Eric Mendelsohn, Ph.D., Full embeddings and the category of graphs with applications to topology and algebra, McGill University. September 1968. (Supervisor: Z. Hedrin)

The problem of graphs with given semigroup is generalized in several directions. It is shown that there are graphs with given semigroup without cycles, without short underlying undirected cycles, of chromatic number α for all $\alpha \geq 2$, and containing a given graph as a full subgraph. Thus the independence of the monoid of endomorphisms of a graph and the monoid of endomorphisms of a subgraph is established. Category theory is used to translate this result to give the independence of the monoid of endomorphisms of a semigroup and that of a subsemigroup; the independence of the monoid of endomorphisms of an algebra of type Δ and that of a subalgebra; and the independence of the group of autohomeomorphisms of a complete metric space and that of a subspace. The completeness of the category of graphs and its full subcategory, undirected graphs, is shown.

Shao-Chien Chang, Ph.D., Summability: Projections, closure properties and consistency problems, Carleton University, Ottawa, Ontario, May, 1968. (Supervisor: M.S. Macphail)

This thesis pursues investigations similar to some recent work of Arterburn and Whitley; Whitley, and Jürimäe.

1. By using a variant of an operator representation theorem due to Thorp and Whitley, we prove that there is no bounded projection of Γ, the Banach space of conservative matrices, onto K^m the sub-space of Γ, consisting of compact matrices. By simple observation, a similar result has been obtained of ℓ-ℓ matrices.

2. Let m be the space of bounded sequences and A a conservative matrix. It was recently proved by Whitley that $c = m \cap c_A$ if and only if Ac is closed in c and the null manifold of A is finite dimensional. With the results of Wilansky and Mazur-Orlicz, we obtain the corresponding result that $c = m \cap c_A$ if and only if Am is closed in m and $A^{\perp}$ is finite dimensional.

3. We say that a conservative matrix A satisfies condition J if $\bar{c} \supset m \cap c_A$ where c_A is the space $\{x | Ax \in c\}$ and $\bar{c}$ is the closure of c with the FK topology for c_A. The matrix A is said to satisfy the condition O if, for each conservative matrix B with

$c_B \supset m \cap c_A$ and $\lim_A = \lim_B$ on c, we have $\lim_B = \lim_A$ on $m \cap c_A$. These conditions were introduced recently by Jürimäe. Although, J and O both hold for coregular matrices, this is not the case for conull matrices in general, as $\lim_B$ may not be continuous on c_A. We have in fact provided a counterexample of a conull matrix which satisfies J but not O, although O implies J. We show, however, that J and O are equivalent if A is multiplicative-o, replaceable, or has maximal inset and that A satisfies O if and only if $\Sigma_k a_k x_k = \lim_A x$ for $x \in m \cap c_A$. So our example shows that not every conull matrix is replaceable, answering a question raised in 1964. We observe that, if A does not satisfy condition J, then A is replaceable; we leave open the question whether every conull matrix which satisfies O is replaceable.

Om Parkash Chandna, Further contributions to the theory of steady rotational flow of gases, University of Windsor. April 1968. (Supervisor: A.C. Smith)

Steady rotational flow of gases is studied in the plane and in three dimensions. Starting from the flow equations in orthogonal curvilinear coordinate system, sets of restrictions on the geometries of flow corresponding to different flow conditions are deduced.

For the planar flow, density, entropy and velocity are eliminated from the flow equations to obtain a pressure equation in natural coordinates. The resulting equation is a linear hyperbolic partial differential equation of the second order. An exceptional case is encountered when the pressure equation is not obtainable from the equations of flow. In this case the stream lines are demonstrated to be straight.

The pressure equation for planar flow is solved explicitly for vortex flow, flow through a parabolic channel and the flow through a hyperbolic channel. Substitution of the solution into the flow equations yields properties of flow, namely velocity, entropy and density.

For three dimensional flow the general flow equations are seen to reduce to two independent pressure equations. Three categories of three dimensional flow are identified. Typical examples of flow for each category are investigated in detail. In the first category flow of gases emanating from a spherical ball and from a cylindrical bar is studied. Flow of gases swirling about the axis of cylinder and through a hyperboloidal tunnel are investigated under the second category. Flow of gases through a tunnel with elliptical cross sections is studied to illustrate the nature of solution encountered in the third category of flow.

Finally, it is proved that the only flow possible, with the stream lines as taken in the example of the third category of flow, is incompressible and irrotational.

Mike P. Closs, Ph.D., On certain G-structures, University of Windsor. April 1968. (Supervisor: H.A. Eliopoulos)

Differential geometry is the study of a differentiable manifold on which we are given some "geometric structure". We shall define a G^r-structure on a differentiable manifold and also a field of geometric objects of order r. The purpose of Chapter I is to show the relation between these two concepts as well as to summarize some of the results bearing on these topics.

In chapter II we define a G_J-structure on a differentiable manifold by a field of linear operators J and consider a riemannian metric G defined on the same manifold. Introducing the compatibility condition $JG = 0$ we obtain a degenerate riemannian structure which is investigated with the help of special bases adapted to the structure. We also define special linear connections on the structure and obtain a characterization of these connections in terms of J and G. Finally we obtain a characterization of these degenerate riemannian structures in terms of the holonomy groups of the linear connections.

Chapter III begins by defining special cases of the G_J-structures which we call r-tangent structures. We introduce the operators C and M of A. Lichnerowicz. Next we construct a tensor determined by the r-tangent structure which we call the torsion tensor and derive the relation

$$C^r dC^r f = f^o T$$

where f is any 1-form and T is the 2-form corresponding to the torsion tensor. Using this relation we are able to obtain an expression of the torsion tensor in local coordinates. A major result of this chapter is the establishment of the fundamental relation

$$C^r df - M^r dC^r f = f o S$$

where f is any 1-form and S is a 2-form depending only on the constants of structure.

In the remaining chapters we consider the almost tangent structures of H.A. Eliopoulos, which are a special case of the r-tangent structures. We restrict ourselves in chapters IV and V to "real" almost tangent structures. Given a riemannian metric G we say that G is hermitian with respect to J if

$$^{t}(JG) + JG = 0,$$

and call the resulting structure an almost hermitian structure subordinate to the almost tangent structure. We define special bases adapted to the almost hermitian structure and special connections which we are able to characterize by conditions on J and G. We prove amongother results, two theorems characterizing the almost hermitian structures with relation to connections and their holonomy groups.

The chapter V consists of a brief study of particular almost hermitian structures. Here we define pseudohermitian and almost kahlerian structures, giving a number of ways in which such structures can be characterized.

In the chapter VI we extend the notion of almost tangent structures to homogeneous almost tangent structures defined on homogeneous Lie spaces and derive necessary and sufficient conditions that such structures exist. We then investigate structures integrable in the sense of Mme Lehmann-Lejeune and obtain similar results for such integrable structures.

Florence S. Gordon, Ph.D., Characterizations of univariate and multivariate distributions through regression properties, McGill University. March 1968. (Supervisor: A.M. Mathai)

The condition that an arbitrary cubic statistic S has cubic regression on a linear one L is imposed. This assumption is used to derive a third order differential equation in the characteristic function of an arbitrary population. For appropriate choices of the coefficients in this equation (that is, for certain relations between the coefficients of S and the regression coefficients), the corresponding solutions lead to a series of characterizations for each of seven populations. Further, a technique is outlined which can be used to study the general polynomial regression of any m^{th} degree statistic on a linear one.

A generalized derivative with respect to a vector variable is introduced and a series of properties which it possesses is developed. Using the assumption of cubic regression for vector statistics S on L, a vector differential equation is obtained. Solutions of this equation, within the class of populations whose characteristic functions of a vector variable can be expressed in a generalized Taylor series expansion, lead to a series of characterizations for the Multivariate Normal distribution.

A generalized derivative with respect to a matrix variable is also introduced together with a number of properties which it possesses. The cubic regression assumption for matrix statistics S on L leads to a matrix differential equation. Further, a generalized Taylor series for matrix functions is presented and, among those populations whose characteristic functions are members of this class, the matrix differential equation is solved to yield a series of characterizations for the Wishart distribution.

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