Lothar Sebastian Krapp, *Algebraic and Model Theoretic Properties of O-minimal Exponential Fields*, Universität Konstanz, Germany, 2019. Supervised by Salma Kuhlmann. MSC: Primary 03C64, 12L12; Secondary 12J25, 12L05. Keywords: o-minimality, ordered exponential fields, real exponentiation, decidability, convex valuations.

Abstract

An exponential exp on an ordered field $(K, +, -, \cdot, 0, 1, <)$ is an order-preserving isomorphism from the ordered additive group $(K, +, 0, <)$ to the ordered multiplicative group of positive elements $(K^{>0}, \cdot, 1, <)$. The structure $(K, +, -, \cdot, 0, 1, <, \exp)$ is then called an ordered exponential field (cf. [6]). A linearly ordered structure $(M, <, \ldots)$ is called o-minimal if every parametrically definable subset of M is a finite union of points and open intervals of M.

The main subject of this thesis is the algebraic and model theoretic examination of o-minimal exponential fields $(K, +, -, \cdot, 0, 1, <, \exp)$ whose exponential satisfies the differential equation $\exp' = \exp$ with initial condition $\exp(0) = 1$. This study is mainly motivated by the Transfer Conjecture, which states as follows:

Any o-minimal exponential field $(K, +, -, \cdot, 0, 1, <, \exp)$ *whose exponential satisfies the differential equation* $\exp' = \exp$ *with initial condition* $\exp(0) = 1$ *is elementarily equivalent to* $\mathbb{R}_{\exp}$.

Here, $\mathbb{R}_{\exp}$ denotes the real exponential field $(\mathbb{R}, +, -, \cdot, 0, 1, <, \exp)$, where exp denotes the standard exponential $x \mapsto e^x$ on $\mathbb{R}$. Moreover, elementary equivalence means that any first-order sentence in the language $\mathcal{L}_{\exp} = \{+, -, \cdot, 0, 1, <, \exp\}$ holds for $(K, +, -, \cdot, 0, 1, <, \exp)$ if and only if it holds for $\mathbb{R}_{\exp}$.

The Transfer Conjecture, and thus the study of o-minimal exponential fields, is of particular interest in the light of the decidability of $\mathbb{R}_{\exp}$. To the date, it is not known if $\mathbb{R}_{\exp}$ is decidable, i.e., whether there exists a procedure determining for a given first-order $\mathcal{L}_{\exp}$-sentence whether it is true or false in $\mathbb{R}_{\exp}$. However, under the assumption of Schanuel's Conjecture—a famous open conjecture from Transcendental Number Theory—a decision procedure for $\mathbb{R}_{\exp}$ exists (cf. [7]). Also a positive answer to the Transfer Conjecture would result in the decidability of $\mathbb{R}_{\exp}$ (cf. [1]). Thus, we study o-minimal exponential fields with regard to the Transfer Conjecture, Schanuel's Conjecture, and the decidability question of $\mathbb{R}_{\exp}$.

Overall, we shed light on the valuation theoretic invariants of o-minimal exponential fields—the residue field and the value group—with additional induced structure. Moreover, we explore elementary substructures and extensions of o-minimal exponential fields to the maximal ends—the smallest elementary substructures being prime models and the maximal elementary extensions being contained in the surreal numbers. Further, we draw connections to models of Peano Arithmetic, integer parts, density in real closure, definable Henselian valuations, and strongly NIP ordered fields.

Parts of this thesis were published in [2–5].

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Xue Li, ***Automating the Repair of Faulty Logical Theories***, Artificial Intelligence Applications Institute, School of Informatics, University of Edinburgh, UK, 2021. Supervised by Alan Bundy, with Alan Smaill, Eugene Philalithis, and Ewen Maclean as secondary supervisors. MSC: 68T05, 68T30 and 68T15. Keywords: automated reasoning, belief revision, abduction, conceptual change, reformation, theory repair.

Abstract

This thesis aims to develop a domain-independent system for repairing faulty Datalog-like theories by combining three existing techniques: abduction, belief revision, and conceptual change. Accordingly, the proposed system is named the ABC repair system (ABC). Given an observed assertion and a current theory, abduction adds axioms, which explain that observation by making the corresponding assertion derivable from the expanded theory. Belief revision incorporates a new piece of information which conflicts with the input theory by deleting old axioms. Conceptual change uses the reformation algorithm for blocking unwanted proofs or unblocking wanted proofs. The former two techniques change an axiom as a whole, while reformation changes the language in which the theory is written. These three techniques are complementary. But they have not previously been combined into one system. We are working on aligning these three techniques in ABC, which is capable of repairing logical theories with better result than each individual technique alone. In addition, ABC extends abduction and belief revision to operate on preconditions: the former deletes preconditions from rules, and the latter adds preconditions to rules. Datalog is used as the underlying logic of theories in this thesis, but the proposed system has the potential to be adapted to theories in other logics.

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Andrew Parisi, ***Second-Order Modal Logic***, University of Connecticut, USA, 2017. Supervised by Marcus Rossberg. MSC: 03B15, 03B45, 03F03, 03F05, 03B20, 03B60, 03A05. Keywords: modal logic, higher-order logic, free logic, sequent calculus, hyper-sequents, inferentialism, proof-theoretic semantics, philosophical logic, quantification, ontological commitment, necessitism, contingentism.

Abstract

The dissertation introduces new sequent-calculi for free first- and second-order logic, and a hyper-sequent calculus for modal logics K, D, T, B, S4, and S5; to attain the calculi for the stronger modal logics, only external structural rules need to be added to the calculus for K, while operational and internal structural rules remain the same. Completeness and cut-elimination are proved for all calculi presented.

Philosophically, the dissertation develops an inferentialist, or proof-theoretic, theory of meaning. It takes as a starting point that the sense of a sentence is determined by the rules governing its use. In particular, there are two features of the use of a sentence that jointly determine its sense, the conditions under which it is coherent to assert that sentence and the