

in value was only about $1\frac{3}{4}$ per-cent. Taking into account, however, the various contingencies that may arise, if the reversion be not converted at the first, the course recommended by the Court seems the most equitable that could be adopted.

I am, Sir,

Your obedient Servant,

J. R. HART, F I.A.

26 *St. Andrew Square,*
Edinburgh,
 8 *March* 1895.

MR. RYAN'S PAPER ON VALUATIONS FOR SOLVENCY.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—In the hope that they may be of interest, I send herewith tables for endowment assurances payable at age 60, corresponding to those for the whole term of life given in Mr. Ryan's paper recently read before the Institute.

In Mr. Ryan's tables, as well as in my own, I am somewhat surprised to find that the percentage of the reserve values to the premiums paid in certain cases first of all diminishes with the increased duration of the policy and then increases again. The point seems to me worthy of further investigation, and I find that a very simple formula will give the value of n when the percentages cease to diminish or commence to increase.

We have
$$\frac{nV_x}{nP_x} > = < \frac{n+1V_x}{(n+1)P_x} \dots \dots \dots (1)$$

according as $(n+1)_nV_x > = < n_{n+1}V_x$

according as
$$\frac{nV_x}{n+1V_x - nV_x} > = < n \dots \dots \dots (2)$$

It should be noted that the tables in Mr. Ryan's paper are concerned with the ratios of the successive terms in the two following series:

$$nP_x, (n+1)P_x, (n+2)P_x, \&c.$$

$$nV_x, n+1V_x, n+2V_x, \&c.$$

The first of these two series is in arithmetical progression, and its differences are a constant quantity ($=P_x$).

The differences in the second series are, however, the results of elements progressing at varying force in different directions, as is evident from the equation

$$({}_nV_x + P_x)(1+i) - q_{x+n}(1-{}_{n+1}V_x) = {}_{n+1}V_x,$$

whence

$$({}_{n+1}V_x - {}_nV_x) = \Delta_n V_x = {}_nV_x(i) + P_x(1+i) - q_{x+n}(1-{}_{n+1}V_x),$$

where, for all the cases in the tables under consideration,

$$\left. \begin{array}{l} {}_nV_x i \text{ increases} \\ P_x(1+i) \text{ is constant} \\ (1-{}_{n+1}V_x) \text{ decreases} \\ q_{x+n} \text{ increases} \end{array} \right\} \text{as } n \text{ increases.}$$

The net effect of these varying elements in the case of whole term and endowment assurances is to cause the series ${}_nV_x$, in most tables to increase as n increases, but not at an even rate; and the value of the above expression in (2) must depend on the rates of interest and of mortality used in the calculations.

Your obedient servant,

28 *King Street,*
Covent Garden, W.C.
30 *March 1895.*

ERNEST WOODS.

VALUATIONS FOR SOLVENCY.

Endowment Assurances payable at 60 or Death.

$$A_{x+n:\overline{60-(x+n)}} - kP'_{x:\overline{60-x}} \left(\frac{1}{2} + a_{x+n:\overline{60-(x+n)-\frac{1}{2}}} \right)$$

1.—*Basis H^M 3½. With-Profit Policies.*

Duration	AGE AT ENTRY 25—OFFICE PREMIUM £2. 15s. 6d.						AGE AT ENTRY 35—OFFICE PREMIUM £4. 3s. 0d.					
	V(1)	V(2)	V(3)	Percentage to Prems. paid of			V(1)	V(2)	V(3)	Percentage to Prems. paid of		
				V(1)	V(2)	V(3)				V(1)	V(2)	V(3)
3	- 1·9	- ·7	+ ·5	6	1·3	2·8	4·2	11	22	34
5	+ 1·7	+ 2·8	4·0	12	20	29	7·2	8·5	9·9	35	41	48
10	11·5	12·6	13·6	42	45	49	24·0	25·1	26·2	58	61	63
20	37·1	37·8	38·5	67	68	69	68·6	69·1	69·5	83	83	84
30	74·0	74·3	74·6	89	89	90

VALUATIONS FOR SOLVENCY—(continued).

Endowment Assurances payable at 60 or Death—(continued).

2.—Basis H^M 3½. Non-Profit Policies.

Duration	AGE AT ENTRY 25—OFFICE PREMIUM £2. 8s. 10d.						AGE AT ENTRY 35—OFFICE PREMIUM £3. 13s. 0d.					
	V(1)	V(2)	V(3)	Percentage to Prens. paid of			V(1)	V(2)	V(3)	Percentage to Prens. paid of		
				V(1)	V(2)	V(3)				V(1)	V(2)	V(3)
3	3·3	4·3	5·3	44	59	73	7·5	8·7	10·0	68	80	91
5	6·6	7·6	8·6	54	62	71	13·0	14·2	15·4	71	78	84
10	16·0	16·9	17·8	65	69	73	28·8	29·7	30·7	79	82	84
20	40·2	40·9	41·5	82	84	85	70·6	71·0	71·4	97	97	98
30	75·3	75·6	75·8	103	103	104

3.—Basis H^M 4. With-Profit Policies.

Duration	AGE AT ENTRY 25—OFFICE PREMIUM £2. 15s. 6d.						AGE AT ENTRY 35—OFFICE PREMIUM £4. 3s. 0d.					
	V(1)	V(2)	V(3)	Percentage to Prens. paid of			V(1)	V(2)	V(3)	Percentage to Prens. paid of		
				V(1)	V(2)	V(3)				V(1)	V(2)	V(3)
3	- 4·0	- 2·8	- 1·7	- .9	+ .5	+ 1·8	...	4	15
5	- .6	+ .5	+ 1·6	...	3	11	+ 4·8	6·1	7·4	23	29	36
10	+ 8·8	9·8	10·8	32	35	39	21·4	22·5	23·5	52	54	57
20	34·0	34·8	35·5	61	63	64	66·9	67·3	67·8	81	81	82
30	72·2	72·5	72·8	87	87	87

4.—Basis H^M 4. Non-Profit Policies.

Duration	AGE AT ENTRY 25—OFFICE PREMIUM £2. 8s. 10d.						AGE AT ENTRY 35—OFFICE PREMIUM £3. 13s. 0d.					
	V(1)	V(2)	V(3)	Percentage to Prens. paid of			V(1)	V(2)	V(3)	Percentage to Prens. paid of		
				V(1)	V(2)	V(3)				V(1)	V(2)	V(3)
3	.9	1·9	2·9	12	25	39	5·0	6·2	7·4	46	57	68
5	4·1	5·0	6·0	33	41	49	10·4	11·5	12·7	57	63	69
10	13·1	13·9	14·8	53	57	61	26·0	26·9	27·9	71	74	76
20	37·1	37·7	38·4	76	77	79	68·8	69·2	69·6	94	95	95
30	73·5	73·8	74·0	100	101	101