MONTE-CARLO CALCULATIONS

J.S. STODÓŁKIEWICZ
N.Copernicus Astronomical Center, Warsaw, Poland

ABSTRACT. The evolution of a nonisolated globular cluster is presented. The binaries (both, tidally captured and formed in three-body interactions), outflow of mass from stellar envelopes and shocks are considered as sources of energy in the cluster.

1. INTRODUCTION

Many theories of dynamical evolution of globular clusters consider these stellar agglomerations as systems of separate mass points which move under the influence of their mutual gravitational attraction. This assumption, justified by small stars radii relatively to their distances in the clusters, simplifies discussion of a majority of proprieties of globular cluster structure and their evolution. One of the most important conclusions, obtained under this assumption, states that globular clusters are not stationary systems, but that their evolution leads to a gravothermal catastrophe (Antonov 1962, Lynden-Bell and Wood 1968). The process of a core collapse is accelerated if stars of different mass are allowed (Spitzer 1969). The process of the gravothermal collapse can be interrupted (Hénon 1975), only when a new (different than the core collapse) source of energy appears in the cluster.

Almost all processes which can supply the clusters with energy operate only, if we reject the above mentioned assumption and take into consideration that stars are evolving objects of nonvanishing sizes and can form at least two-component systems interacting with other stars of the clusters. There are following sources of energy in globular clusters considered in literature:

J. Goodman and P. Hut (eds.), Dynamics of Star Clusters, 361–372.

1/ binaries formed in three-body interactions (Gurevich and Levin 1950, Heggie 1975),

2/ binaries produced in two-body tidal interactions (Fabian, Pringle, Rees 1975),

3/ outflow of matter from evolving stars (Angeletti and Giannone 1977, 1980, Stodółkiewicz 1982),

4/ shocks, when the cluster passes the galactic plane (Spitzer 1975, Ostriker, Spitzer, Chevalier 1975),

5/ a massive black hole in the cluster centre (Bahcal and Ostriker 1975, Silk and Arons 1975).

The aim of this paper is to examine the influence of heating (or cooling) of the cluster by three first processes from the above list. The heating by shocks is also taken into consideration; however the cluster is assumed to be far from the galactic centre $R_C \approx 10$kpc and thus this effect is not essential for this cluster.

2. METHOD

A Monte Carlo scheme, which is a version of Henon's (1971) method, described by Stodółkiewicz (1982), was expanded by an incorporation of binary-single star and binary-binary interactions. The whole cluster is divided into 1000 superstars. If the number of superstars declines below 500 during the evolution, each superstar is doubled. So the number of superstars in the cluster is always contained between 500 and 1000.

The most serious problem is to represent correctly the very central region of the core, where, as a result of the collapse, the density in a small volume reaches high values. This cusp of the density is caused by only several hundreds of stars and is represented by a few (3-6) superstars. We cannot expect that it will be possible to elaborate any credible statistical theory which could describe the proprieties of such a small and nonuniform agglomeration of stars interacting with the rest of the cluster. The most promising method to treat the very centre of the core is the hybrid Monte-Carlo-N-body code described by McMillan and Lightman (1983) and presented at this Symposium. It seems that faster methods of globular cluster evolution calculations will incorporate in the future the models of the deep core obtained with this code. In this paper I imposed however very simple requirements on the motion of the innermost superstars. These conditions are the same as described by Stodółkiewicz (1982, Model C1b), moreover, if the two innermost superstars form a bound system the sum of their kinetic energies is divided in equal parts between them and their velocities are "isotropised".

The calculations performed under these assumptions gave reasonable results concerning the evolution of the bulk of the cluster. We shall return to this problem at the end of the paper.

Mass loss from stars is simulated by a single, instantaneous, spherically symmetrical outflow of a stellar envelope at the top of the red giant branch of the evolutionary track. It is assumed that the gas from stellar envelopes immediately leaves the cluster. The data on stellar evolution are taken from Bartelli, Bolton, Chiosi, Nasi (1979) and Alcock, Paczyński (1978). We adopt that all stars more massive than 2.8 solar masses have compact remnants of mass equal to 1.4 $M_{\odot}$. Lighter stars reject half of their mass during the evolution.

The influence of the gravitational field of the Galaxy is considered in two ways. Firstly, the size of the cluster is limited by its tidal radius. Secondly, the cluster is heated by shocks induced by its passages through the galactic plane.

The influence of binaries on the evolution of globular clusters has been investigated by many authors. Most of them have used the analytical formulae in order to estimate the mean values of energy released in the binary-single star interactions. In a few papers only the life of population of binaries was investigated together with the evolution of the cluster: Hills (1975), Spitzer and Mathieu (1980), Aarseth (1980), Stodółkiewicz (1983). McMillan and Lighman (1983) using the "hybrid" code obtained the binaries produced in three-body encounters for the first time. This result is a convincing illustration of formation of binaries in large stellar systems. In this paper we adopt (similarly as in Stodółkiewicz 1983) that binaries are born in three-body encounters in the rate given by Heggie's (1975) formula (5.6). We consider only binaries with initial binding energies greater than $8\beta^{-1}$ (where $\frac{3}{2}\beta^{-1}$ is the local mean kinetic energy per particle).

The binaries produced by tidal capture play also very important role in the life of the cluster. They cool the system because during their formation the energy of relative motion of two interacting stars is transformed into the energy of oscillations of components. Direct collisions of stars will lead to dissipation of kinetic energy of the cluster. But we can expect that these binaries, produced in two-body tidal interactions, play also some role in heating of the system. It is realized in two ways. Firstly, in the very dense central part of the core the probability of interaction between binary and field star can reach high values, even for very tight binaries. Secondly, the mass loss from the close and contact binaries or from single, but massive, stars formed as a result of collisions should be more effective than from separate stars.

The dissipative interactions between two stars lead to formation of two types of objects:

(a) Gingold and Monaghan (1980) showed that, if a star passes the other one at the distance even a little shorter than the radius of its Roche sphere, we can expect the essential deformation of that star which will cause an exchange of angular momentum and loss of energy of orbital motion, or even a disruption of the star. The future of the newly formed object strongly depends on its initial parameters, but its sizes probably will be comparable to the separation of components at the moment of their first periastron passage. The chance of the interaction of field star with such systems, before essential loss of mass from them, is small. In this paper we only count the number of this type of systems appearing in the cluster but do not follow them in further computations. The energy of relative motion of reacting stars we subtract from the energy of the nearest superstar, but we do not take into account any supply of energy from these systems.

(b) When the periastron distance of the first passage grows, the result of interactions is less dramatic. The binary system which can be formed in this case has a circular orbit with semiaxis two times greater than the distance between stars at the moment of the first periastron passage. The system is well detached, both stars lie deep inside their Roche lobes. The cross-section for such capture depends on the structure of both stars. We adopt the results given by Press and Teukolsky (1977) to calculate the probability of formation of binaries, separately for the pairs: unevolved-unevolved star and unevolved-compact star. Each newly produced binary is tested, whether it has enough time for the circu-larization of its orbit before the interaction with a field star; if not, the binary is excluded from the computations.

The binary-single star interactions are computed numerically by integration of three body motion equations. Probability of interaction with a field star is evaluated for each binary in the each step of calculation. If the interaction occurs, the impact parameter, the orientation of the binary orbit and the direction of velocity of the field star are chosen randomly, other characteristics of the interaction are determined by the binary and conditions in the vicinity of this event. We take into consideration only these interactions which are expected to give a recoli energies greater than $1/2\ \beta^{-1}$. During the interaction all stars are treated as mass points. Results of Mikkola (1984) paper are incorporated in the used scheme in order to evaluate the effects of binary-binary interactions. No stable configurations containing more than two stars are considered.

3. EXAMPLE OF GLOBULAR CLUSTER EVOLUTION

In this chapter we investigate the evolution of globular cluster which has the following initial characteristics:

1/ total mass equals 10^5 solar masses,
2/ continuous distribution of stellar masses, given by Salpeter formula in the interval 0.1 and 3.0 solar masses,
3/ density distribution according to King (1966) model with r_c = 1pc and r_t = 31pc,
4/ no mass segregation.

The initial mean relaxation time in the cluster is equal to $1.7 \cdot 10^9$ years.

The calculations are interrupted after $29.7 \cdot 10^9$ years, when the mass of the cluster declined to 2,000 solar masses and the binding energy of the cluster was reduced to 0.0006 of its initial value.

During the first phase of the evolution we observe the growth of the central concentration of unevolved stars in the cluster. This process is accompanied with the very effective creation of binaries produced by two-body tidal capture mechanism. In the first $8 \cdot 10^9$ years 83 percent of all tidally formed binaries are created. At the same time the most massive stars end their evolution and are transformed into neutron stars or white dwarfs. Both, the mass loss from stars and interactions of binaries with field stars, cause heating of the cluster and lenghtening of this phase of its evolution. After about this time the central density of unevolved stars begins to decrease and when the cluster is about 14 billion years old the density of unevolved stars drops below its initial value.

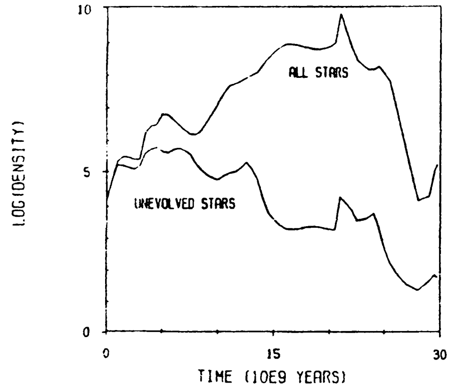

Figure 1. Logarithm of central density (number of stars per pc^3) as a function of time.

The evolution of central density is presented in Fig. 1. From this time, further on, the activity of the cluster declines for external observer: the density of shining stars is low, tidally captured binaries cease their opperation (Fig. 2).

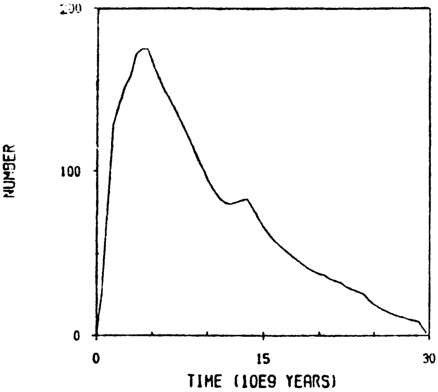

Figure 2. Number of tidally captured binaries actually existing in the cluster as a function of time. Binaries whose components passed the Roche sphere are not presented.

But we are still far from the moment when the collapse is completed!

First binaries formed in the three-body encounters appear in the cluster after $4.5 \cdot 10^9$ years from the beginning of the computations. But the number of these objects is small for a long time. We observed only 10 three-body formed binaries within the first $19 \cdot 10^9$ years. At the end of this time the density of the central core, populated now mostly by evolved stars, crosses the value of 10^9 stars per pc^3. Then a rapid production of the three-body captured binaries and their interactions with field stars stops further collapse of the core. But at the time of the maximum of central density ($21 \cdot 10^9$ years) only about 26 percent of initial mass survives in the cluster and its binding energy decreases tenfold. The last period of the cluster life, after the core collapse, is characterized by decline of density in the whole cluster volume under the influence of still operating three-body captured binaries.

During the evolution of the cluster we observed 94 binaries formed in three-body capture mechanism almost all their components were the evolved stars which took part in 1167 interactions with field stars. The number of stars which were thrown out of the cluster due to encounters with these binaries was 374 (including components of binaries). The tidally captured binaries were much more numerous. There were 1177 such objects (belonging only to the second group), and 3969 interactions of these binaries with field stars. As a result of these interactions 2380 stars were removed from the cluster. During the whole cluster evolution we observed 3658 passages of a star through the Roche sphere of another star.

TABLE 1. Numbers of binaries lost from the system

The way	2-body binaries	3-body binaries
a/	708	72
b/	12	3
c/	417	10
d/	38	8

a/ escape after binary-single star interaction;
b/ disruption after binary-binary interaction;
c/ disruption after binary-single star interaction;
d/ escape in the relaxation process.

The ways of binary loss is presented in Table 1. The interactions of binaries with field stars (the most effective process of loss of binaries) cause that the life-time of tidally captured binaries is short ($2.2 \cdot 10^9$ years) during the first 5 billion years of the cluster evolution. In the latter half of the cluster evolution, when these interactions are much more rare, the life-time of these binaries increases to about $7 \cdot 10^9$ years. The disruption of tidally captured binaries takes place only during the phase of circularization of their orbits, just after the capture of the components and in interactions with the other binaries .

The efficiency of three main sources of the energy in the cluster is presented in Fig. 3. The energy released in the interactions of the two-body captured binaries is much greater than the initial binding energy of the cluster, but only small part (about 10 percent) of this energy is absorbed by the cluster, the rest is carried away by the stars and the binaries leaving the cluster after the interactions. These binaries heat the cluster only during the first phase of the cluster evolution; during first 10 billion years the cluster receives 90% of energy obtained from these systems. It is evident that their action cannot interrupt the core collapse, but, of course, can retard its development. These binaries play also some role in the expultion of unevolved stars from the deep core. After 15 billion years the binaries produced in three-body interactions start to heat the cluster, and later on they become the only source of energy in the cluster. Although the amount of the energy

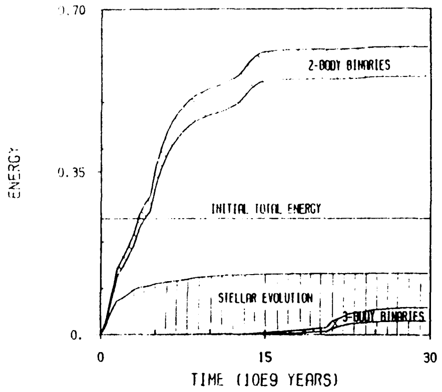

Figure 3. Heating by binaries and outflow of matter from stars. The curves represent the amount of energy integrated up to a given time. Upper curves for binaries represent energy released, lower curves - the energy carried away from the cluster; the curve for stellar evolution describes energy absorbed by the cluster.

received by the cluster from these systems is small comparatively to the initial value of the energy of the cluster , but it is supplied to the much less massive cluster and much loosely bound (except for the deep core) than at the time of its birth. The energy released by these binaries is consumed to prevent the further collapse of the deep core (which contains now most of the binding energy of the cluster) and afterwards to the complete dissolution of the cluster. The most abundant source of energy is the mass outflow from evolving stars. It delivers more than a half of the energy needed to disrupt the cluster. This source it self is sufficient to suppress the effects of collapse of the radius of a sphere containing 10 percent of cluster mass. But it operates effectively only during about the first 5 billion years and, of course, is not able to prevent the collapse of the inner core.

The balance of energy (including all its sources) during the whole evolution of the cluster is presented in Fig. 4 and in Table 2. The energy carried away by the escaping stars is small. But it is a by-product of the most important process enforcing the cluster evolution, the process of relaxation, in which the energy is transported from the core, through the bulk of the cluster, to its envelope, and which causes the steady evaporation of stars. The energy gained from the shocks is small too; it is a consequence of our assumption that the cluster moves far from the galactic centre.

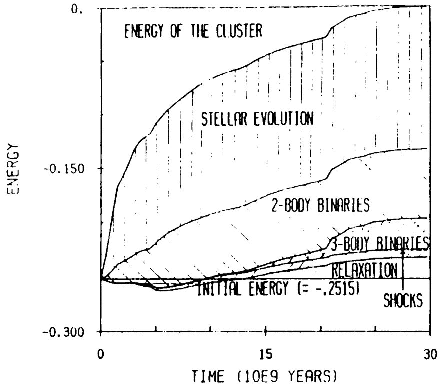

Figure 4. Balance of energy. Amounts of energy absorbed by the cluster until a given time.

TABLE 2. Energy balance

Source / Age 10^9y	10	20	29.7
stellar evolution	.1254	.1316	.1321
2-body binaries:			
energy released	.5354	.6136	.6181
energy absorbed	.0577	.0637	.0637
3-body binaries:			
energy released	.0007	.0143	.0583
energy absorbed	.0004	.0075	.0284
shocks	.0044	.0071	.0076
relaxation	.0035	-.0113	-.0195
mass of the cluster	.6174	.3064	.0197

The role of the same mechanisms for the balance of mass (Fig. 5) is quite different. The relaxation is the most effective process of the mass loss. 82 precent of the cluster mass (but 98.6 percent of stars) escapes due to encounters with field stars. It should be noted that this loss is practically constant during the whole cluster life.

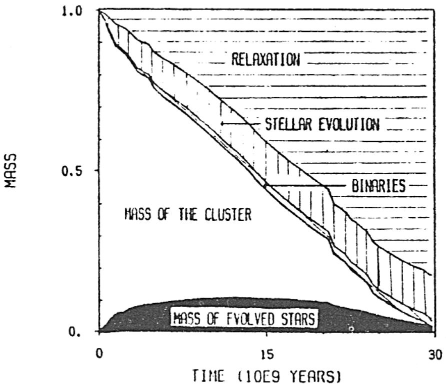

Figure 5. Balance of mass. Mass loss caused by each mechanism until a given time.

The same result was obtained by Hénon more than twenty years ago (Hénon 1961). This phenomenon has been also observed in the n-body calculations of the evolution of the open clusters. Here it is fully maintained, although the cluster mass decreases through two decades. The second process which is essential for the mass loss from the cluster, outflow of stellar envelopes, is much less efficient. Only 13 percent of the mass leaves the cluster this way. The mass of stars thrown out of the system as a result of interactions with binaries is quite negligible (2.6 percent of the initial cluster mass). The relative contribution of the dead stars systematically grows. 13 billion years after its birth the mass of the evolved stars is equal to 22 percent and reaches 85 percent at the end of the computations.

The evolution of this cluster, in which we took into account all essential sources of energy, can be compared with the evolution of globular clusters computed earlier (Stodółkiewicz 1982), where we neglected the action of binaries. In Models C1a and C1b from the quoted paper the main source of energy was the steady contraction of the inner core. The mass loss in all models and the mass distribution (except for the central part of the clusters) is very similar (Table 3). We see the evolution of the bulk of the cluster is very insensitive to the energy source. The energy demanded by the cluster evolution is delivered from this source, which can operate in the actual conditions. This is a reason that we were bold to calculate the cluster evolution without the special method to examine the structure of the central deep core.

TABLE 3. Comparison between Models C1a, C1b$^{x/}$ and presented in this paper. Total mass and radii of spheres containing 20, 40, 60 and 80 percent of cluster mass at the age of 20 billion years.

Model	r_{20}	r_{40}	r_{60}	r_{80}	M
C1a	.105	.225	.366	.541	.356
C1b	.100	.233	.387	.537	.305
This paper	.133	.240	.354	.544	.306

$^{x/}$see text

The results presented here concern the globular cluster of initial parameters given above and cannot be directly applied to the other clusters.

REFERENCES

Aarseth, S.J. 1980, IAU Symp. 85, 325.
Angeletti, L., and Giannone, P. 1977, Astron. Astrophys., 58, 363.
Angeletti, L., and Giannone, P. 1980, Astron. Astrophys., 85, 113.
Alcock, Ch., and Paczyński, B. 1978, Astrophys. Journ., 223, 244.
Antonov, Y.A. 1962, Bull.Univ.Leningrad, 7, 135.
Bahcal, J.N., and Ostriker, J.P. 1975, Nature, 256, 23.
Bartelli, G., Bolton, A., Chiosi, C., and Nasi, E. 1979, Astron. Astrophys. Suppl., 36, 429.
Gingold, R.A., and Monaghan, J.J. 1980, M.N.R.A.S., 191, 897.
Gurevich, L.E., and Levin, B.Yu. 1950, Astron. Żurn., 27, 273.
Fabian, A.C., Pringle, J.E., and Rees, M. 1975, M.N.R.A.S., 172, 15P.
Heggie, D.C. 1975, M.N.R.A.S., 173, 729.
Hénon, M. 1961, Ann. d'Astrophys., 24, 369.
Hénon, M. 1971, Astrophys. Sp. Sci., 14, 151.
Hénon, M. 1975, IAU Symp., 69, 133.
Hills, J.S. 1975, Astron. Journ., 82, 626.
King, I. 1966, Astron. Journ., 71, 64.
Lynden-Bell, D., and Wood, R. 1968, M.N.R.A.S., 138, 495.
McMillan, S.L.W., and Lightman, A.P. 1983, Cambridge, Mass., Preprint 1941.

Mikkola, S. 1984, M.N.R.A.S., 206, 115.
Ostriker, J.P., Spitzer, L., and Chevalier, R.A. 1975, Astrophys. Journ. /Lett./, 176, L51.
Press, W.H., and Teukolsky, S.A. 1977, Astrophys. Journ., 213, 183.
Silk, J., and Arons, J. 1975, Astrophys. Journ. /Lett./, 200, L131.
Spitzer, L. 1969, Astrophys. Journ. /Lett./, 158, L139.
Spitzer, L. 1975, IAU Symp., 69, 3.
Spitzer, L., and Mathieu, L.D. 1980, Astrophys. Journ., 241, 618.
Stodółkiewicz, J.S. 1982, Acta Astron., 32, 63.
Stodółkiewicz, J.S. 1983, Publ. Astron. Inst. Czechoslovac Acad. Sci., 56, 103.

DISCUSSION

SPITZER: What is the radial distribution of the formation rate for tidal-capture binaries?

STODÓŁKIEWICZ: Most of these binaries is formed in the deep core, 85 percent of them was born inside the sphere containing the innermost 3 promille of the cluster stars.

LUPTON: How does the mass function evolve in your calculations?

STODÓŁKIEWICZ: The rate of escape of massive stars is smaller than - of the others. But the rate of evaporation of all stars with masses lower than about the mean stellar mass in the cluster is practically not dependent on their mass.