values already known; and assuming that but one is left alive at the extreme age (whatever that may be fixed upon to be), the number alive at all the younger ages may be found. The D and N columns may then be found in the usual way, except that it will be found desirable to carry on the calculation of the l_x, D_x, and N_x at once, so that any error (not unlikely to arise in so many successive dependent operations) may be at once detected.

In conclusion, the only source of accumulative error may be pointed out, viz., that arising from the value of v being not exact. This may be found in extent, and corrected by a periodic alteration in the last figure; and for those who may employ 3 per cent. as the basis of their calculations, it may be mentioned that an error in excess will occur in each of the operations involving v, the process being, of course, a dependent one.

<div align="right">Your obedient servant,
WILLIAM WYLIE.</div>

Scottish Widows' Fund Assurance Society,
  Edinburgh, May 12, 1852.

ON DE WIT'S HYPOTHESIS AS TO THE RATE OF MORTALITY.

To the Editors of the Assurance Magazine.

GENTLEMEN, — All who study the science of life contingencies, and especially those who take an interest in the history of its rise and progress, are under great obligations to Mr. Hendriks for his notices in the *Assurance Magazine* regarding two remarkable works new to the English reader — that of Tetens, who had anticipated Barrett in his method of simplifying assurance calculations, and who was also in possession of more powerful methods of practical computation than any known in this country until the last few years; and, more recently, the restoration of the earliest treatise on life annuities, by John De Wit, which, although printed, had from the time of its publication eluded the search of all previous inquirers.

The earliest English writing upon the subject of life annuities is the paper by Dr. Halley in the *Philosophical Transactions* for 1693. The date of De Wit's treatise is 1671, but from its suppression Halley could not have seen or obtained any information from it, so that his paper is as original as if De Wit had not written. Halley there gave the first Life Table ever constructed upon true principles from real observations, and his knowledge of the properties of life tables, and the mode of applying them to the computation of life annuities on one or more lives, was as perfect as ours is at the present day. The actual computation of life annuity tables was, however, much facilitated by deducing the value of an annuity upon age x from that on age $x+1$, the rule for which was given by De Moivre.

Such is the nature of Halley's paper; and now, having De Wit's treatise restored to us through Mr. Hendrik's perseverance and labour, it will be interesting to examine if it is of the same order of merit, and if its publication in England would have destroyed the originality of Halley's paper. I have examined it with this view, and have come to the conclusion that De Wit's treatise is so vitiated by fundamental errors, that it does not contain the true method of calculating life annuities, and that its reputation could not at any time have long survived its perfect publication.

Before proceeding to point out the errors in De Wit's work, I would observe that although his style is very verbose and unmathematical, these points are not fair objects of criticism, since it is a Report to the States General of Holland (a political and not a scientific body), and therefore attempted to be made obvious to ordinary persons.

He sets out with the following hypothesis of mortality. Take fifty years of a man's life when he is in his full vigour, say from 3 up to 53 years of age; it is as likely that this man will die in any one year or half-year of this period as in another; that in a second period, from 53 to 63 years of age, the probability of his dying in a year or half-year will be to the same probability for the first period as 3 to 2; in a third period, from 63 to 73, as 2 to 1; and in a fourth period, from 73 to 80, as 3 to 1; at which latter age the life becomes extinct.

Although there is some ambiguity in the statement of the hypothesis, there can be no doubt that the idea of it in De Wit's mind was, that the *risk of mortality* was equal for persons of all ages from 3 to 53, was equal from 53 to 63, but increased as 3 to 2, &c. This is evident from the phrase, " his full vigour"—he supposing, as most persons in his age did, that during that term one life was for a short period as good as another; also, he shows that, according to his hypothesis, the expectation of receiving 2,000 florins if a life age 58 (in the second period) dies within six months, is equal to an expectation of 3,000 florins if a life aged 40 (in the first period) should die within the same period of time. These expectations are equal if the *mortality* of the second period is to that of the first as 3 to 2, and in no other case. And in his concluding observations he observes that, although he has assumed each of the first 50 years to be equally destructive, yet that there is in nature a perceptible gradation, so that the destructiveness of the last years of a period is greater than that of the first years, from the gradual waste of vigour; and makes other remarks which are inconsistent with any other interpretation of his hypothesis than that I have given.

And here it may be well to inquire if this hypothesis was derived from observation of mortality records. Mr. Hendriks says, yes; but if De Wit's treatise is given entire, there is not the shadow of a proof of this. Nothing is said by him of such observations, and any collection of them would have shown it to be radically erroneous. Moreover, its accordance with the notions of mortality prevalent in almost all ages previous to the accurate investigation of the subject, strengthens the probability that it was a mere numerical statement of common notions.

Such is De Wit's hypothesis, and his calculation should have been in accordance with it; but they are not. In his calculations he has taken the age of 3 years, because he supposes that the best age at which to select a life for an annuity, and he assumes that *this person* is as likely to die in any one year of age as another up to 53 years of age. This is the hypothesis of *equal decrements*, a totally distinct thing from *equal mortality*, although much more in accordance with real life tables. This shifting of the hypothesis arose from a radical confusion in De Wit's mind between the probability of a person now aged x years dying in the $(x+n+1)$th

year of age, which is $\dfrac{l_{x+n}-l_{x+n+1}}{l_x}$, and the probability of a person now

aged $x+n$ dying in the $(x+n+1)$th year, which is $\dfrac{l_{x+n}-l_{x+n+1}}{l_{x+n}}$. This

misinterpretation by De Wit of his own hypothesis when applied to the computation of the value of a life annuity, is alone fatal to the reputation of his treatise. There is another part of his calculations equally erroneous; but instead of entering into detail, I will simply show it to be at variance both with *equal decrements* and *equal mortality* for the different periods. It will be observed on reference to his treatise, that after taking each of the first half-yearly terms once, he takes $\frac{2}{3}$ of each of the next 20 terms of the second period, $\frac{1}{2}$ of the terms of the third period, and $\frac{1}{3}$ of the terms of the final period. Now, according to his own hypothesis, as misinterpreted by him from mortality to decrements, these multipliers should obviously be $\frac{3}{2}$, 2, and 3; and lowering the decrement from 1 to $\frac{2}{3}$ instead of increasing the mortality in the proportion of 3 to 2, actually decreases it. The same reasoning applies to the other periods.

I think it is now shown that De Wit's hypothesis of mortality was not founded upon any observations, and was therefore unworthy of any confidence as a basis for the valuation of life annuities; and that, as a mathematician, he committed two serious errors in the application of it to his subject.

Although truth should never be sacrificed to national partialities, it can hardly be displeasing to us to find Halley still alone in his glory as the founder of our science.

WILLIAM ORCHARD.