*Relativity in Fundamental Astronomy*
*Proceedings IAU Symposium No. 261, 2009*
*S. A. Klioner, P. K. Seidelman & M. H. Soffel, eds.*

doi:10.1017/S1743921309990603

# Practical relativistic clock synchronization for high-accuracy space astrometry

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**Abstract.** Future high-accuracy space astrometry missions, such as Gaia and SIM, will need a time-tagging of observations consistent with General Relativity nowadays used as standard background for global data processing scheme. In this work, we are focusing on the realization of the onboard time scale. The onboard clock, being not ideal and consequently tainted with systematic biases, has to be carefully calibrated to the ideal relativistic proper time of the satellite. We present here a modeling of this essential step to provide a reliable relation between the onboard time and TCB, a time scale suitable for global data processing.

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## 1. Introduction

Future space astrometry missions are expected to reach an accuracy of several microarcseconds ($\mu$as) for the determination of positions, parallaxes and proper motions of celestial objects. This high accuracy requires subtle relativistic modeling to be used for the data processing. First of all, it is crucial to be able to give a relativistic formulation of astrometric observations, which is usually performed by a resolution of the null geodesics equations for the light propagation from the celestial object to the observer. Then it is also indispensable to control the attitude of the satellite in the four-dimensional spacetime, which requires to construct a particular tetrad or to use a description of the satellite's attitude in a comoving center-of-mass reference system, ideally defined as kinematically non-rotating. Another issue is crucial: the realization and the use of relativistic time scales (Le Poncin-Lafitte 2008).

For instance, the whole data processing of Gaia observations will be done in $TCB$, the coordinate time scale of the Barycentric Celestial Reference System (Soffel *et al.* 2003). From practical point of view, an onboard clock will produce a time scale, called OnBoard Time $OBT$, used to tag all kind of observations. In particular, specific tasks, such that observations of variable phenomena, will need precise absolute timing and require that the $OBT$ time scale have to be stabilized with an accuracy of roughly one microsecond. However, if $OBT$ can be viewed as a practical realization of the relativistic ideal Gaia proper time $TG$ along the worldline of the satellite, the onboard clock is not perfect and $OBT$ will be contaminated by some technical clock errors which means that formally $OBT$ and $TG$ time scales will be different. Moreover because of the motion of Gaia and non-zero gravitational potential at the spacecraft location around the Earth/Sun $L_2$ point, it is not straightforward to relate $TG$ and $TCB$ and a complete relativistic time transformation to go from $TG$ to $TCB$, and vice versa, is needed (Klioner 1992). The problem is worse when one thinks that in fact we need a relation between $OBT$ and $TCB$ to be able to do data processing in a correct way.

## 2. Operational strategy

This issue can be achieved by synchronizing the onboard clock with Earth ground clocks. However, it must realized that a satellite is not necessarily continuously observable from the ground during the full time of the mission. Then, the synchronization will be only possible during a period of visibility where many time telemetry procedures are performed at regular intervals. This procedure consists in the interrogation of the onboard clock to create a tag $OBT_k$. After delay due to the packaging by the onboard computer of that tag into a time telemetry package, the latter is sent to the Earth. After the flight time of the signal between the satellite and the Earth, the package is received by the antenna of a ground station. A new time delay is then necessary to transfer the package from the antenna to a computer where a tag $UTC_k$ *of reception*, in Universal Coordinate Time, is created and stored. We finally obtain a pair $(OBT_k, UTC_k)$ for each procedure. All these pairs constitute the initial data set for the synchronization. The question is now how to deduce from that pairs a new set of $(OBT_k, TG_k)$. A method, illustrated on the next figure, is proposed in this paper to perform this task.

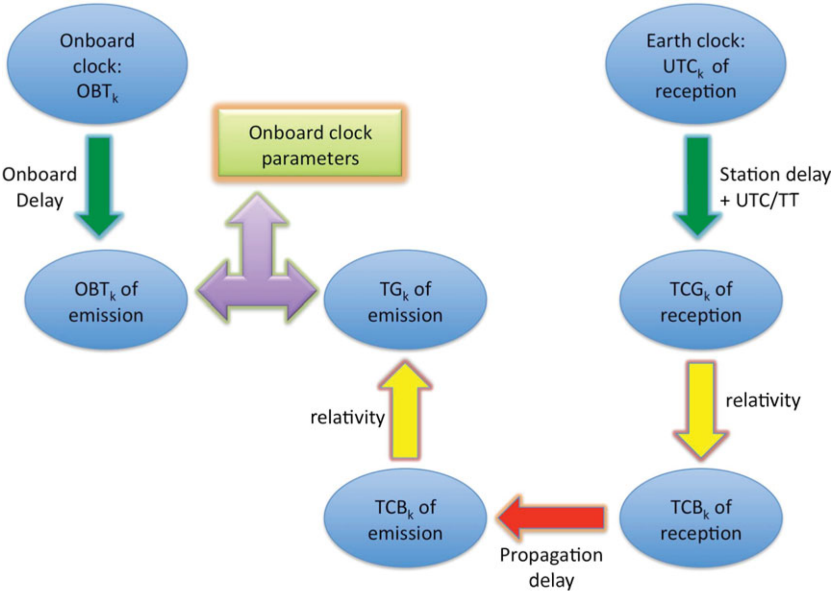

**Figure 1.** Time correlation process during a period of visibility of the satellite.

Let us detail the steps involved in the procedure:

- $UTC_k$ *of reception* is first transformed into $TCG_k$ *of reception*. Taking into account the instrumental ground delay and using usual relations between terrestrial time scales, it is possible to perform the following transformation $UTC_k \rightarrow TAI_k \rightarrow TT_k \rightarrow TCG_k$ *of reception*.
- $TCG_k$ *of reception* to $TCB_k$ *of reception*. This is a pure relativistic step which is achieved by solving an ordinary differential equation along the wordline of the geocenter.
- $TCB_k$ *of reception* to $TCB_k$ *of emission*. The purpose here is to convert a time tag relative to the reception of the packet on Earth ground station to a time tag relative to the emission time tag from the satellite. This step requires knowing all kinds of time delays during the propagation of the signal between the satellite and the Earth. It can be written as the sum of several contributions involving the instantaneous BCRS distance of the satellite to the Earth based station (it requires precise ephemeris of the probe as well

as the knowledge of the position of the Earth based station in the BCRS), the relativistic Shapiro delay, the propagation delay due to the Solar plasma, the troposphere and the ionosphere of the Earth.

- $TCB_k$ *of emission* to $TG_k$ *of emission*. This is the second pure relativistic step of the scheme. It is achieved by solving an ordinary differential equation along the worldline of the satellite.
- $OBT_k$ to $OBT_k$ *of emission*. This step just consists of taking into account some instrumental onboard delays between the interrogation of the clock, producing one tag $OBT_k$, and the operational instant of sending a time telemetry packet, $OBT_k$ *of emission*, to the Earth ground station.

## 3. Modeling the errors of the clock

Because the onboard clock can obviously not be ideal for technical reasons, the practical time-scale $OBT$ realized onboard will not be stable. Consequently the time-tagging of all observations will be contaminated with technical errors. It means that the formal difference $OBT - TG$ along the satellite worldline can not be exactly zero ; so for each pair $(OBT_k, TG_k)$ we will get $OBT_k - TG_k = f(OBT_k)$. The whole question is to find a simple expression of the function $f$. To determine this, a modeling of the clock errors is then indispensable. Usually the frequency of a clock is represented by

$$\frac{d}{d\tau}(\tau - \tau_{Rb}) = A + B\tau + C\sin(2\pi f + \phi) + \frac{D}{\tau_0}F(\tau_0)\,, \tag{3.1}$$

where $d\tau$ is the ideal local proper time interval between two events on the worldline of the onboard clock, $d\tau_{Rb}$ is the time interval between the same two events as measured by the clock, $A$, $B$, $C$, $D$, $f$, $\phi$ and $\tau_0$ are some constants to be determined and $F(\tau_0)$ is a random distribution (white noise) of points at interval $\tau_0$ with unitary standard deviation. Here, it is assumed that the stochastic behaviour of the clock (last term in 3.1) is stationary, *i.e.* independent of which period of data is chosen. That is likely the case, except for malfunctions. The value of $A$ is an arbitrary frequency offset, its value has a priori no effect on the stability and it will have to be estimated in flight. Additionally, we are practically interested in the integral of (3.1), *i. e.* in phase signal, which implies an integration constant (time offset) again to be estimated in flight. The frequency drift $B$ and periodic effect $C$ can be highly correlated and both will also have to be estimated in flight. The synchronization is then achieved when all available pairs $(OBT_k, TG_k)$ have been used to numerically constraint the parameters $A$, $B$, $C$...

## References

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