sary. That is, a system which satisfies Hermes's axioms would be a system of special relativity, but not every system of special relativity would satisfy Hermes's axioms. Hence Hermes's axioms do not properly constitute "an axiomatization of general mechanics."

The difficulty seems to lie mainly in axiom A8.1, which says that the corpuscles of matter behave in certain very peculiar fashions. It is quite possible that, in writing A8.1, Hermes really had in mind some sort of conditional statement to the effect that if the corpuscles behave in certain very peculiar fashions, then certain other things would happen. However, as stated, A8.1 is distinctly not conditional. BARKLEY ROSSER

I. M. BOCHEŃSKI. *Notes historiques sur les propositions modales.* ***Revue des sciences philosophiques et théologiques*** (Paris), vol. 26 (1937), pp. 673–692.

The logic of modal propositions is at best a decadent development. Great confusion arises from the use of "possible" to mean, sometimes, "not impossible," and sometimes, "neither impossible nor necessary." The author proposes to use "contingent" for the latter sense.

Aristotle uses "possible" in the sense of "contingent" and Theophrastus uses it in its strict sense. Herein lies their main point of difference with regard to modal propositions, causing Theophrastus to think he had a new system of logic which corrected the "errors" of Aristotle. Albertus Magnus followed Aristotle; "Pseudo-Scotus" expounded the distinction; Ockham combined the two systems and derived 1000 valid forms.

"Pseudo-Scotus" produced interesting proofs of the propositions, "A false proposition implies any proposition" and "A true proposition is implied by any proposition."

S. K. LANGER

R. FEYS. *Les logiques nouvelles des modalités.* ***Revue néoscolastique de philosophie,*** vol. 40 (1937), pp. 517–553, and vol. 41 (1938), pp. 217–252.

A clear, concise systematization of modern contributions to the study of elementary, abstract, modal logics; a summarization and comparison of (0) the classical true-false logic, e.g., that of ***Principia mathematica,*** (1) logics of the traditional modal concepts of necessity, possibility, etc., e.g., Lewis's logic of "strict implication," (2) "intuitionistic" logics, e.g., Heyting's, (3) many-valued logics, e.g., those of Łukasiewicz. The essay provides a valuable basis and stimulus for further investigations in the field.

I would question one remark. Following Wajsberg, the author states that logics of type (1) can be translated into logics of general propositions or classes. For example, he would say that the analogue of Ap, or better, $A(\phi x)$, "ϕx is necessary," is $(x)\phi x$, "for every x, ϕx." But in a calculus of classes or general propositions there can be strict as opposed to material relations, analogous to strict as opposed to material implication as distinguished by Lewis. In the usual extensional treatment of classes and general propositions, where only material relations are explicitly used, $(x)\phi x$ means merely "for every actual x, ϕx" instead of "for every possible x, ϕx," and is not equivalent to "ϕx is necessary." Only a calculus of classes or general propositions using strict relations would contain analogues of the traditional modal forms. CHARLES A. BAYLIS

L. CHWISTEK and W. HETPER. *New foundation of formal metamathematics.* ***The journal of symbolic logic,*** vol. 3 (1938), pp. 1–36.

The primitive signs comprise just the letter "*c*" and a two-place operator "*". These are combined according to a principle familiar to readers of Łukasiewicz: "*cc", "**ccc", "*c*cc", "**cc*cc", etc. The two signs receive meaning only mediately, through interpretation of certain arbitrary complexes thereof. For example, complexes of the form "******cc**cc*cc*EE*FF*GG*HH" (with the capitals supplanted by any expressions built in turn from "*c*" and "*") are abbreviated "$(EFGH)[c]$" and explained as meaning that H is the result of substituting G for F in E. Another arbitrary mode of combination of "*c*" and "*" is identified with alternative denial (Sheffer's stroke); and in terms of this the other truth functions are developed in familiar fashion. The syntactical notion of substitution and the logical truth functions are thus gathered into a single system. On this basis further syntactical notions are defined, among them the notions of expression and statement, the identity of expressions, and the occurrence of one expression in another. Rules of procedure are adopted which yield both logical and syntactical theorems.

Natural numbers are introduced by explaining "0", "1", "2", ⋯ as abbreviations of "*cc", "*00", "*11", ⋯ . The statement that E is a natural number, in this sense, is itself expressed within the formal system, in terms of substitution; in effect, E is a natural number if *EE results from substituting 0 for 1 in **EE*EE.

Quantification is introduced much in the manner of substitution and alternative denial: certain arbitrary complexes of "c" and "x" are interpreted as variables, and certain further complexes containing these variables are interpreted as quantifiers. A distinction is maintained between logical and so-called semantic variables, and also between variables of different logical types.

The authors restrict quantification in a manner similar to Poincaré's repudiation of "impredicative" definitions, or to Russell's ramified theory of types without the axiom of reducibility. They introduce a hierarchy of "elementary systems," in each of which an upper limit is imposed upon the admissible types of variables; and they allow quantification only within one or another definite elementary system, specified numerically in the quantifier. But some relief is gained through metamathematical channels; it is possible within the formal notation to deal with successive elementary systems, and to state that such and such is or is not a theorem of each.

Under the head of applications, an elementary arithmetic is outlined. The definitions of sum, product, and power are essentially the traditional recursive ones, transformed into direct formal definitions. But Frege's method of transforming recursive definitions into direct ones, through the medium of hereditary classes (or relations), is not followed here; Frege's use of the class variable is avoided in favor of the expression variable. Where Frege's construction refers to membership in all infinite classes of a certain sort, the present authors accomplish the same purpose by speaking rather of peculiarly environed occurrence within all finite expressions of a certain sort. (For a fuller account of the essential procedure, see the reviewer's *On derivability* (III 53(1)), and *Definition of substitution* (I 116)).

Likewise under the head of applications, the authors sketch the beginnings of a theory of classes, a theory of relations, and a metamathematical calculus. Classes are arbitrarily identified with universally quantified statements; roughly, the class $\hat{x}(\ldots)$ is identified with the statement "$(x)(\ldots)$". Huntington's postulates for Boolean algebra are derived. In the metamathematical calculus, the principal notion constructed is that of the arithmetized syntax corresponding to a given elementary system.

Constructions are obscured by failure to distinguish clearly between use and mention of expressions. This difficulty pervades the paper; by way of a sample instance, however, it will suffice to consider the explanation of "$(EFGH)[c]$". In this explanation, as observed earlier, the authors use the idiom ". . . is the result of substituting . . . for . . . in . . . ." Now grammar would direct us to fill the blanks here with nouns—names of the things talked about. But the things talked about here are expressions in turn. The blanks should thus be filled by names of the expressions with which the substitution deals; not by the expressions themselves. (Illustration: substitution of alpha for eta in zeta yields zeta, not zalpha.) It would appear, then, that the complexes of "c" and "*" which supplant the four capitals, in any specific instance of "$(EFGH)[c]$", are *names of* the expressions involved in the denoted substitution. But the use made of the form "$(EFGH)[c]$" in subsequent constructions denies this; we find that the intended objects of substitution are the expressions which actually appear in the statement of substitution. (Cf. the definition of "E is a natural number," mentioned above.) Does this mean that all complexes of "c" and "*" are to be construed as names of themselves? But then how can the authors give those same complexes other meanings in addition—meanings of alternative denial, substitution, etc.? It is difficult to determine how much of the essential theory of the paper could be reconciled with a strict distinction between use and mention of expressions. (Cf. Church's review of Chwistek, this JOURNAL, vol. 2 (1937), p. 170.) W. V. QUINE

JIRÔ HIRANO. *Einige Bermerkungen zum v. Neumannschen Axiomensystem der Mengenlehre.* ***Proceedings of the Physico-mathematical Society of Japan,*** 3 s. vol. 19 (1937), pp. 1027–1045.