POSSIBILITIES OF INCREASING THE ACCURACY IN THE DETERMINATION OF REFRACTIONAL ANGLES WITH TENGSTRÖM'S IDM

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ABSTRACT

The main purpose of the IDM research of prof. Tengström was to confirm the possibility of determining instrumentally the refraction angle in the field even over long distances.

An appreciable increase of the accuracy could only be achieved by further improvement of the instrument. Even with the first laboratory model it seems possible to achieve an increase of the accuracy in the final results. This paper discusses the above possibilities and proposes some methods to achieve this improvement in the accuracy.

1. INTRODUCTION

Experience from investigations in the field indicates that the angular refraction is varying with amounts, which can rarely be calculated from theoretical formulas, containing measured meteorological parameters, valid along the ray at the instant of observation.

Even if the theoretical model atmosphere is correct, which probably will certainly never be the case, the discrete meteorological measurements along the path above a non homogeneous surface will not be able to give information for an accurate calculation of the curvature integral. In practice it is also extremely difficult to collect data which all correspond to the epoch of observation.

Formulas containing various meteorological parameters have been elaborated by several scientists (Angus-Leppan, P.V., 1971, Brunner, F.K. 1977 Brunner, F.K. & Fraser, C.S., 1977, Rinner, K., 1977, Saastamoinen, J., 1974) and their achievements are of great importance.

But a severe difficulty when applying such formulas is the more or less stochastic behaviour of grad n for certain terrain conditions (topographical features, vegetation, variation of wind and cloud cover etc.),

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especially near the ground.

From what is said above one must conclude, that a direct determination of the whole atmospherical integral at the epoch of observation is necessary in order to master the problem of eliminating the refractional influences both in EDM measurements and, above all, in angular measurements.

The n-integral at EDM and the $\frac{dn}{dh}$ - integral at vertical angle measurements can be evaluated by multi-wave approaches.

The success in EDM has recently been demonstrated by Hugget and Slater (Hugget, G.R., Slater, L.E., 1977), whose terrameter, using He-Ne and He-Cd lasers, are soon available on the market by Terra Technology, Redmond, Washington (Hugget, G.R., 1978).

It seems also, that the two-colour systems (Glissman, T., 1977, Prilepin, M.T., 1974, Tengström, E., 1967, Tengström, E., 1977, Williams, D.C., 1977) promise to solve the refraction angle problem with high accuracy in the nearby future. Prilepin (Prilepin, M.T., 1975) has also suggested the use of a single-wave system. Perhaps Tengström's IDM-system can also be used under favourable atmospheric conditions with a small stellar interferometer set up, which will increase the accuracy.

Of all above mentioned methods I have had the possibility to learn exactly in practice the achievements of the IDM method. On this base I think that this method have certain possibilities to increase the accuracy and reliability of its result. The proposal for such possibilities are the essence of this paper.

2. ANALYSIS OF CURRENT ACHIEVEMENT OF THE REFRACTION ANGLE USING IDM

To have out of account the whole description of practical realization of setting of the refraction angle by the dispersion given detailed earlier (Tengström, E., 1967) we only would analyse the possible sources of errors and the total error resulting by them.

From the principle of IDM measurement the vertical angle of refraction is calculated

$$\alpha = K_{12}^{(1)} \delta_{12} \qquad (1)$$

where

$$K_{12}^{(1)} = \frac{N_{01}}{N_{02} - N_{01}} \qquad (2)$$

is the dispersion coefficient dependent only on the refractive index n_{oi}

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$$N_{oi} = n_{oi} - 1 \quad (3)$$

and δ - the dispersion angle

$$\delta_{12} = \alpha_2 - \alpha_1 \quad (4)$$

In prof. Tengström's IDM measurements, laser sources of monochromatic light are used, He-Ne (red 0.6328 µm) and He-Cd (UV 0.3250 µm). They should be in the same horisontal plane and in a straight line perpendicular to the direction of wave propagation.

As a result of dispersion and refraction in atmosphere a difference δ appears between the refraction angles of the two light beams. It is the angle between the planes of the R and UV wave fronts. After interference in the 6 slit grating and focusing by the Cassegrain camera (f = 6260 mm) the interference fringes produced are photographed and then δ is calculated as below

$$\delta_{RUV} = \frac{z_{RUV}}{f} = \frac{\lambda_R \; z_{RUV}}{l_R \; d} \quad (5)$$

where λ_R the length of the red beam

z_{RUV} the distance between the central fringes of red and UV (fig. 1) on the photograph

l_R the distance between succeeding fringes of red

d the slit distance

The z and l are measured on precise comparator. Let us discuss the accuracy of this method. From (1) we have

$$\left(\frac{m_\alpha}{\alpha}\right) = \sqrt{\left(\frac{m_K}{K}\right)^2 + \left(\frac{m_\delta}{\delta}\right)^2} \quad (6)$$

From the Edlen's formula, for these laser sources (dry air at T = 273.16°K, p = 760 Torr, contain of 0.03% CO_2)

$$N_{0_{UV}} = 304.24 \times 10^{-6} \pm (<4 \times 10^{-8})$$

$$N_{0_R} = 291.76 \times 10^{-6} \pm (<4 \times 10^{-8})$$

whence $\Delta N = N_{0_{UV}} - N_{0_R} = 1248 \times 10^{-8} \pm (<6 \times 15^{-8})$

$$K = K_{RUV}^{(R)} = 23.38$$

and
$$\frac{m_K}{K} = \sqrt{\left(\frac{m_N}{N}\right)^2 + \left(\frac{m_{\Delta N}}{\Delta N}\right)^2} \simeq 5 \times 10^{-3} \qquad (7)$$

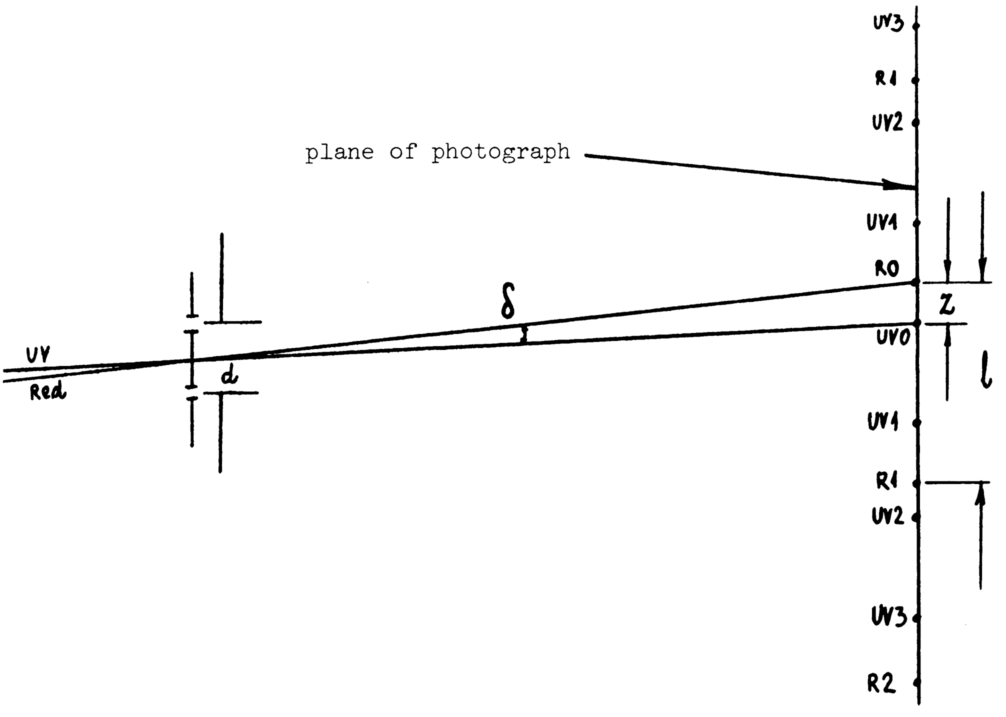

Fig. 1

Nowadays, neglecting the influence of the humidity of the air (Tengström, E., 1974), this value limits the relative accuracy of the refraction angle determination. The magnitude corresponds with an absolute accuracy of about 0.03^{cc}/km with average meteorological conditions.

From (5) we get

$$\left(\frac{m_\delta}{\delta}\right)^2 = \left(\frac{m_\lambda}{\lambda}\right)^2 + \left(\frac{m_d}{d}\right)^2 + \left(\frac{m_z}{z}\right)^2 + \left(\frac{m_l}{l}\right)^2 \qquad (8)$$

where $\lambda = \lambda_R = 0.6328\ \mu m$

with the spectral width of about ± 1.5 GHz

Since $\lambda = \frac{c}{f}$

and $\frac{m_c}{c} \simeq 4 \times 10^{-9}$ $\frac{m_f}{f} = 3 \times 10^{-6}$

hence $$\left(\frac{m_\lambda}{\lambda}\right)^2 = 9 \times 10^{-12} \tag{9}$$

In the current IDM measurements d = 5 mm is obtained directly with an accuracy of about 0.05 mm, whence

$$\left(\frac{m_d}{d}\right)^2 \simeq 1 \times 10^{-4} \tag{10}$$

Estimation of the influences of obtaining of the z and l errors requires more penetrating consideration.

These distances are measured on 35 mm film. In this case the following sources of partial errors are possible:

a. The change of z and l result from a non perpendicular situated photosensitive plane relative to the main optical axis of Cassegrain camera (fig. 2). If we describe this deviation by means of its component angle, θ lengthwise to the fringe lines and ϕ perpendicular to these lines, then;

a.1. The θ deviation causes a change in the distances z and l to $z' = z \sec\theta$ and $l' = l \sec\theta$; these changes do not introduce any errors since in δ exists only the quotient

$$\frac{z'}{l'} = \frac{z}{l}$$

a.2. The ϕ deviation causes a change in the scales between the lines of red and UV fringes. Assuming C for the scale along the red fringes, the scale along the UV fringes is

$$C' = C(1 + \frac{t}{f} \sin\phi)$$

where t is the distance between the red and UV fringe lines ≃ 200 μm, and f is the focal length of camera = 6260 mm.

ϕ can be established approximately from the required conditions of

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perpendiculary of the optical axis in 35 mm cameras of about 1° whence

$$\left(\frac{C_1}{C}\right)_{max} = 1 + \frac{0.2}{6260} \sin 1^{\circ} \simeq 1 + 6 \times 10^{-7} < 1 + 1 \times 10^{-6}$$

Because $z_{max} < 100$ µm, then the maximum error caused by this change of scale gives a value of $z_{max} < 10^{-4}$ µm which can be neglected.

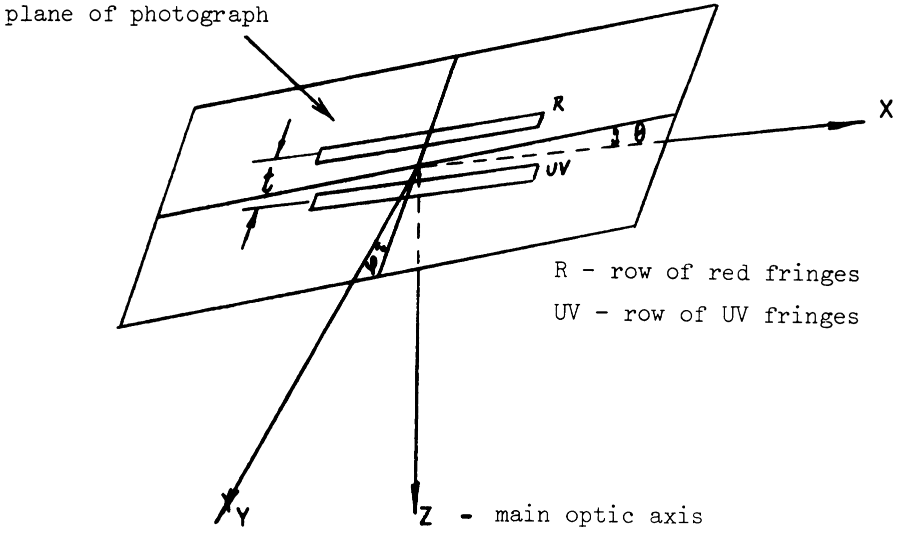

Fig. 2

b. The deviation θ (fig. 3) of the direction of line of fringes according to the direction of the motion of comparator carriage. In this case we obtain instead of z and l, z' = l sec θ and l' = l sec θ and it results as the case described in a.1.

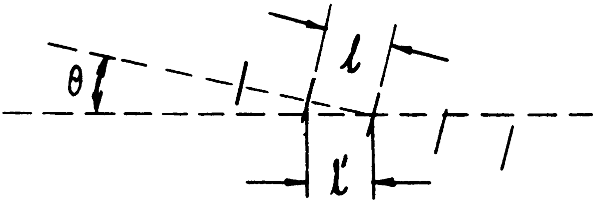

Fig. 3

c. The error of positioning the stadia hairs of the comparator on the image of the fringe which are more or less sharp.

It is the main most decisive source of error, especially in obtaining z, which is measured between the central fringes of two different spectra-images. Each of them is situated in two different lines separated by distance of about 200 μm.

As result of the unequal film sensitivy and coefficient of absorption of the atmosphere to the red and UV the shape, the extent and the intensity of the red and UV fringes are rather different.

Now the accuracy of obtaining z and l by one observer is estimated at about 1-2 μm (internal). In spite of this internal accuracy there can occur divergences of about a few μm between the results of two different observers. For further consideration we assume $D \simeq 20$ km, $l = 800$ μm, $z \simeq 67$ μm and

$$m_z = m_l = 2\ \mu m \qquad (11)$$

$$\text{whence} \quad \left(\frac{m_z}{z}\right)^2 \simeq 9 \times 10^{-4} \quad \text{and} \quad \left(\frac{m_l}{l}\right)^2 \simeq 6.25 \times 10^{-6} \qquad (12)$$

Let us consider one more possible factor of error in z as result of non exactly adjusting the two laser sources with each other. Some deviations of the required equal level (fig. 4a) and from the plane perpendicular to the direction to IDM (fig. 4b) are possible. The deviations appear when $h \neq 0$ or $p \neq 0$.

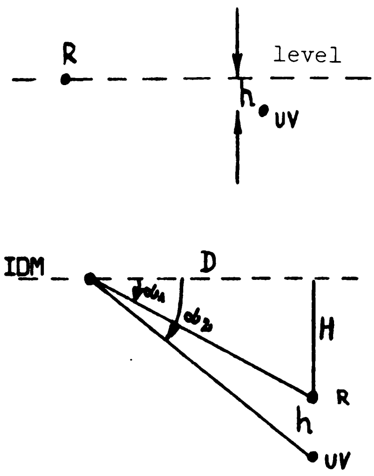

Fig. 4a

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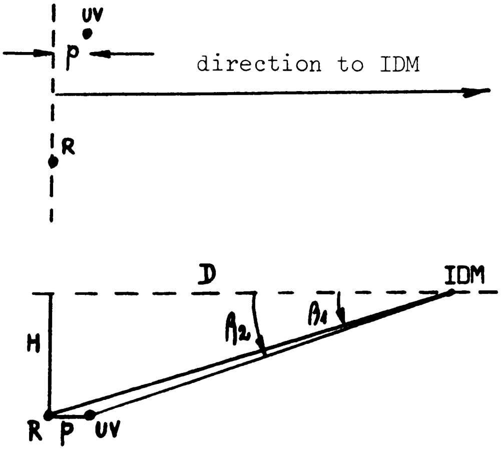

Fig. 4b

In this instance we obtain

$$tg\alpha_1 = \frac{H}{D} \quad , \quad tg\alpha_2 = \frac{H+h}{D} \quad , \quad tg\beta_1 = \frac{H}{D+p} \quad , \quad tg\beta_2 = \frac{H}{D}$$

and after simple transformation, expansion in series and assuming that $h \ll D$, $p \ll D$, $H+h \simeq H$, $D+p \simeq D$, we obtain

$$\begin{aligned} \Delta\delta_1^{cc} &= \alpha_2 - \alpha_1 \simeq \frac{h}{D}\left(1 - 2\,\frac{H}{D}\right)\rho^{cc} \\ \Delta\delta_2^{cc} &= \beta_2 - \beta_1 \simeq \frac{pH}{D^2}\left(1 - 2\,\frac{H}{D}\right)\rho^{cc} \end{aligned} \tag{13}$$

and the resulting systematic error in δ is

$$S^{cc} = \Delta\delta_1 + \Delta\delta_2 = \left(h - \frac{pH}{D}\right)\left(1 - 2\,\frac{H}{D}\right)\frac{\rho^{cc}}{D} \tag{14}$$

To ensure the error in α less than 1.0^{cc} one must require an accuracy in the positioning of the laser sources (by $D \simeq 20$ km, $H \simeq 0.06$ km)

$$h < 1.25 \text{ mm} \; , \; p < 0.4 \text{ m} \tag{15}$$

It is very easy to preserve these requirements and in the following the influence of S from (14) can be neglected.

But it seems that it is not to neglect a possible difference in z as a result of the possible differences of atmosphere medium in non spaceidentical light path of the red and UV beams.

After taking into account the above consideration and neglecting the influence of humidity (Tengström, E., 1974, Tengström, E., 1977) we obtain from (6) and (8)

$$\frac{m_\alpha}{\alpha} = \sqrt{A + B} \qquad (16)$$

where $A = \left(\frac{m_K}{K}\right)^2 + \left(\frac{m_\lambda}{\lambda}\right)^2 + \left(\frac{m_d}{d}\right)^2 \simeq (5 \times 10^{-3})^2 + (3 \times 10^{-6})^2 +$

$$+ (1 \times 10^{-2})^2 \simeq (1.1 \times 10^{-2})^2 \qquad (17)$$

$$B = \left(\frac{m_z}{z}\right)^2 + \left(\frac{m_l}{l}\right)^2 \simeq (3 \times 10^{-2})^2 + (2.5 \times 10^{-3})^2 \simeq$$

$$\simeq (3.0 \times 10^{-2})^2 \qquad (18)$$

A is the part of relative error of absolute value of refraction angle. It depends only on the accuracy of Edlen's formula and the IDM's construction and adjustment. It does not change during repeated measurements of the same α or different α angles by this same instrument. It does not influence the variations in the refraction angle. But it introduces a systematic error into absolute value of the refraction angle.

Because the A-value is limited by a magnitude of

$$\frac{m_K}{K} \simeq 5 \times 10^{-3} \quad \text{(Edlen's formula limitation)}$$

we can consider that:

- the highest possible theoretical accuracy of the IDM is limited to approximately $\pm\, 5 \times 10^{-3}\alpha$,
- it would be useful to increase the accuracy of the term $\frac{m_d}{d}$ so that the term will be in a magnitude of about $1 - 2 \times 10^{-3}$.

B is the part of random relative error in absolute value of refraction angle.

The accuracy of z is of importance for this part of error. It is clear that increasing of the accuracy of z has the crucial significance and decides the accuracy of the whole method.

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3. PROPOSAL OF ACCURACY INCREASING OF THE MAIN FACTOR OF IDM METHOD

The discussed analysis showed that the main factor of accuracy is the accuracy of obtaining z. It would also be advisable to increase the accuracy of obtaining of the value d.

A. The method that have been used up to the present to obtain z and l described above, is characterized by many negative aspects like:

A.1. The necessity of the displacement of the red and UV laser sources and their inconvenient adjustment.

A.2. The measurement of distance between fringes which are not lying in one straight line.

A.3. Pointings at fringe shape images that does not guarantee the best obtainable positions.

A.4. Relatively large deviation between values of z as determined by two different observers.

The reasons mentioned in A.3. and A.4. have the greatest weight in the budget error. They have been caused on the one hand by the unsharpness of the fringe shape produced by short time variation of refraction, vibration of IDM and light sources, imperfect optics and film, and on the other hand by imperfect and subjectivity between different observers. An additional factor is the difference between the images of the red and UV fringes being caused by the fact that the photographic film has different sensitivity to the red and to the UV spectrum.

Apart from the above mentioned reasons the best definition of the fringe could be obtained by the objectively positioning for maximum opacity. Such a position could be determined by the measurement of the fringe image's optical density.

The equality I_2 of the energy produced in each part of its spectrum by one energetic constant light beam crossing through partially transparent medium, is dependent on its blackness. The extent E of blackness is dependent of quantity of received photoenergy I_1. The relation between the above mentioned values are not directly or inversely proportional but they are given by monotonic functions,

$E = f(I_1)$ increasing function

$I_2 = g(E)$ decreasing function

hence $(I_2)_{min} = F(I_{1_{max}})$, $(I_2)_{max} = F(I_{1_{min}})$

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where $F(x) = g(f(x))$

The I_1 is the effecting stream by exposure. Its mean average position estimates E_{max} on the film. Therefore the received $(I_2)_{min}$ by photometric scanning points to the right definition of fringe.

This value can be realized using the microphodensitometer in two ways, one by plotting a graphical curve, or two by directly obtaining the position of maximum by comparison with the standard curve. One proposed method of improvement is to have all the red and UV fringes in one straight line. This excludes the source of error mentioned in A.1 and A.2.

Let us consider the proposed device. After plotting a photodensity curve which is the sum of red and UV diffraction curves deviated from each other by the value z (fig. 5).

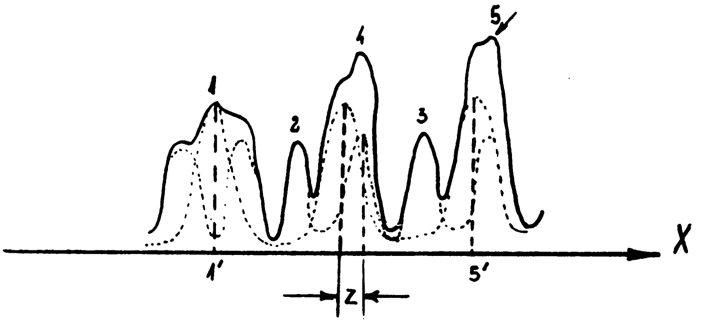

Fig. 5

Then the z value can be obtained

$$z = \frac{x_3 + x_2}{2} - \frac{x_5 + x_1}{2} \tag{19}$$

and
$$l_r = \frac{x_5 - x_1}{2} \tag{20}$$

or
$$l_r = \frac{\lambda_r}{\lambda_{UV}} l_{UV} = \frac{\lambda_r}{\lambda_{UV}} \cdot \frac{x_3 - x_2}{2} \tag{21}$$

where for current used sources of light $\frac{\lambda_r}{\lambda_{UV}} = 1.94708$, that means

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$l_r \approx 2l_{UV}$.

Therefore, by $z \ll l_{UV}$ the difficulty appears in obtaining the right position of the red fringes (fig. 5 points N = 1',5'), because every red maximum is lying near each second maximum of UV fringes, shifted only by means of z.

It seems to be possible to solve the question in two different ways.

One is to increase the resolving power of IDM grating to about $\frac{1}{10}$ z, that means for the red and UV laser a value of about

$$\Delta\lambda = \frac{1}{10}(0.6328 - 0.325) \simeq 0.031\ \mu m \qquad (22)$$

According to (Born, M., Wolf, E., 1959) the resolving power of a grating is defined by the ratio

$$R = \frac{\lambda}{\Delta\lambda} = mN \qquad (23)$$

where m the order of fringe image

N the number of grating slits.

Substituting in (23) for $\lambda = 0.6328\ \mu m$, $\Delta\lambda = 0.031\ \mu m$, $m = 1$ (the principal maximum) we obtain

$$N \simeq 20 \qquad (24)$$

which proposes to use grating of non less than 20 slits, but to achieve comparable light power on film surface it must keep hold the same sum of transparent surface of grating.

It is clear that the above mentioned proposal depends only on theoretical considerations and the practical investigations in this question are of great importance.

A second way one can try is analyzing the image of the film density curve of the photometric poorly devided red fringe maxima. From such an image each second maximum (fig. 5 points 2 and 3) is apparently divided, which gives an accurate position of the other UV maximum position.

The relative ratio of the maximum magnitude of red can be obtained from the red fringes alone. In the same way one can also obtain the ratio of UV alone.

Now one has:

B.1. The ratios $I_1:I_2:I_3:I_4:I_5$ for maximum of red fringes alone.

B.2. The means of I_1', I_2', ... I_{11}' for the UV fringe maximums calculated from the relative ratio of UV fringes only and from the absolute magnitudes of selected well divided UV fringes and red together.

B.3. The mean f(x) of sum density function of red and UV, where x is the argument of position (fig. 5).

By simple Fourier's analysis we obtain

C.1. The mean of UV density function A(x).

C.2. The mean of red density function B(x) = f(x) - A(x).

C.3. The maximum position x_i of function B(x)

$$J_i^{(r)} = \max(B(x_i))$$

The results calculated in such a manner can be objectively controlled by:

D.1. $$J_r = \frac{1}{2}(x_c - x_{c'}) = \text{const.} \tag{25}$$

where x_c and $x_{c'}$ are the suitable fringe maximum position lying symmetrical to the central fringe.

D.2. The ratios of $J_i^{(r)}$ should be equal to those obtained experimentally.

Then z from formula (19) is given by

$$z = J_{UV} - J_r \tag{26}$$

It can be assumed that the proposed procedure should give the z mean with a higher accuracy. But even if the accuracy would only be comparable with that from the comparator reading the absolute value of z would be much more objective and independent of the observer.

B. The d value was obtained by direct measurement of slits. In the formula (5) there acts some actual value which effects the interference. Therefore it is proposed to obtain the value of d from observations of the laser source interference image produced by the IDM grating. When we use for measurement the micrometer of the Wild T4 theodolite we can determine the angle θ with an accuracy of about 0.5^{cc}.

Because $$d = \frac{n\lambda}{\sin\theta} \quad (27)$$

where n is the order of interference fringes, the relative accuracy of d when $d \simeq 5$ mm and $n = 3$ is

$$\frac{m_d}{d} \simeq 2 \times 10^{-3} \quad (28)$$

Such a measurement can be realized in the laboratory under controlled conditions.

A further step to increase the possibility of an absolute test of IDM measurements is to correlate fully the observed vertical angle with the determinated dispersion. In this case it seems to be advantageous to make simultaneous exposures of the fringes of IDM and of the image of the red laser source at the focal plane of the theodolite.

In this way one observes both the actual time of observation and average time of the derived phenomena thus excluding the instrumental sources of error connected with the direct measurement of the vertical angle. Of course, it is clear that such a change involves some increase of the following evaluations for achieving the final result of the measured vertical angle.

4. CONCLUSIONS

1. The above mentioned proposals should increase the accuracy of the results of the present IDM measurement.

2. The increase of the accuracy in the present results can facilitate the explanation of the most important causes of problems in the IDM method and would assist in its further improvement.

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DISCUSSION

D.G. Currie: I wonder, if your humidity problem could perhaps be amenable to the techniques that we look forward to use, and that is: there are absorption bands in both the 6000 and the 8000 Ångström area which are accessable with silicon diods, and in our case where it is not gradients that we are interested in but actual water vapor content, we would sample the entire beam and switch between the in the band and the out of the band. And as long one does not operate in the saturated band, that can give a rather straight-forward determination of the water vapor in that column. In your case, where you have two separate columns, and you are asking for the gradient between these, it seems that you might also be able perhaps to switch between these two columns, giving the water vapor gradient in the same direction, which you are interested in, giving that number for a humidity correction on your vertical angle.

J. Milewski: I think, that the main part of the refraction angle dependent on humidity can be calculated by the meteorological data. But I suppose it must be investigated which parameters explain the occasional great uncorrelation between dispersion and refraction when using widely separated spectral lines. It is therefore very important also to study the humidity factor. I believe it would be possible by investigating simultaneously the fluctuations of microwave beams.

E. Tengström: We have studied the humidity question rather carefully since the time as de Munck pointed out to me the importance of the vertical water vapor gradient. We investigated wet models using Laplace formula, and found that during inversions with great positive temperature gradients the humidity corrections can take negative values which can not be neglected. Meteorological experiences here show, however, that the humidity gradient can reach values more than 20 times those predicted from the temperature gradient by Laplace, so that already at an inversion of +2°C/100 m and a humidity gradient of -30 mm/km, the correction amounts to -8", the total refraction being as high as 130". Great refraction values, which occur during inversions in the evening some hours before midnight, seem to correspond to high negative humidity gradients, at least in our experiments, and it is therefore - during such conditions - necessary to have relevant gradient observations done, so that the humidity integral can be computed with sufficient accuracy. We have started a cooperation with the Meteorological Institute here, and our common plan is to use balloons evenly spread out along the line of sight of our test base for assembling meteorological informations at short intervals of time, to be transmitted from the sensors in the balloon sondes and recorded at the observation site. From the vertical gradient distribution of humidity, pressure and temperature obtained, we may calculate the humidity correction under various conditions of inversion with high accuracy. In the future field work we have to avoid measuring during such inversions, which correspond to high positive values of the vertical gradient of temperature. A rough observation of this gradient before the measurement can help us to decide if we shall observe or not. Under normal conditions ($\sim -1^{\circ}C/100$ m) with medium horizontal windspeed, the humidity gradient has been found to be small and the humidity correction is negligable or can be derived from rough data. To calculate the total refraction angle by means of merely meteorological data seems to be impossible to realize even in the future, because we shall never in practise be able to achieve simultaneity with the angle observations and a sensor spacing along the line of sight, which satisfy our needs for deriving refraction with an accuracy aimed at for the moment (that is to within 0".2 ≡ mean error of today's star catalogues). What the turbulence effects concerns, we have to choose integration times of dispersion measurements equal to the angle observation times. And these integration times have to be evaluated according to the specific use of the results. With lasers it will - to my opinion - also be necessary to focus a wider part of the incoming red and blue wavefronts, which have different irregularities in shape, and then by means of a small concave collimator mirror direct the integrated plane fronts into the small aperture of the observing instrument. I believe that this spatial integration (see my previous remark under Mårtensson's paper) will reduce the occasional bad correlation between dispersion and refraction, now observed with our short laser exposures. I don't think the definition of the fringes will be essentially deteriorated, because we will have the same situation as in the case of ordinary light of various wavelengths, the wavefronts of which are very regular at great distances.

J. Dommanget: I would like to inform you about some research I made on the effect of water vapor pressure in the atmosphere on the image quality observed with our 45 cm aperture Cooke-Zeiss refractor. At Brussels, the Royal Observatory and the Royal Meteorological Institute are located at the same spot. So this was a good opportunity for observing image quality (Danjon's estimator scale) - refractor diameter shut down to 25 cm - when radiosounding were made. A statistical study showed a good correlation between water vapor pressure and image quality when in the same time shearing (important wind gradient) between two contiguous layers was observed. This seems to support the idea just considered, of the importance of water vapor pressure also into the general problem of atmospheric refraction. (See: Communication de l'Observatoire Royal de Belgique, no 231, 1964).

E. Tengström: Mr Kahmen and Mr Brunner said, that there is a clear correlation between the magnitude of the refraction angle and the fluctuations caused by the turbulence. Then your statement would mean that the refraction is strongly correlated to the humidity pressure itself, which I find a little strange.

J. Milewski: I think it is rather clear, because there is a very close correlation between the gradient of temperature and the gradient of humidity. One may riskily admit that it should also be a close correlation between the humidity pressure and the turbulence.

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