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CORRIGENDUM

Thermodynamics of the Katok map – CORRIGENDUM

Y. PESIN, S. SENTI and K. ZHANG

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1. *Introduction*

While all main results of the paper [**PSZ**] remain correct as stated, the proofs of the technical Lemmas 5.2 and 5.3 contain mistakes. This affects the proof of Lemma 6.4 and we provide necessary modifications to the proofs of these lemmas.

2. *Lemma 5.2*

In Lemma 5.2, the third and fourth inequalities should be replaced by the single inequality

$$|s_1(t)| \geq |s_1(b)|(1 + 2^\alpha C_1 s_1^{2\alpha}(b)(b-t))^{-1/2\alpha}, \quad T_1 \leq t \leq b \leq T.$$

While equation (12) is correct, it is not longer needed and can be ignored.

The proof goes as follows. To show the second and fourth inequalities, observe that, as shown in the paper (see page 774),

$$\frac{ds_1(t)}{dt} \geq \frac{\log\lambda}{r_0^\alpha}\, s_1^{2\alpha+1}(t), \qquad \frac{ds_2(t)}{dt} \leq -\frac{\log\lambda}{r_0^\alpha}\, s_2^{2\alpha+1}(t).$$

Integrating the first of the above inequalities from t to b and the second one from a to t yields

$$s_1^{-2\alpha}(b) - s_1^{-2\alpha}(t) \leq -C_1\,(b-t), \quad s_2^{-2\alpha}(t) - s_2^{-2\alpha}(a) \geq C_1\,(t-a),$$

with $C_1 = 2\alpha\log\lambda/r_0^\alpha$, and the second and the fourth inequalities follow. The proof of the first and third inequalities is similar.

3. *Lemma 5.3*

In Lemma 5.3 the second inequality should be replaced by

$$\Delta s_2(t) \leq \frac{\Delta s_2(T_1)}{s_1(T_1)} s_1(t) \left(\frac{1 + 2^\alpha C_1 s_1^{2\alpha}(b)(b-t)}{1 + 2^\alpha C_1 s_1^{2\alpha}(b)(b-T_1)} \right)^\beta, \quad T_1 \leq t \leq b \leq T.$$

Indeed, a sign mistake in the application of Gronwall's inequality to the proof of the second inequality appears in the original version of the proof. The correct proof is as follows:

$$\begin{aligned}\chi(t) &\leq \chi(T_1)\exp\bigg(-\frac{(1-\mu)\log\lambda}{2r_0^\alpha}\int_{T_1}^t s_1^{2\alpha}(\tau)\,d\tau\bigg)\\ &\leq \chi(T_1)\exp\bigg(-\frac{(1-\mu)C_1}{4}s_1^{2\alpha}(b)\int_{T_1}^t (1+2^\alpha C_1 s_1^{2\alpha}(b)(b-\tau))^{-1}\,d\tau\bigg)\\ &\leq \chi(T_1)\exp\bigg(\frac{(1-\mu)C_1}{4\alpha}\frac{1}{C_1 2^\alpha}\log(1+2^\alpha C_1 s_1^{2\alpha}(b)(b-\tau))\bigg|_{\tau=T_1}^t\bigg)\\ &= \chi(T_1)\bigg(\frac{1+2^\alpha C_1 s_1^{2\alpha}(b)(b-t)}{1+2^\alpha C_1 s_1^{2\alpha}(b)(b-T_1)}\bigg)^{(1-\mu)/\alpha 2^{\alpha+2}},\end{aligned}$$

thus proving the second inequality.

4. *Lemma 6.4*

The changes in the statements of Lemmas 5.2 and 5.3 require a modification of the proof of case 2 in estimating δ_n; see page 789. Lines 19–21 on this page should read:

By (22), (39) and Lemma 5.3 for all $T_l \leq t \leq m_{l+1}$ and $0 \leq \tau \leq 1$,

$$\begin{aligned}\|\Delta s(n+\tau)\| &\leq 2\Delta_s(n+\tau)\\ &\leq 2\frac{\Delta_2(T_l)}{s_1(T_l)}s_1(n+\tau)\bigg(\frac{1+2^\alpha C_1 s_1^{2\alpha}(m_{l+1})(m_{l+1}-n-\tau)}{1+2^\alpha C_1 s_1^{2\alpha}(m_{l+1})(m_{l+1}-T_l)}\bigg)^\beta.\end{aligned}$$

Page 790, line 1 should now read:

Applying Lemma 5.3 on the time interval $[m_l, n+1]$ and using (37) and the above estimates,

$$\begin{aligned}\|A_n - B_n\| &\leq C\sup_{0\leq\tau\leq 1}[s_1^{2\alpha-1}(n+\tau)\|\Delta s(n+\tau)\|]\\ &\leq 2C\frac{\Delta s_2(T_l)}{s_1(T_l)}\sup_{0\leq\tau\leq 1}\bigg[s_1^{2\alpha}(n+\tau)\bigg(\frac{1+2^\alpha C_1 s_1^{2\alpha}(m_{l+1})(m_{l+1}-n-\tau)}{1+2^\alpha C_1 s_1^{2\alpha}(m_{l+1})(m_{l+1}-T_l)}\bigg)^\beta\bigg].\end{aligned}$$

The proof should continue as follows. By the fourth inequality in (the new version of) Lemma 5.2, we obtain

$$\begin{aligned}\|A_n - B_n\| \leq{}& C\frac{\Delta s_2(T_l)}{s_1(T_l)}s_1^{2\alpha}(m_{l+1})\\ &\times\sup_{0\leq\tau\leq 1}[(1+C_1 s_1^{2\alpha}(m_{l+1})(m_{l+1}-n-\tau)^{-1})]\\ &\times\sup_{0\leq\tau\leq 1}\bigg[\bigg(\frac{1+2^\alpha C_1 s_1^{2\alpha}(m_{l+1})(m_{l+1}-n-\tau)}{1+2^\alpha C_1 s_1^{2\alpha}(m_{l+1})(m_{l+1}-T_l)}\bigg)^\beta\bigg].\end{aligned}$$

Since $s_2(m_l)$ and $s_1(m_{l+1})$ are both uniformly bounded,

$$\begin{aligned}\frac{|\Delta s_2(T_l)|}{s_1(T_l)}s_1(m_{l+1}) &= \frac{|\Delta s_2(T_l)|}{s_2(T_l)}s_1(m_{l+1})\\ &\leq \frac{|\Delta s_2(m_l)|}{s_2(m_l)}s_1(m_{l+1}) \leq C|\Delta s_2(m_l)|,\end{aligned}$$

It is shown in the paper that $|\Delta s_2(m_l)|/d(x_{n_{2l+1}}, y_{n_{2l+1}})$ is uniformly bounded, and hence,

$$\frac{\|A_n - B_n\|}{d(x_{n_{2l+1}}, y_{n_{2l+1}})} \le C\left[\frac{(1 + 2^\alpha C_1 s_1^{2\alpha}(m_{l+1})(m_{l+1} - n))}{(1 + 2^\alpha C_1 s_1^{2\alpha}(m_{l+1})(m_{l+1} - T_l))}\right]^\beta \times (1 + C_1 s_1^{2\alpha}(m_{l+1})(m_{l+1} - n - 1)^{-1}.$$

Finally, we have

$$\sum_{n=T_l}^{m_{l+1}} \frac{\|A_n - B_n\|}{d(x_{n_{2l+1}}, y_{n_{2l+1}})} \le C(1 + 2^\alpha C_1 s_1^{2\alpha}(m_{l+1})(m_{l+1} - T_l))^{-\beta} \times \sum_{n=T_l}^{m_{l+1}} \frac{(1 + 2^\alpha C_1 s_1^{2\alpha}(m_{l+1})(m_{l+1} - n))^\beta}{1 + C_1 s_1^{2\alpha}(m_{l+1})(m_{l+1} - n - 1)}.$$

The expression on the right-hand side is bounded by a constant which is independent of l and numbers T_l and m_l. This completes the proof of case 2 in estimating δ_n.

5. *Some typos*

We have discovered some typos in the original version of the article which we correct here.

- In equation (32) on page 784 a term $\sqrt{1+\mu^2}$ should be added in the middle inequality.
- Equation (40) should read

$$\begin{aligned}\|\Delta s(n+\tau)\| &\le 2\Delta s_2(n+\tau)\\ &\le 2\Delta s_2(m_l)\frac{s_2(n+\tau)}{s_2(m_l)}(1 + 2^\alpha C_1 s_1^{2\alpha}(m_l)(n + \tau - m_l))^{-\beta}.\end{aligned}$$

6. *References*

The following references should be updated:

[PSS17] Ya. Pesin, S. Samuel and F. Shahidi. Area preserving surface diffeomorphisms with polynomial decay rate are ubiquitous. *Preprint*, 2020, http://arxiv.org/abs/2003.08503.

[SZ17] F. Shahidi and A. Zelerowicz. Thermodynamics via inducing. *J. Stat. Phys.* **175**(2) (2019), 351–383.

After our paper was published we learned about the preprint by T. Wang [**Wan**], in which the author shows that for sufficiently small values of the parameters $\alpha > 0$ and $r > 0$ the Katok map has unique equilibrium measure μ_t corresponding to the geometric potential φ_t for all values of $t < 1$. It is, however, not known if the measure μ_t has exponential decay of correlations for the values of $t \le t_0$ where the number t_0 is determined by formula (10) in the paper.

References

[**PSZ**] Y. Pesin, S. Senti and K. Zhang. Thermodynamics of the Katok map. *Ergod. Th. & Dynam. Sys.* **39**(3) 764–794.

[**Wan**] T. Wang. Unique equilibrium states, large deviations and Lyapunov spectra for the Katok Map. *Preprint*, 2019, https://arxiv.org/abs/1903.02677v2.