'The Schuler Pendulum and Inertial Navigation'

Frank Coffman Bell

MR. J. A. LEE by his commentary (*Journal*, **22**, 267) rightly suggests that instead of saying in my par. 1 (*Journal*, **21**, 504), as I did after quoting Mr. Carr and Sq.-Leader Scott, R.A.F. (*Journal*, **20**, 406, sec. 2, par. 1) my words '—and from this deducing velocity and position of the vehicle,' I could better have simply continued their words, 'From this acceleration we can then, by successive mathematical integrations, deduce first velocity and secondly the displacement of the vehicle.' Furthermore, it was superfluous and careless of me to add my quite worthless opinion as to soundness and virtue of expression on their part, but I comfort myself, and I hope also Mr. Lee, with the knowledge that there is no evidence that my opinions have ever previously had any effect whether for good or ill. To the extent, if any, that I can understand Mr. Lee's learned commentary, I believe I quite agree with him and am especially grateful for the clear definition given in his final sentence. If, however, an answer has been given to the question I raised as to the connection of Schuler's 1923 paper with inertial navigation, I am not presently aware of this.

I will take this opportunity to mention a statement in terms of elementary servo theory and bearing on my question. The Mathieu differential equation, which for infinitesimal θ and in one and the same plane can be written

$$\frac{d^2\theta}{dt^2}+(\alpha+\beta\cos t)\,\theta=0,$$

is the general system equation for both Schulerian and inertial navigations. In Schulerian systems, both α and β vanish identically. In inertial navigation systems β vanishes identically and $\alpha\equiv g/R$. This statement is mine, but the ideas on which it rests (if it does) are, as far as I am concerned, entirely due to Professor Hsue Shen Tsien, Ph.D., who has set them forth in his 1954 book *Engineering Cybernetics*, McGraw-Hill Book Company, Inc., New York, Toronto, London, esp. Chapter 11, 'Nonlinear Systems', pp. 160–167.

Position Finding with Spirit Level

F. W. G. Smith

THE use of the Sun's shadow is a very old method for position finding. It is easy to adapt a small spirit level to produce a shadow on a millimetre scale which will give results of sufficient accuracy to add to the interest in learning to work out position lines for the Sun; less accurately for the Moon. Schools might find this useful for theoretical and practical classes.

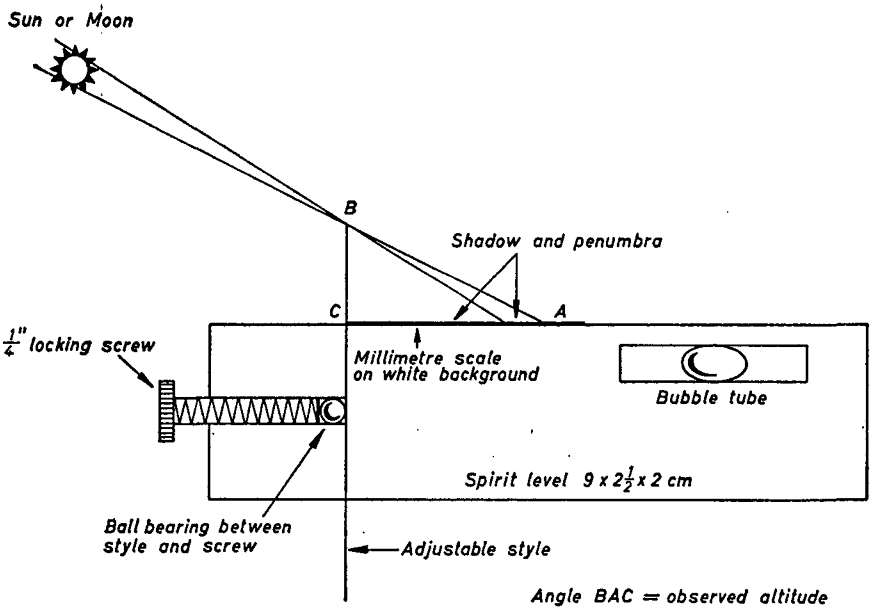

FIG. 1. General principles and sectional lateral view

The illustrations explain the principles involved and the constructional details. With practice it is fairly easy to set the style to the height necessary to give a clear shadow and to record its length to one tenth of a millimetre on the scale. To avoid errors of parallax in setting the style, the pinhole sight is held at the required height at the end of the spirit level opposite to the style while the latter's height is measured with the millimetre rule which should be held, vertical, about where the shadow is found to end on the scale. The metal parts are not appreciably effected by temperature changes. If possible five sights, at over a

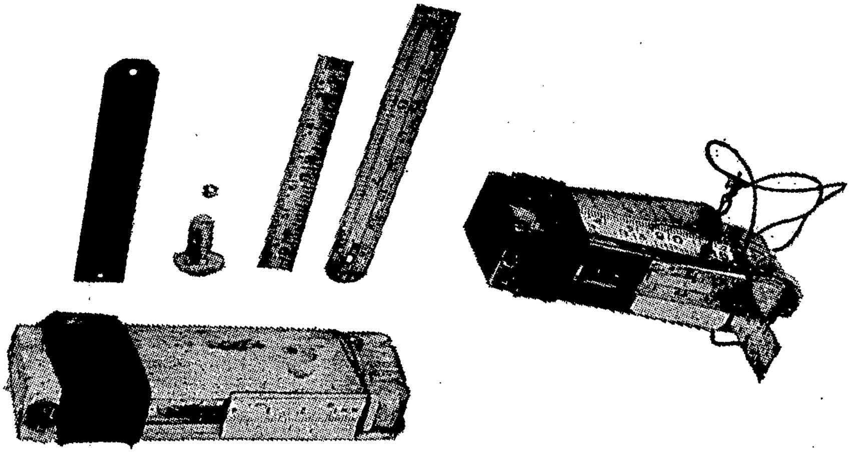

FIG. 2. The equipment

minute's interval, are recorded and averaged as it was noted that varying sky conditions could produce sudden small alterations in the shadow length. Unsteady shadows suggested a poor sight. The instruments were calibrated, as in the backstaff, by taking meridian altitudes of the Sun at a known position. SL 2 appeared to require no correction for the Sun's semi-diameter, as applied in the backstaff, but SL 3 had a constant error which was corrected by the addition of the Sun's semi-diameter. It was considered that the penumbra produced by low altitude readings cancelled correction for refraction. For Moon sights a transparent scale is necessary, as noted in SL3; which can be used also for Sun sights. SL3 was levelled up on a small adjustable table, the bubble being illuminated by a weak red light from a pencil torch. Shadows can be recorded from a quarter to full Moon.

Sun sights were taken sitting down, resting the hands on the thighs, or with the instrument held level on the ground. I found it best to watch the bubble, with both eyes open, and take glances at the shadow length on the scale, keeping the head steady. Anisometropes or those with a strong master eye should do what suits them best and estimate their own working error. One must avoid background reflections which can alter the length of the shadow. Side shades will accentuate a poor shadow. Ex-Meridian, *Burton*'s 4-figure or computed altitude and azimuth tables were used for working the sights; bubble sextant corrections for the Moon. A uniform sky, clear or overcast, gave good sights, contrasting conditions near the Sun may give poor results. Sun altitudes were recorded from 6°–62°, lengths of style 1 mm. to 350 mm. and shadows 9 mm. to 300 mm., approximately. Over 2300 Sun and 79 Moon sights were taken. The tables show the results of 266 Sun and 79 Moon sights, after the technique had been developed. They were taken under varied weather conditions; not necessarily at regular times or on successive days, which might unconsciously influence judgment when working from a known position, especially with meridian altitudes. Sights taken during gales, rain, fog, snow, &c. did not necessarily give poor results.

Analysis of Sun sights

A Conditions: (1) Sun clear of big clouds or cloud bank.
(2) Fairly uniform sky, which can be overcast.
(3) Good to medium shadows.

B Conditions: Poor shadow, excluding big cloud or cloud bank near Sun.

C Conditions: (1) Big clouds or cloud bank near Sun.
(2) Good, medium or poor shadows.

Error from true position in nautical miles.

Conditions	Number of sights	Altitudes	0′ – 5′	+ 5′ – 10	+ 10′ – 20′	+ 20′
(A)	172	15° – 62°	50%	29%	20%	1%
(B)	29	,,	38%	28%	24%	1%
(C)	65	,,	6%	20%	54%	20%

Analysis of Moon sights

A Conditions: (1) Fairly quiet conditions or uniform sky.
(2) Good to medium shadows.

B Conditions: (1) Poor shadow.
(2) Haze, gales, &c.

Error from true position in nautical miles.

Conditions	Number of sights	Altitudes	0′ – 5′	+ 5′ – 10′	+ 10′ – 20′	+ 20′
(A)	42	18° – 62°	26%	26%	24%	24%
(B)	37	,,	27%	19%	8%	46%

Compared to modern navigational methods the results are poor, but when a Sun shadow is obtainable, position lines within ten miles, and quite often better, can be obtained under most conditions with an improvised spirit level. No horizon is required and the device can be carried in the waistcoat pocket. Under quiet conditions sights can be obtained at sea and in fog. With household odds and ends, each instrument costs half-a-crown.

If made by an instrument maker, with an easier and more accurate method of setting the style, the results would be appreciably better as it was noted that more errors arose from setting the style than from judging the length of the shadow to a tenth of a millimetre.

I am indebted to Lieutenant Commander A. P. H. Crosbie, R.N., and to Mr. A. Stimson, of the National Maritime Museum, for their interest and help with this paper.

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Helicopter Servicing and the Collision Regulations

C. L. Pielow

THE servicing of ships off Cape Town by helicopter with stores and spares is now established practice. In addition, test exercises are in progress with the Rotterdam Port Authorities to embark pilots by helicopter, and preliminary enquiries are under way for helicopter servicing in the Arabian Gulf.