CORRESPONDENCE

The Joint Editors 24 *November* 1961

The Journal of the Institute of Actuaries Students' Society

Dear Sirs,

In his paper 'Options in Life Assurance' published in *J.S.S.* **16**, 3 Mr Souness honoured me by recalling my demonstration of how simple it is to produce a new theory of select mortality. There was, however, a 'sting in the tail', for he went on to say that we are no nearer the truth.

I should be grateful for a little of your space to comment.

All I would claim to have demonstrated, or perhaps to have endeavoured to demonstrate, in *J.S.S.* **4**, 3 can be put into one sentence—and that a negative one. It is simply that the select table model does not imply certain death within a designated time to any predetermined individual or group in it.

I find it difficult to accept Mr Souness's suggestion that if selection were observed to last exactly '*t*' years, then the theory of predetermined deaths would be a little harder to refute. Certainly, as Messrs Hooker and Longley-Cooke say in their text-book, the conclusion is at variance with experience.

If it be admitted that an aggregate table could represent the facts of life, and that such a table, being completely dependent on rates of mortality which are less than unity, cannot imply certain death within a designated time to a predetermined individual or group in it, why should it be thought that a select table, which also could represent the facts of life, and is completely dependent on rates of mortality, should carry such an implication? Selection *might* be such that those rejected were all condemned to death within a designated time, but no such claim is ever made for it, and in my view the select table model, correctly interpreted, applies to the normal type of selection involving fractional rates of mortality for all concerned, as well as to the extreme type involving certainties in respect of those rejected.

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In order to get to the bare bones of the matter let us suppose that a stationary population results from an l_0 of 1000, a q_0 of ·6 and a q_1 of $\cdot\dot{9}$. Then this population may be represented by $l_0 = 1000$, $l_1 = 400$ and $l_2 = 0$.

Now if we can identify, from the moment of birth, some of the individuals who will die in the first year of life we can reject them, and only them. Their rate of mortality will be unity and I will symbolize it by $q''_{[0]}$. Let us suppose we so reject 200 out of the 1000 then the rate of mortality of the 800 not rejected or, if you will, selected must be ·5 (symbolized by $q'_{[0]}$) in order to give 400 deaths which, together with the 200 deaths of the rejected will produce the 600 deaths for the population as a whole.

The position so far reached may be displayed in the form of a life table as follows:

Progress of selected lives	Progress of aggregated lives	Progress of rejected lives
—	l_0 (1000)	—
$l'_{[0]}$ (800) → $l'_{[0]+1}$ (400)	l_1 (400)	$l''_{[0]+1}$ (0) ← $l''_{[0]}$ (200)

A similar but more extensive table was given in my paper in *J.S.S.* 4, 3 at page 183.

The above table shows how l_0 may be split into $l'_{[0]}$ and $l''_{[0]}$ and how, consequently, l_1 is split into $l'_{[0]+1}$ and $l''_{[0]+1}$.

The division into selected and rejected lives may be applied to l_1 just as to l_0, the symbols being $l'_{[1]}$ and $l''_{[1]}$. In the circumstances of this abbreviated table, as q_1 is $\cdot\dot{9}$ both the rejected and the selected lives will all die within a year, but the principle is not affected and the table may be completed as follows:

Progress of selected lives	Progress of aggregated lives	Progress of rejected lives
—	l_0 (1000)	—
$l'_{[0]}$ (800) ← $l'_{[0]+1}$ (400)	l_1 (400)	$l''_{[0]+1}$ (0) ← $l''_{[0]}$ (200)
$l'_{[1]}$ (0) ← $l'_{[1]+1}$ (0)	l_2 (0)	$l''_{[1]+1}$ (0) ← $l''_{[1]}$ (400)

Let us call this table E (extreme type).

Now suppose we give up the idea of basing our selection on a rejection of some of the individuals who will die in the first year and let us divide the l_0 births into two groups with different mortality, still using the symbols $q'_{[0]}$ and $q''_{[0]}$. Then, the population being unaltered, we must have $l'_{[0]}$ plus $l''_{[0]}$ equal to $l_0(1000)$ and $q'_{[0]}\ l'_{[0]}$ plus $q''_{[0]}\ l''_{[0]}$ equal to d_0 (600). Let us suppose we can divide the l_0 births into two groups with $q'_{[0]} = \cdot 5$ and $q''_{[0]} = \cdot 75$ then $l'_{[0]}$ will be 600 and $l''_{[0]}$ 400 and a life table may be constructed as follows:

Progress of selected lives	Progress of aggregated lives	Progress of rejected lives
—	l_0 (1000)	—
$l'_{[0]}(600) \rightarrow l'_{[0]+1}$ (300)	l_1 (400)	$l''_{[0]+1}$ (100) $\leftarrow l''_{[0]}$ (400)
$l'_{[1]}(0) \rightarrow l'_{[1]+1}$ (0)	l_2 (0)	$l''_{[1]+1}$ (0) $\leftarrow l''_{[1]}$ (400)

Let us call this table N (normal type)

Table E and table N both show the progress through life of l_0 births when taken as a whole, and also when subdivided in two different ways. The tables also represent a stationary population similarly subdivided.

Now if (*a*) we are concerned with what happens to the selected lives only and (*b*) we are not concerned with the stationary population, but only with the progress through life, and (*c*) we want to save printing space, then we may rewrite both table E and table N as follows:

Table T (traditional type)

Select	Ultimate
$l'_{[0]} = 800$	$l_1 = 400$
	$l_2 = 0$

What can table T by itself tell us about the mortality of the rejected lives? To my mind the answer is nothing at all. But what if we know that $l_0 = 1000$? Then we can say that $q''_{[0]}$ exceeds $\cdot 6$ but we cannot say what $q''_{[0]}$ actually is.

Yours faithfully,

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P.S.

In order not to be accused of being merely critical—or on the other hand of flogging my own hobby horse, I should like to offer to fellow students a possible solution to a connected, and so far as I am aware, unsolved problem, namely the calculation of assessment premiums.

In their book *The Practice of Life Assurance* Messrs Coe and Ogborn deal with assessmentism at an early stage. After dismissing yearly contracts as neither practical nor popular, they go on to consider 1-year temporary assurance premiums so computed as to include the right of the assured to renew the contract at a premium appropriate to his attained age without evidence of health, but they find that in practice it is not possible to obtain a firm basis for assessing the cost of the option involved and conclude that generally speaking assessmentism does not provide an adequate financial foundation for life assurance.

I do not know what attempts have been made to obtain assessment premiums, but it may be of interest to see whether they could in fact be produced from a select table of the traditional type.

Let us consider in the first place what is almost a contradiction in terms, namely, an n year temporary assessment assurance, and let me suggest as a possible definition of assessment premium business that the prospective reserve on a select basis must always be zero, that is to say the office must not be worried if any of the assured lives drop out. On the basis of this definition, and representing the assessment premium by $z_{[x]+t}$, n equations of the following form will give us the n values of z

$$M_{[x+t]}-M_{x+n} = D_{[x+t]}z_{[x]+t}+D_{[x+t]+1}z_{[x]+t+1}+\dots + D_{[x+t]+n-t-1}z_{[x]+n-1}$$

($t = 0$ to $n-1$ inclusive).

On A24/29 3% with $x = 40$ and $n = 5$ the five equations give:

t	$z_{[40]+t}$ %
0	·387
1	·410
2	·401
3	·370
4	·295

It will be seen that $z_{[40]+4} = P^{\,1}_{[44]:\overline{1}|}$

The six equations for $x = 39$ and $n = 6$ will produce the same values with $t = 1, 2, 3, 4, 5$ as were produced by the five equations for $x = 40$ and $n = 5$ with $t = 0, 1, 2, 3, 4$ and so on. Starting therefore with $P^{\,1}_{[44]:\overline{1}|}$ we can produce assessment premiums back to any age (subject to the limitations of the functions available) and the one scale will apply to any age at entry, the assurance terminating at $x = 45$.

There would be a scale for each terminating age, but in fact these scales coincide if the last few years before the termination ages are ignored.

The highest age at which $q_{[x]}$ is shown on A24/29 is 80 and I give below an indication of how the scale would run with a termination age of 81.

x	z_x %
30	·236
40	·387
50	·771
60	1·992
70	5·222
80	6·330

W. J. COURCOUF

The Joint Editors, 16 *October* 1961

The Journal of the Institute of Actuaries Students' Society

Dear Sirs,

In the Editorial at the beginning of *J.S.S.* **16**, 1, you stated that you aim to stir interest through your correspondence columns. A letter certainly appeared at the end of the same number, since which there have been three numbers without the vestige of a letter. As this particular object of yours appears to be in danger of being unfulfilled, perhaps I may be permitted through your correspondence column to exhort members—and particularly younger ones—to respond to the amended Resolution which was passed this month at the Society's A.G.M.

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It may be recalled that the original wording of the Resolution, before amendment, would have earmarked a few pages of certain issues to a 'Random Muse' section. One speaker was afraid I had in mind only the 'frankly hilarious' side of 'Random Muse'; but this was not the intention, even though frank hilarity is nothing to be ashamed of. Had I spoken to the amendment I would have pointed out that this lowest type of contribution would serve the purpose of encouraging the more timid members to send in their semi-serious contributions, their more light-hearted ideas, even their germs of ideas. Indeed, a slogan to encourage them might well be 'Send your germs to the Editors of *J.S.S.*'

This is the jet age, the teenage age, the rock 'n' roll age, the beat age, even the off-beat age. May I appeal to those who spoke in support of my Resolution (and those who didn't)—as many as possible of them—to send you their 'off-beat' articles; perhaps this process might become known as 'the method of least Squares'. If they support me in this effort to leaven future issues of your *Journal* then my crusade will not have been in vain, and the 'Random Muse' experiment will live on. There may even be a hilarious issue on the occasion of the Society's Diamond Jubilee.

If I have descended to frank hilarity in parts of this letter, I offer no apologies whatsoever, and remain frankly unashamed, and

Yours truly,

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