isomorphism relation for a class of compact metrizable structures. This provides a more direct proof of the theorem above and allows one to view the earlier results of Sabok and of Clemens, Gao, and Kechris as consequences of it.

Abstract prepared by Joseph Zielinski
E-mail: zielinski.math@gmail.com
URL: http://hdl.handle.net/10027/21320

ANTON BOBKOV, ***Computations of Vapnik–Chervonenkis Density in Various Model-Theoretic Structures***, University of California, Los Angeles, 2017. Supervised by Matthias Aschenbrenner. MSC: Primary 03C, Secondary 05C. Keywords: VC-density, trees, Shelah–Spencer graphs, superflat graphs, p-adic numbers.

Abstract

Aschenbrenner et al. have studied Vapnik–Chervonenkis density (VC-density) in the model-theoretic context. We investigate it further by computing it in some common structures: trees, Shelah–Spencer graphs, and an additive reduct of the field of p-adic numbers. In the theory of infinite trees we establish an optimal bound on the VC-density function. This generalizes a result of Simon showing that trees are dp-minimal. In Shelah–Spencer graphs we provide an upper bound on a formula-by-formula basis and show that there isn't a uniform lower bound, forcing the VC-density function to be infinite. In addition we show that Shelah–Spencer graphs do not have a finite dp-rank, so they are not dp-minimal. There is a linear bound for the VC-density function in the field of p-adic numbers, but it is not known to be optimal. We investigate a certain P-minimal additive reduct of the field of p-adic numbers and use a cell decomposition result of Leenknegt to compute an optimal bound for that structure. Finally, following the results of Podewski and Ziegler we show that superflat graphs are dp-minimal.

Abstract prepared by Anton Bobkov
E-mail: antongml@gmail.com
URL: https://escholarship.org/uc/item/5xg6m05f

ATHIPAT THAMRONGTHANYALAK, ***Extensions and Smooth Approximations of Definable Functions in O-minimal Structures***, University of California, Los Angeles, 2013. Supervised by Matthias Aschenbrenner. MSC: Primary 03C64, Secondary 14P10, 32B20. Keywords: o-minimal structures, Whitney Extension Theorem.

Abstract

A jet of order m on a closed set $E \subseteq \mathbb{R}^n$ is an indexed family $(f_\alpha)_{\alpha\in\Lambda}$, where $\Lambda = \{(\alpha_1,\dots,\alpha_n) \in \mathbb{N}^n : \sum_{i=1}^n \alpha_i \leq m\}$. In 1934, H. Whitney proved Whitney's Extension Theorem, which gives a necessary and sufficient condition on the existence of C^m-extensions of a jet of order m on a closed subset of $\mathbb{R}^n$. In the same year, he asked how one can determine whether a real-valued function on a closed subset of $\mathbb{R}^n$ is the restriction of a C^m-function on $\mathbb{R}^n$ and gave an answer to the case $n = 1$. Later, the case $m = 1$ was proved by G. Glaeser using the concept of "iterated paratangent bundles". A complete answer to Whitney's Extension Problem was provided much later in early 2000s by C. Fefferman.

In the first part of this thesis, we study the above questions in an o-minimal expansion of a real closed field. We prove a definable version of Whitney's Extension Theorem. In addition, we solve the C^1 case of Whitney's Extension Problem in o-minimal context.

In the rest of this thesis, we discuss the following question: Suppose R is a real closed field and U is an open subset of R^n. If $f : U \to R$ is continuous, definable in an o-minimal expansion of R, and $\varepsilon \in R^{>0}$, is there a definable C^m-function $g : U \to R$ such that $|g(x) - f(x)| < \varepsilon$ for all $x \in U$? We gave a positive answer to this question. This result was inspired by a series of articles by A. Fischer.

Abstract prepared by Athipat Thamrongthanyalak
E-mail: athipat.th@chula.ac.th
URL: https://escholarship.org/uc/item/3046c8hr

ERIK WALSBERG, ***Metric Geometry in a Tame Setting***, University of California, Los Angeles, 2015. Supervised by Matthias Aschenbrenner. MSC: Primary 03C64. Keywords: o-minimal structures, metric spaces, tame geometry.

Abstract

The thesis is about the topology and geometry of metric spaces definable in an o-minimal expansion $\mathcal{R}$ of an ordered field $(R, <, +, \cdot)$. A definable metric space is a pair (X, d) consisting of a definable set $X \subseteq R^k$ and a definable $(R, +, <)$-valued metric. If $X \subseteq R^k$ is definable and e is the restriction of the usual euclidean metric on R^k to X then (X, e) is a definable metric space, in this way the geometry of definable sets may be considered as a special case of the geometry of definable metric spaces. Examples of definable metric spaces whose geometry is unlike that of any definable set are given by the hyperbolic plane ($\mathbb{R}_{\exp}$-definable) and certain subriemannian spaces ($\mathbb{R}_{\text{an}}$-definable). The main theorem of the thesis is the following: *Let* (X, d) *be a definable metric space. Then one of the following holds*:

1. *There is an infinite definable* $A \subseteq X$ *such that* (A, d) *is discrete.*
2. *There is a definable set* $Z \subseteq R^l$*, for some* l*, such that* (X, d) *is definably homeomorphic to* Z *equipped with its induced euclidean topology.*

If $(R, <, +, \cdot)$ is the ordered field of real numbers, then a definable set A is infinite if and only if it is uncountable. As a separable metric space cannot contain an uncountable discrete subset the theorem above shows that a separable metric space definable in an o-minimal expansion of the real field is definably homeomorphic to a definable set equipped with its induced euclidean topology. This reduces the topology of separable definable metric spaces in o-minimal expansions of the real field to the topology of definable sets. Perhaps surprisingly, there are interesting examples of nonseparable metric spaces definable in $(\mathbb{R}, <, +, \cdot)$, geometric realizations of Cayley graphs of "definable group actions".

Later in the thesis, the theory of imaginaries in real closed valued fields is used to prove the following: *If* $\mathcal{X}$ *is an* $(\mathbb{R}, <, +, \cdot)$*-definable family of compact metric spaces then the collection of Gromov–Hausdorff limits of sequences of elements of* $\mathcal{X}$ *forms an* $(\mathbb{R}, <, +, \cdot)$*-definable family of metric spaces.* This theorem is an analogue of a result proven by van den Dries on Hausdorff limits of definable families of sets. Its proof gives a connection between the model theory of valued fields and the geometry of definable metric spaces.

Abstract prepared by Erik Walsberg
E-mail: erikw@illinois.edu
URL: https://escholarship.org/uc/item/9z92f967

ANTON FREUND, ***Type-two well-ordering principles, admissible sets, and*** Π^1_1***-comprehension***, University of Leeds, UK, 2018. Supervised by Michael Rathjen. MSC: 03B30, 03D60, 03F05. Keywords: well-ordering principles, admissible sets, Π^1_1-comprehension, dilators, beta-proofs, Bachmann-Howard ordinal, primitive recursive set theory, slow consistency, proof length, Paris-Harrington principle.

Abstract

This thesis introduces a well-ordering principle of type two, which we call the Bachmann-Howard principle. The main result states that the Bachmann-Howard principle is equivalent to the existence of admissible sets and thus to Π^1_1-comprehension. This solves a conjecture of Rathjen and Montalbán. The equivalence is interesting because it relates "concrete" notions from ordinal analysis to "abstract" notions from reverse mathematics and set theory.

A type-one well-ordering principle is a map T which transforms each well-order X into another well-order $T[X]$. If T is particularly uniform then it is called a dilator (due to Girard). Our Bachmann-Howard principle transforms each dilator T into a well-order $\mathrm{BH}(T)$.