide range of N and single and two-mass star clusters, and for a standard parameter choice, very well with expectations from direct solutions of the orbit-averaged Fokker-Planck equation and averaged direct N-body simulations. The underlying assumptions (Fokker-Planck approximation, small angle two-body encounters dominate evolution in core collapse, and their energy transport properties can be modelled by a heat transfer equation analogous to gas dynamics, but with a specially tailored conductivity to account for the stellar dynamical relaxation timescale) and a statistical treatment of the average heating rate by formation and hardening of three-body binaries are supported by this result.

Differences between gaseous models and other solutions still remain in the case of the anisotropy in the outer halo regions and and in the stochastic behaviour of *individual* N-body models compared with the statistical gaseous model. The first problem might be overcome in higher order moment models, because it is related to the anisotropy decay in the collisional terms, which rely on certain assumptions on the functional form of the velocity distribution. The second problem is tackled by a stochastic treatment of binaries as in a Monte-Carlo model (cooperation with M. Giersz in progress, see a very similar approach for isotropic Fokker-Planck models by Takahashi & Inagaki 1991). Fig. 6 shows a gaseous model with stochastic binaries as compared with the N-body simulation; the characteristic behaviour of the real N-body system is matched very well. Including stochastic binaries and other effects like stellar evolutionary mass loss into the model in the near future will generate a very efficient realistic model of a star cluster.

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