JOURNAL OF FINANCIAL AND QUANTITATIVE ANALYSIS Vol. 58, No. 8, Dec. 2023, pp. 3420–3449

doi:10.1017/S0022109023000029

The Design and Welfare Implications of Mandatory Pension Plans

Linda S. Larsen
Copenhagen Business School Department of Finance, Pension Research Centre (PeRCent), and Danish Finance Institute
lsl.fi@cbs.dk

Claus Munk
Copenhagen Business School Department of Finance, Pension Research Centre (PeRCent), and Danish Finance Institute
cm.fi@cbs.dk (corresponding author)

Abstract

In a rich, calibrated life-cycle model, we show that well-designed mandatory pension plans significantly improve the welfare of individuals procrastinating on savings, and even improve most rational individuals' welfare through a return tax advantage and fair annuitization. For a group of heterogeneous savers, in terms of preferences and sophistication, the best plan has contributions of 10% of income from age 30, a glidepath investment strategy, payouts following a variable lifelong annuity, and options to choose a different investment strategy and to modify the annuitization feature. This plan generates an average welfare gain of $175,000 per individual.

I. Introduction

Large population groups build up insufficient savings for retirement. Close to half of adult Americans do not have access to a workplace retirement saving plan, and more than half worry that they will not have enough money for retirement.[1] Gomes, Hoyem, Hu, and Ravina (2020) estimate that, in a sample of U.S. workers in a pension plan, 75% save too little. Retirement saving is further challenged by the predicted rise in longevity, possible cuts in Social Security, soaring health care costs, and the increase in self-employed and gig workers.

Evidently, mandatory pension plans can reduce the under-saving problem. Mandatory defined contribution (DC) saving plans exist for the majority of workers

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We appreciate comments and suggestions from an anonymous referee, Marcel Fischer, Thierry Foucault (the editor), Bjarne Astrup Jensen, Bruno Kamden, Matti Keloharju, David Laibson, Ellen McGrattan, Alexander Michaelides, Lasse Heje Pedersen, Jesper Rangvid, and Roine Vestman, and participants at presentations at the 2019 PeRCent Conference in Copenhagen, the City University of Hong Kong, Aalto University Helsinki, the Danish Society of Actuaries, Lund University, the 2020 FMA Virtual Conference, and the European University Institute and Banco de España's 2021 Conference on Social Security Reforms. We gratefully acknowledge support from the Danish Finance Institute (DFI) and from PeRCent, which receives base funding from the Danish pension funds and Copenhagen Business School.

[1] See 2018 Bureau of Labor Statistics at https://www.bls.gov/ncs/ebs/benefits/2018/benefits_retirement.htm and a 2019 Gallup survey at https://news.gallup.com/poll/249164/americans-feel-generally-positive-own-finances.aspx.

in many countries (e.g., the United Kingdom, the Netherlands, Australia, and Denmark), but with considerable variation in the plan design. In the United States, several states are introducing or expanding such programs. While some economists and politicians push for a universal mandatory retirement saving program, for example, the plan by Ghilarducci and James (2018), opponents point out, among other things, that mandatory programs harm individuals capable of accumulating sufficient savings on their own. These observations give rise to important questions that we address in this article: How do various pension plan designs affect the individual participants? What is the best mandatory plan design for a heterogeneous group of participants? In particular, which fraction of their income should workers contribute to the pension plan?

We embed a mandatory DC pension plan in a rich life-cycle model of individual consumption and investment decisions with Epstein–Zin preferences, uncertain labor income, mortality risk, Social Security benefits, and possible out-or-pocket medical costs. The model features both an illiquid pension saving account and a liquid private account, and both private and pension savings can be invested in stocks and bonds. Calibrating the model to U.S. data, we show that a mandatory plan greatly improves the welfare of individuals procrastinating on savings by generating a more balanced life-cycle consumption-saving profile closer to the rational ideal. Rational individuals can to a large extent undo the undesired consequences of a mandatory plan by adjusting private saving and investment decisions. In fact, a well-designed mandatory plan can even improve the welfare of rational individuals through a more lenient taxation on returns on pension savings than private savings and through access to annuitization at better terms than in the current annuity market.[2]

Our analysis points to designing the plan as follows: Each member contributes 10% of income from age 30 until retirement to an individual pension account. The default choice is that the balance of the account is invested in a glidepath strategy with 90% stocks until 25 years before retirement, sliding to 30% 10 years after retirement and staying there (similar to target date funds available in the industry). The pension savings are paid out as a life annuity with constant expected payouts until death. The default option is a 100% solidarity factor so that, upon death, the entire balance of the member's account is distributed to surviving members' accounts.

What are the welfare implications of this plan relative to the case with only private savings and the option to annuitize retirement savings at a realistic 20% cost? Consider a 25-year-old individual with initial wealth and life-cycle income profile similar to the median U.S. worker. The individual has a relative risk aversion (RRA) of 4, an elasticity of intertemporal substitution (EIS) of $1/4$, and modest bequest motive. The mandatory pension plan outlined above leads to a welfare gain of 30.3% or roughly \$262,000 in present value terms for an individual procrastinating on savings to an extent that she expects to build up only around one third of

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[2] Positive welfare effects could also come from the pension fund generating a better pre-tax risk–return trade-off by having access to better diversification, additional asset classes (commercial real estate, infrastructure, and foreign investments), and maybe superior asset selection skills, but the empirical evidence indicates that pension fund managers have a hard time beating low-cost ETFs.

the retirement savings that a fully rational individual does (about $165,000 instead of $463,000). An otherwise identical rational individual experiences a welfare gain of 2.0% or about $18,000 in present value terms with most of the gain stemming from the automatic and fairly priced annuitization. The welfare gain is generally increasing with the risk aversion of the individual.

Rational individuals with low risk aversion, low EIS, and a small bequest motive prefer accumulating little wealth and may thus experience a modest welfare loss when forced into the pension plan. To accommodate such individuals, we suggest allowing each member at enrollment to choose a different, riskier investment strategy and a lower solidarity factor so that a share of the deceased member's balance is distributed to designated beneficiaries. We also show that an option to pay out up to a certain fraction of pension savings some years before retirement is particularly valuable to these risk-tolerant rational individuals. Such options reduce the welfare loss of risk-tolerant rational participants considerably, and sometimes turn a loss into a small gain. At the same time, these options do not allow procrastinators to avoid building up considerable retirement savings.

When designing a mandatory pension plan for a heterogeneous group of individuals, the share of procrastinators is important. A large psychology literature has documented that many individuals procrastinate when faced with various decisions (Steel (2007), Klingsieck (2013)). Many individuals seem to postpone or refuse to set money aside for retirement (Bernheim, Skinner, and Weinberg (2001), Benartzi and Thaler (2007), Choi, Laibson, and Madrian (2011), and Heimer, Myrseth, and Schoenle (2019)), but a precise estimate of the fraction of people procrastinating on retirement saving is unavailable. Among employees in a major U.S. company, Choi, Laibson, Madrian, and Metrick (2002) found that 68% recognize that they save too little, 24% plan to increase savings within the next 2 months, but only 3% do so. For our model to match the wealth–income ratio seen in U.S. data, around two-thirds of the population must procrastinate on savings, which implies that our suggested plan generates an average welfare gain of around $175,000 per individual in a group with heterogeneous preferences and sophistication. While the estimate of the procrastinator frequency depends on the assumed preference parameters, the optimal pension plan design is relatively insensitive to the exact share of procrastinators.[3]

Our main analysis assumes Social Security retirement benefits at the current level. However, according to the Apr. 2020 report of the Social Security and Medicare Boards of Trustees, the Social Security system lacks funding and benefits have to be cut by 24% to make the system sustainable (source: https://www.ssa.gov/OACT/TR/2020/index.html, accessed on Aug. 7, 2020). We find that with a 24% reduction of Social Security, the optimal contribution rate increases from 10% to 11%, and the average gain goes up by about $18,000 per individual.

A significant driver of the welfare gains generated by the mandatory pension plan is the built-in annuitization of retirement savings. Annuitization implies that individuals share lifetime risks to avoid ending up consuming too little if living

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[3] Brown and Previtero (2020) also discuss the role of procrastination for various pension-related decisions of individuals enrolled in a retirement saving scheme, and they identify procrastinators as individuals postponing a certain health care decision to the last possible day.

long (or bequeathing too much if dying early) and is backed by economic research (Yaari (1965), Davidoff, Brown, and Diamond (2005), Koijen, Nijman, and Werker (2011) and Yogo (2016)). However, surveys estimate that only 4.3% of U.S. households (Lockwood (2012)) and 5.9% of English households (Inkmann, Lopes, and Michaelides (2011)) participate in the annuity market. This annuitization puzzle can be partly explained by the costs of private market annuities: Mitchell, Poterba, Warshawsky, and Brown (1999) report that for a 65-year-old annuitant, the money's worth ratio of U.S. life annuities is 75%–85% depending on the annuitant's gender and the discount rate used, and assuming population mortality rates. Households may perceive costs to be higher due to adverse selection issues, product opacity, and limited financial literacy. With few people annuitizing, the issuers cannot rely on diversification of lifetime risk across customers, which can explain high direct costs. A broad mandatory plan can implement annuitization at much lower costs.

Our article builds upon life-cycle models (e.g., Viceira (2001), Cocco, Gomes, and Maenhout (2005), and Gomes and Michaelides (2005)) that, among other things, conclude that most individuals should invest all their savings in stocks early in life and then gradually replace stocks by bonds.[4] Our model adds a mandatory illiquid pension scheme, which complicates the solution as the optimal private decisions depend on the accumulated pension savings in addition to the level of labor income and private savings.

Only few articles explicitly model an illiquid pension account. Campbell, Cocco, Gomes, and Maenhout (2001) assume a predetermined, constant contribution rate and fund asset allocation, and derive the individual's optimal consumption and private investments over the life cycle. They compare welfare and optimal decisions for two fund allocation strategies, namely i) 100% in the risk-free asset versus ii) 50% in stocks, 50% risk-free. We extend the analysis by deriving the optimal combination of contribution rate and fund allocation strategy, and we discuss various option features. As we do, Blake, Wright, and Zhang (2014) investigate the optimal contribution rate and stock–bond allocation of the pension plan. However, they disregard bequest and taxes as well as free savings outside the plan, which fixes consumption at a fraction of current income and thus prevents consumption smoothing.[5]

The most closely related article is Dahlquist, Setty, and Vestman (2018), who set up a model of the Swedish pension system and calibrate it to register data. They fix the contribution rate at the current Swedish level and search for the optimal default investment strategy of the pension fund, allowing each investor to switch

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[4] The canonical life-cycle model has been extended to labor supply flexibility (Bodie, Merton, and Samuelson (1992)), housing (Cocco (2005)), time-varying investment opportunities (Koijen, Nijman, and Werker (2010)), unemployment risk (Branger, Larsen, and Munk (2019)), income–stock market co-integration (Benzoni, Collin-Dufresne, and Goldstein (2007)), habit formation in preferences (Gomes and Michaelides (2003)), and stock market entry/participation costs (Fagereng, Gottlieb, and Guiso (2017)).

[5] Other articles focus on how rational investors for a given contribution rate can exploit the differential taxation of pension returns and private returns (cf., e.g., Dammon, Spatt, and Zhang (2004), Gomes, Michaelides, and Polkovnichenko (2009), and Fischer and Gallmeyer (2017)). These articles do not discuss the optimal contribution rate or, more generally, how the pension system should be designed.

(at a certain cost and only at age 25) to an alternative strategy.[6] The investor's private stock market participation is also an active decision with an entry cost. While our model disregards switching and stock market entry costs, we incorporate taxes, procrastination, and quantify the welfare implications of mandatory schemes, and we search for both the optimal contribution rate and fund investment policy.

The behavioral household finance literature has documented that the financial decisions of many individuals deviate systematically from what standard theoretical models prescribe (cf. the surveys by Campbell (2006), (2016), Guiso and Sodini (2013), and Beshears, Choi, Laibson, and Madrian (2018)). Behavioral biases translate into welfare losses of a moderate or large magnitude depending on the specific setting (Calvet, Campbell, and Sodini (2007), Bhamra and Uppal (2019)). We show that biases against retirement savings and annuitization induce substantial welfare losses, but also that a well-designed mandatory pension plan can reduce these welfare losses considerably by generating consumption at a higher level and with a balanced life-cycle profile. In fact, a mandatory pension plan with a significant, stable stock market investment can also improve the welfare of households who invest too little in risky assets, are under-diversified, or trade too frequently (Barber and Odean (2000), Calvet et al. (2007)), but we focus on procrastination in this article.

The rest of the article is organized as follows: Section II sets up the model and fixes the baseline parameter values. Section III presents the optimal decisions of a rational individual without a mandatory scheme, which serves as a benchmark in our welfare analysis. Section IV identifies pension plan designs that improve the welfare for a range of individuals with both different preference parameters and different degrees of financial sophistication. Section V discusses limitations and possible extensions of our setting. Finally, Section VI concludes.

II. The Modeling Framework

This section describes our model and introduces our baseline parameter values that are summarized in Table 1. Figure 1 illustrates the sequence of events in a typical year in our model. Each variable is described in detail below.

A. Mortality and Income

We use a model with annual time steps for the decision problem of an individual who has just turned $t_1 = 25$ years old, retires when she turns $t_R = 67$ years old (the Social Security full-benefit retirement age when born 1960 or later), and may live on until the end of her year $t_M = 100$. Being alive at age t, the probability of being alive at age $t+1$ is p_t with $p_{t_M} = 0$, and $P_{t,s} = p_t \times p_{t+1} \times \ldots \times p_{s-1}$ is the probability of being alive at age s conditional on being alive at age t, with $P_{t,t} = 1$. We use mortality rates from the 2017 life table for the U.S. population (Arias and Xu (2019)).

As illustrated in Figure 1, the individual receives income Y_t at the beginning of year t from labor, a state pension, or other sources. The income dynamics are

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[6] In an extension, Schlafmann, Setty, and Vestman (2020) consider that the implications of letting the contribution rate depend on the income level and the stock market participation of the saver.

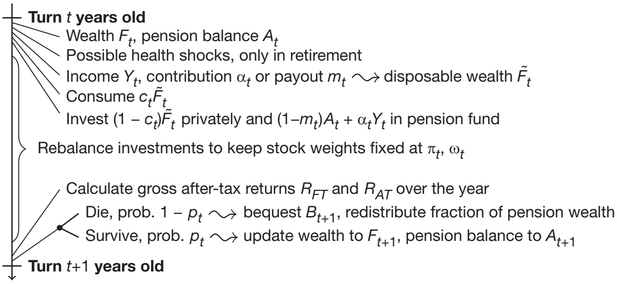
TABLE 1
Baseline Parameter Values

The parameter values listed in Table 1 are motivated in the main text.

Parameter	Description	Value
Financial assets		
r	Risk-free interest rate	0.01
μ_S	Expected excess stock return	0.04
σ_S	Stock volatility	0.157
Horizon, preferences, and initial wealth		
t_1	Initial age in years	25
t_R	Retirement age in years	67
t_M	Maximum age in years	100
γ	Relative risk aversion	4
ψ	Elasticity of intertemporal substitution	0.25
β	Subjective discount factor	0.96
ξ	Bequest strength parameter	2
F_{t_1}	Initial financial wealth (thousand USD)	5
A_{t_1}	Initial pension wealth (thousand USD)	0
Income		
Y_{t_1}	Initial annual income (thousand USD)	40
σ_Y	Income volatility	0.1
ρ_{YS}	Income–stock correlation	0
ζ	Social Security relative to final salary	0.45
Tax rates		
τ_Y	Income tax rate	0.30
τ_F	Tax rate on private returns	0.20
τ_A	Tax rate on pension returns	0.00

FIGURE 1
The Sequence of Events in a Typical Year in Our Model

The notation and events shown in Figure 1 are described in detail in the main text.



$$Y_{t+1} = Y_t R_{Yt}, \tag{1}$$

where

$$R_{Yt} = \begin{cases} \exp\left\{\mu_{Yt} - \frac{1}{2}\sigma_Y^2 + \sigma_Y \varepsilon_{Yt}\right\}, & \text{for } t = t_1, \ldots, t_R - 2, \\ \zeta, & \text{for } t = t_R - 1, \\ 1 - \phi_t h - \Phi_t H, & \text{for } t = t_R, \ldots, t_M - 1. \end{cases} \tag{2}$$

The shocks are $\varepsilon_{Yt} \sim N(0,1)$ and independent over time. We take an income volatility of $\sigma_Y = 10\%$ and a zero income–stock correlation, consistent with typical estimates in the literature (e.g., Davis and Willen (2000), Cocco et al. (2005) and Fagereng et al. (2017)). The initial annual pre-tax labor income is \$40,000, and, following Cocco et al. (2005) and others, the expected labor income growth $\mu_{Yt} = \ln(E_t[Y_{t+1}/Y_t])$ is described by a third-order polynomial; we determine the coefficients of the polynomial so that expected labor income peaks at age 55 at a value 50% above initial income and subsequently drops by 10% until retirement. The income numbers are broadly consistent with the median earnings of full-time workers in different age groups reported by the Bureau of Labor Statistics in 2020:Q1.[7]

The constant ζ is the ratio of the annual state pension to preretirement income. We let $\zeta = 0.45$, leading to an expected after-tax annual state pension of \$17,328. Social Security benefits depend nonproportionally on the average of the 35 career-highest inflation-adjusted annual earnings, but our model requires proportionality for tractability. Due to the hump-shaped income over life, the average salary over the best 35 years is typically not far from the final salary. In our baseline case, this salary level is likely to be \$50–60,000, and the annual Social Security benefits in year 2020-dollars are then between 45.5% and 43.1% thereof, justifying our value of ζ.[8]

Out-of-pocket medical costs can reduce disposable retirement income and significantly impact saving and risk taking (e.g., De Nardi, French, and Jones (2010)). To match cost estimates at different ages reported by De Nardi, French, Jones, and McCauley (2016) and Koijen, van Nieuwerburgh, and Yogo (2016), together with the observed large dispersion across individuals, we set up a two-shock structure. We let $\phi_t, \Phi_t \in \{0,1\}$ indicate whether a health shock with a small cost $h = 3\%$ (e.g., for prescription medicine) (resp. large cost $H = 85\%$ (e.g., for nursing home spending)) occurs at age t. The small-shock probability is the constant $q = \text{PROB}(\phi_t = 1) = 15\%$, whereas the large-shock probability $Q_t = \text{PROB}(\Phi_t = 1) = \min\left\{0.03 \times \frac{t - t_R}{t_M - t_R} + \left(\frac{(t - t_R - 15)^+}{t_M - t_R - 15}\right)^2, 0.5\right\}$ grows linearly until 15 years into retirement where it accelerates until reaching 50%. Based on simulations of our model, medical costs are expected to be 2.9%, 10.4%, 26.6%, and 85.3% of Social Security pay at ages 72, 79, 86, and 93, in line with De Nardi et al. (2016) and Koijen et al. (2016).[9]

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[7] The median annual earnings are \$31,460 (20–24 years), \$45,344 (25–34 years), \$56,160 (35–44 years), \$57,252 (45–54 years), \$56,264 (55–64 years), and \$48,776 (65+ years), respectively. Source: https://www.bls.gov/news.release/wkyeng.t03.htm, accessed on Apr. 28, 2020.

[8] Using 2020 as the year of eligibility, the monthly benefits are $0.9x + 0.32\max\{0, \min\{x - 960, 4825\}\} + 0.15\max\{0, x - 5785\}$, where x is the monthly salary. Source: https://www.ssa.gov/oact/cola/piaformula.html, accessed on Apr. 28, 2020.

[9] Koijen et al. ((2016), Table III) report health expenses in 2005-dollars at selected ages from 51 to 93. Our model has medical costs in retirement, so we take their mean expenses at ages 72, 79, 86, and 93 in excess of expenses at age 65, and adjust for inflation by multiplying by the ratio 257.971/190.7 of CPI in Jan. 2020 to Jan. 2005 (see https://www.usinflationcalculator.com, accessed on May 20, 2020). The mean excess expenses constitute 2.3% of expected after-tax Social Security benefits at age 72, 10.9% at age 79, 28.9% at age 86, and 181.9% at age 93. A similar exercise based on De Nardi et al. ((2016), Figure 3) leads to average expenses of 3.2%, 9.9%, 20.0%, and 41.7% at the same ages, that is, much lower late-life expenses. Our expected expenses at each age level fall in between the estimates from the two articles. For parsimony, we assume that any health-related reductions in disposable income are

B. Pension Savings and Private Wealth

The individual has a private, liquid wealth F_t and pension savings A_t at the beginning of year t. The initial pension savings are 0, $A_{t_1}=0$, and the initial liquid wealth is $F_{t_1}=5{,}000$ to broadly match the median 25-year-old U.S. worker (cf. the family net worth statistics of the 2016 Survey of Consumer Finances (SCF) and the online Net Worth Percentile Calculator of Personal Finance Data).[10]

The individual pays the fraction $\alpha_t \in [0,1)$ of pre-tax income into the pension fund and withdraws a fraction $m_t \in [0,1)$ of the balance of the fund. We restrict α_t and m_t to depend only on age and assume that

$$\alpha_{t_R} = \alpha_{t_R+1} = \cdots = \alpha_{t_M} = 0, \quad m_{t_1} = m_{t_1+1} = \cdots = m_{t_R-1} = 0,$$

so that contributions to the pension fund are made only before retirement and withdrawals only in retirement. The income after pension contribution and any withdrawals from the pension fund are subject to a proportional tax given by the rate $\tau_Y = 30\%$, which is in the range of tax rates across U.S. states.

The disposable wealth (a.k.a. Cash-on-hand) at time t is therefore

$$\tilde{F}_t = F_t + (1-\tau_Y)[(1-\alpha_t)Y_t + m_t A_t].$$

Of disposable wealth, she decides to consume a fraction $c_t \in (0,1)$ and to invest the remainder $(1-c_t)\tilde{F}_t$ in financial assets with a share of $\pi_t \in [0,1]$ in the stock market index and the rest in the risk-free asset. The private wealth dynamics are thus

$$F_{t+1} = (1-c_t)\tilde{F}_t R_{Ft}, \tag{3}$$

where R_{Ft} is the after-tax gross return over year t on the private investments.

We assume a constant annual log risk-free rate of r, that the log stock market return over any period dt is normally distributed with expectation $\left(r+\mu_S-\frac{1}{2}\sigma_S^2\right)dt$ and standard deviation $\sigma_S\sqrt{dt}$, and that returns are independent in the time dimension. The expected annual rate of return on the stock is thus $\exp\{r+\mu_S\}-1$, that is, μ_S captures the excess expected stock return. We assume the standard parameter values $r=1\%$, $\mu_S=4\%$, and $\sigma_S=15.7\%$.

By assuming that the private portfolio is continuously rebalanced through year t to maintain a constant stock weight of $\pi_t \in [0,1]$, the log return on the portfolio over the year is normally distributed with expectation $r+\pi_t\mu_S-\frac{1}{2}\pi_t^2\sigma_S^2$ and standard deviation $\pi_t\sigma_S$.[11] All returns on private investments (realized or not) are taxed at year-end at a proportional rate of $\tau_F = 20\%$, so that the after-tax gross return is

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permanent (transitory shocks have little impact anyway), and we do not model transitions between different health states as computation time would grow proportionally with the number of states.

[10] See SCF Table 2 in Bricker, Dettling, Henriques, Hsu, Jacobs, Moore, Pack, Sabelhaus, Thompson, and Windle (2017) and https://personalfinancedata.com/networth-percentile-calculator/, accessed on Apr. 28, 2020.

[11] The continuous-time stock price dynamics are $dS_t = S_t[(r+\mu_S)dt+\sigma_S dz_t]$, where z is a standard Brownian motion. With a fraction π_t of wealth in the stock and the rest in the risk-free asset, the wealth dynamics are $dW_t = W_t[(r+\pi_t\mu_S)dt+\pi_t\sigma_S dz_t]$, which with a constant $\pi_t=\pi$ implies $W_{t+\Delta} = W_t \exp\left\{\left(r+\pi\mu_S-\frac{1}{2}\pi^2\sigma_S^2\right)\Delta + \pi\sigma_S(z_{t+\Delta}-z_t)\right\}$, where $z_{t+\Delta}-z_t \sim N(0,\Delta)$. Hence, the pre-tax

$$R_{Ft} = 1 + (1-\tau_F)\left[\exp\left\{r + \pi_t\mu_S - \frac{1}{2}\pi_t^2\sigma_S^2 + \pi_t\sigma_S\varepsilon_{St}\right\} - 1\right],$$

where $\varepsilon_{St} \sim N(0,1)$ is uncorrelated with the income shock ε_{Yt} as mentioned above.

We explore different pension fund designs. One design aspect is what happens to the balance of the pension account upon death. We let $I \in [0,1]$ denote the fraction of the balance, which is distributed across accounts of surviving members of pension fund, with the remaining balance being paid out to the heirs of the deceased member. We refer to I as the "solidarity factor." With $I=0$, the fund member has a personal plan, isolated from the accounts of the other members. With $I=1$, the plan is fully solidary. For a large group of members of age t with the same balance, the balance of each member surviving until age $t+1$ will be added a fraction $d_t = I(1-p_t)/p_t$ of the balance at the end of year t due to transfers from deceased members. The pension fund invests $(1-m_t)A_t + \alpha_t Y_t$, that is, the time t balance plus net contribution, over year t with a fixed share (continuously rebalanced) of w_t in the stock market index and the rest in the risk-free asset. The after-tax gross returns on pension investments in year t are thus

$$R_{At} = 1 + (1-\tau_A)\left[\exp\left\{r + w_t\mu_S - \frac{1}{2}w_t^2\sigma_S^2 + w_t\sigma_S\varepsilon_{St}\right\} - 1\right], \tag{4}$$

where τ_A is the tax rate on pension returns with $\tau_A = 0$ as the baseline value. If the individual survives year t, next year's opening balance of the pension account is

$$A_{t+1} = [(1-m_t)A_t + \alpha_t Y_t]R_{At}(1+d_t). \tag{5}$$

We fix $m_{t_M} = 1$ so when the individual turns t_M years old, the fund pays out the remaining balance A_{t_M}. We define the payout rates recursively by

$$m_t = \left(1 + \{m_{t+1}\,\mathrm{E}_t[R_{At}](1+d_t)\}^{-1}\right)^{-1}, \quad t = t_R, t_R+1, \cdots, t_M-1, \tag{6}$$

which implies that the monetary payouts in year t and $t+1$ are linked by

$$m_{t+1}A_{t+1} = m_t A_t \frac{R_{At}}{E_t[R_{At}]}, \quad t = t_R, t_R+1 \ldots, t_M-1, \tag{7}$$

so that monetary payouts are constant in expectation through retirement.[12] The expected annual payout increases in I through d_t.

With the mortality rates and parameter values assumed above, the expected annual payout from a \$100,000 investment at retirement in a fair-priced variable annuity with 50% stocks is \$4,622 for $I=0$, \$6,130 for $I=1/2$, and \$7,297 for $I=1$.

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rate of return over a 1-year period can be written as $\exp\left\{r + \pi\mu_S - \frac{1}{2}\pi^2\sigma_S^2 + \pi\sigma_S\varepsilon_S\right\} - 1$, where $\varepsilon_S \sim N(0,1)$.

[12] The plan can also be designed to have expected payouts being either increasing or decreasing through retirement, but the welfare gains are only marginally different to the case with constant expected payouts. See the Supplementary Material for more information.

FIGURE 2

Annuity Values and Survival Probabilities Through Retirement

In Figure 2, the individual's age is displayed along the horizontal axis. The gray lines show the value of an annuity in thousands of USD for different values of the solidarity factor *I*. The annuity starts at age 67 with a value of \$100,000 and is a variable annuity with 50% stocks. The orange lines are to be read off the right-hand axis and show the probability of being alive at the end of a given year conditional on being alive when turning 67 (solid line) and the probability of dying in year *t* conditional on being alive at the beginning of year *t* (dashed line). These lines are based on the 2017 life table for the U.S. population (Arias and Xu (2019)) with an imposed maximum age of 100. Please see online version for correct colors.

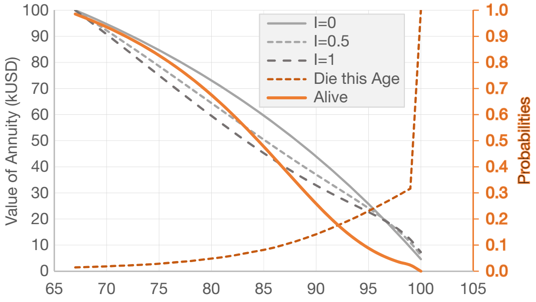

Figure 2 shows the mortality frequency at each age (dashed orange; maximum age of $t_M = 100$) and the probability of being alive at the end of each year conditional on being alive when turning 67 (solid orange).[13] The gray lines show the value of the annuity portfolio for the three values of I. For $I = 1$, annual payouts are larger, so the annuity value first declines steeply, but when the mortality rate picks up, more wealth is transferred from deceased portfolio holders causing the survivor's annuity value to decline less steeply. The downside of a higher I is the lower bequest upon death. With $I = 1$, the payout stream matches that of lifelong variable annuities (e.g., Charupat and Milevsky (2002), Horneff, Maurer, Mitchell, and Stamos (2010)).

C. Decisions and Preferences

The individual chooses c_t and π_t for $t = t_1, t_1 + 1, \ldots, t_M$ to maximize lifetime utility. We let J_t denote the indirect utility at time t, conditionally on being alive, and this includes the utility of consumption in year t and subsequent years, as well as any bequest utility. At the end of any year t, after receiving the returns on investments over the year, the individual dies with a probability of $1 - p_t$. The private wealth is then F_{t+1}, which is passed on to the heirs. The pension wealth is $[(1 - m_t)A_t + \alpha_t Y_t]R_{At}$ of which a fraction $1 - I$ is passed on to the heirs (subtracting income tax as contributions were made out of pre-tax labor income) and the rest is distributed to surviving fund members. The total bequest is thus

$$B_{t+1} = F_{t+1} + (1 - \tau_Y)(1 - I)[(1 - m_t)A_t + \alpha_t Y_t]R_{At}.$$

Should the individual reach the maximum age, the pension account has already been paid out, so the bequest is then $B_{t_M+1} = F_{t_M+1}$.

We assume Epstein–Zin utility with J_t satisfying the recursive relation

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[13] Readers of the printed journal are referred to the online version for colors.

$$J_t = \max_{c_t, \pi_t} \left\{ \left(c_t \tilde{F}_t\right)^{1-\frac{1}{\psi}} + \beta \mathrm{CE}_t^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}}, \tag{8}$$

where

$$\mathrm{CE}_t = \left(p_t \mathrm{E}_t\left[J_{t+1}^{1-\gamma}\right] + (1-p_t)\mathrm{E}_t\left[\overline{U}_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}} \tag{9}$$

is the certainty equivalent of next period's utility, which is J_{t+1} if surviving and the bequest utility $\overline{U}_{t+1}$ if not. We assume that $\overline{U}_t = \xi^{\frac{1}{\psi-1}} B_t$, where $\xi \geq 0$ measures the strength of the bequest motive: If death at the end of the year is certain, the individually chooses consumption in that year so that the bequest is roughly ξ times consumption (see the Supplementary Material). In addition to ξ, preferences are characterized by the relative risk aversion (RRA) $\gamma > 0$, the elasticity of intertemporal substitution (EIS) $\psi > 0$, and the subjective discount factor $\beta > 0$.[14]

The baseline values are RRA 4, EIS 1/4, and bequest weight 2, but we also consider other values. The subjective discount factor is $\beta = 0.96$. These parameter values are similar to those used in related articles and consistent with empirical estimates. Based on a Swedish administrative panel data set, Calvet, Campbell, Gomes, and Sodini (2021) estimate the cross-sectional distribution of preferences among middle-aged, stock-owning households. The median estimates include an RRA of 5.30, an EIS of 0.42, and a discount factor of 0.9600 (they report $\ln\beta = -0.0408$). Using Italian data on household income and wealth, Chiappori and Paiella (2011) report a mean RRA of 2.5–4.2 depending on which households are included. Based on decisions of U.S. investors on a person-to-person lending platform, Paravisini, Rappoport, and Ravina (2017) find a mean RRA estimate of 2.81. Both articles find substantial cross-sectional variation in the RRA.[15] In Merton's no-income setting, the optimal stock weight is 81.1%, 40.6%, and 27.0% with RRA equal to 2, 4, and 6, respectively, showing that the 2–6 RRA range we consider covers a wide range of risk attitudes. In our framework, without imposing a mandatory pension plan, an individual with RRA 2 (and EIS 1/4 and bequest weight 2) optimally invests 100% in stocks all life, which supports the view that RRA 2 represents a strong risk tolerance in the context of our model. While varying substantially across studies, most EIS estimates based on microdata are positive and below 1. For example, Vissing-Jørgensen (2002) reports 0.3–0.4 and Bonaparte and Fabozzi (2017) report 0.33–0.56, both using U.S. stockholder data. In settings related to ours, Dahlquist et al. (2018) assume an EIS of 0.5, and Gomes and Michaelides (2005) consider 0.2 and 0.5. The combinations of RRA and EIS we consider involve cases with a preference for early resolution of uncertainty

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[14] We assume that $\gamma \neq 1$ and $\psi \neq 1$, but cases with $\gamma = 1$ or $\psi = 1$ or both can be studied separately with appropriate adjustments of (8) and (9).

[15] To obtain a match between life-cycle model predictions and observed savings and investment decisions of Scandinavian households, Fagereng et al. (2017) and Dahlquist et al. (2018) estimate γ in the range 11–15 (to produce low stock weights) and β in the range 0.75–0.93 (to produce low savings). However, these estimates may reflect behavioral biases rather than genuine preferences, and such values fall outside the intervals generally considered reasonable based on introspection, experiments, and other empirical studies.

($\psi > 1/\gamma$) or late resolution of uncertainty ($\psi < 1/\gamma$) or an indifference toward the timing ($\psi = 1/\gamma$; time-additive power utility).

As argued in the Introduction, some individuals procrastinate on savings. For convenience, we model procrastination as follows: The utility the individual derives from any consumption plan is associated with the baseline discount factor $\beta = 0.96$. However, due to lack of self-control, the individual applies a lower value, $\beta = 0.85$, when making decisions, so she consumes too much early in life and builds up too little wealth as illustrated below.[16] In Section III.B, we argue that with $\beta = 0.85$ approximately two-thirds of the population procrastinate on retirement saving.

Given our setup, the indirect utility is a function $J_t = J_t(F_t, Y_t, A_t)$ of private wealth, current income, and pension wealth. We show in the Supplementary Material that the dimension of the state space can be reduced by 1 by exploiting a homogeneity property; our assumptions on proportional taxation and that the state pension is proportional to preretirement income are needed here. More precisely,

$$J_t = (F_t + [1 - \tau_Y]A_t)G_t(y_t, a_t), \tag{10}$$

where

$$y_t = \frac{[1 - \tau_Y]Y_t}{F_t + [1 - \tau_Y]A_t}, \qquad a_t = \frac{[1 - \tau_Y]A_t}{F_t + [1 - \tau_Y]A_t}, \tag{11}$$

and we solve for G and optimal decisions by backward dynamic programming on a grid of points $\left(y_i, a_j\right)$. We simulate 10,000 paths forward and report averages at each age to indicate an expected life-cycle pattern. The Supplementary Material has details on the numerical solution approach.

D. Welfare Metric

In our setting, the pension fund design is characterized by a sequence of contribution rates $\alpha_{t_1}, \alpha_{t_1+1}, \ldots, \alpha_{t_R-1}$ and stock weights $w_{t_1}, w_{t_1+1}, \ldots, w_{t_M-1}$, as well as the solidarity factor I. For simplicity, we represent such a design by (α, w, I).

A key element of our analysis is to compare the utility or welfare that an individual can generate with a certain mandatory pension plan to the utility without a pension plan, but under the same assumptions about income, initial wealth, and so forth. Let $J_{t_1}(F, Y, A; \alpha, w, I)$ denote the indirect utility with a pension plan (α, w, I), and let $J_{t_1}(F, Y, A; 0)$ denote the indirect utility without a plan. We can quantify how much better off the individual is with (α, w, I) than without a plan by the fraction λ of additional lifetime labor income and initial wealth that the individual without a plan would need to receive to obtain the same lifetime utility as with the plan (α, w, I). With the additional income and wealth, the indirect utility without a plan is

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[16] To be clear, we assume that $c_t(a_t, y_t)$ and $\pi_t(a_t, y_t)$ are derived by the dynamic programming technique assuming $\beta = 0.85$ and then evaluated with $\beta = 0.96$. The evaluation of a given strategy (c_t, π_t) follows the same backward iterative approach as when deriving the optimal strategy, except that no maximization is performed. Procrastination can also be modeled using (quasi-)hyperbolic discounting (Laibson (1997)), but this is computationally more demanding.

$$J_{t_1}([1+\lambda]F,[1+\lambda]Y,[1+\lambda]A;0) = (1+\lambda)J_{t_1}(F,Y,A;0). \tag{12}$$

Equating this with $J_{t_1}(F,Y,A;\alpha,w,I)$, we find

$$\lambda = \frac{J_{t_1}(F,Y,A;\alpha,w,I)}{J_{t_1}(F,Y,A;0)} - 1. \tag{13}$$

Of course, $\lambda < 0$ means that the pension plan leads to a welfare loss. To give a better sense of the size of the gain/loss, we transform it into a dollar amount by multiplying λ by the sum of the initial financial wealth and the present value of after-tax income (from labor and Social Security less medical expenses). Since the income is not spanned by traded assets, there is no unique way to fix the discount rate for future expected income. To be specific, we compare with the stock market where the risk premium is assumed to be 4% for a standard deviation of 15.7%. Scaling by the 10% standard deviation of income, we apply a risk premium of 2.55%, that is, a discount rate of 3.55%, giving a present value of income equal to \$859,722 on top of the financial wealth of \$5,000. Hence, a welfare gain λ of 1% corresponds to \$8,647.[17]

E. Pension Plan Design Features

To streamline the analysis, we focus on simple plan designs; simplicity is also important for transparency and thus public support of a broad or even universal mandatory retirement saving program. We assume a constant contribution rate α either applying immediately from age 25 or from a later age and always until retirement, and we consider only integer contribution percentages. We focus on the following fund investment policies each characterized by how the stock weight varies with age:

IP1: 50% stocks all life;
IP2: stock weight at age t is $(120-t)\%$;
IP3: 90% stocks until age $t_R - 25$, slopes to 30% at age $t_R + 10$ and stays there;
IP4: 100% stocks until age $t_R - 20$, slopes to 50% at age $t_R + 20$ and stays there;
IP5: 100% stocks all life.

IP1 and IP5 feature a constant stock weight, whereas the other policies have a glidepath element. IP2 is the "120-minus-age" strategy seemingly popular among some financial advisors. IP3 emulates Vanguard's target date funds.[18] The more aggressive IP4 resembles the life-cycle pattern of the average optimal stock

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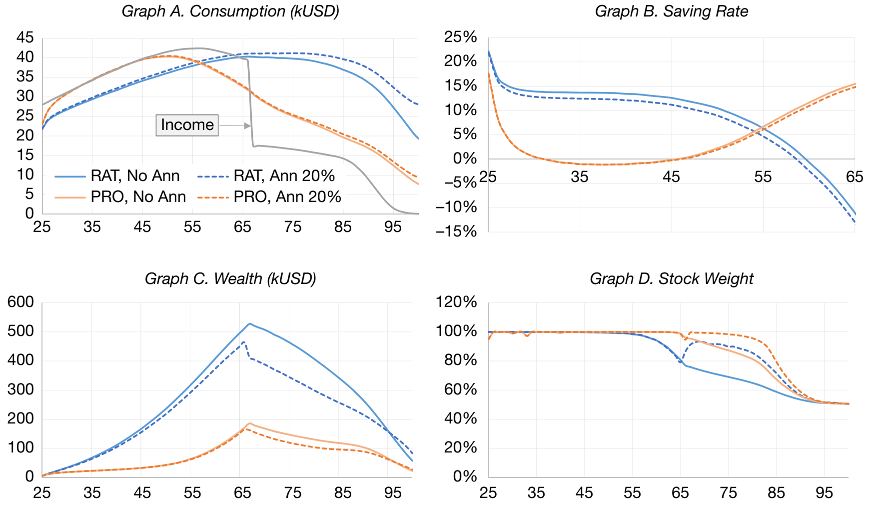
[17] Using the risk-free rate as a discount rate, as Guiso and Sodini (2013) and others do, a 1% welfare gain corresponds to a present value gain of \$15,184. Using the unscaled equity premium and thus a discount rate of 5%, a 1% welfare gain corresponds to \$6,677.

[18] Source: https://institutional.vanguard.com/web/c1/investments/product-details/fund/1691, accessed on Mar. 12, 2021. Vanguard's description of the asset allocation glidepath involves U.S. stocks, international stocks, U.S. nominal bonds, international bonds, and short-term TIPS (Treasury Inflation Protected Securities). Based on the 2060 fund, we add U.S. and international stocks to get the stock weight used in our model. Vanguard's downward-sloping stock weight is slightly nonlinear, but we approximate it with a linear relation.

FIGURE 3

Life Without a Pension Plan

In Figure 3, the horizontal axis shows the individual's age. The blue lines refer to a rational individual, and the orange lines refer to a procrastinator. The solid lines are for the case without annuitization, and the dashed lines are for the case with access to variable life annuities (50% stocks) with 20% cost. All the lines show means across 10,000 simulated paths. Graph A shows the annual consumption and, in gray, labor income and state pension after taxes and medical costs. Graph B displays the saving rate. Graph C shows the financial wealth, including the value of the annuity portfolio after cost. Graph D illustrates the fraction of (nonannuitized) wealth invested in the stock market. The baseline parameter values listed in Table 1 are used. Please see online version for correct colors.



Graph A. Consumption (kUSD)

Graph B. Saving Rate

Graph C. Wealth (kUSD)

Graph D. Stock Weight

weight for rational individuals without a pension plan (cf. Graph D of Figure 3). We consider the values of the solidarity factor I in multiples of 0.1 within the interval $[0,1]$.

We assume that a broad plan covering heterogeneous individuals stipulates a default choice of i) the contribution rate and starting age, ii) the investment policy IPn for some $n \in \{1, \ldots, 5\}$, and iii) the solidarity factor I. We discuss the implications of adding various options to deviate from the default choice. We stick to options that are computationally tractable so that the homogeneity property (equation (10)) is preserved. This includes options to change the investment policy and the solidarity factor from the initial date, without the possibility to revise the choice later. We also consider options that can be exercised at a later age, namely the option to end contributions or to pay out a fraction of the pension savings prematurely.

III. Benchmark: No Mandatory Pension Plan

This section studies the case without a mandatory pension plan which serves as a benchmark for the subsequent analysis of the design and implications of such a plan. Compared to mainstream life-cycle models (Viceira (2001), Cocco et al. (2005)), our model includes taxes on income and returns, Social Security retirement benefits, out-of-pocket medical costs, and the possibility to annuitize retirement savings. We consider both rational individuals and individuals procrastinating on saving.

We derive decisions and utility both without and with access to annuities. In the latter case, the individual decides at retirement upon the fraction of wealth to spend on a variable life annuity with an equal mix of stocks and bonds. The Supplementary Material extends our model with the option to annuitize. An annuity is fairly priced if payments are based on the population mortality rate and the annuity issuer makes a zero expected profit. We consider both fair annuities and (as motivated in the Introduction) annuities with a 20% cost meaning that payouts are 20% lower.

A. Life-Cycle Patterns and Annuitization

Figure 3 illustrates the life-cycle patterns generated by the model for the baseline preference parameters, and Table 2 lists selected results also for other combinations of preference parameters. For now, we assume the baseline parameters.

TABLE 2

Annuitization

In Table 2, the individual is given the option to choose a fraction of wealth to annuitize at retirement through a variable life annuity with 50% in stocks and 50% in the risk-free asset. The annuity is either actuarially fair (0% cost) or has a 20% cost. Panel A reports the average fraction of wealth annuitized across 10,000 simulated paths. Panel B reports the percentage welfare gain from the access to annuitization. Panel C reports the average value of the ratio of wealth to after-tax income at age 60 in the case with access to annuities with 20% costs. The results are shown for different combinations of the investor rationality, RRA γ, EIS ψ, and the bequest weight ξ. The numbers with a gray background are for the baseline preference parameter values.

		$\gamma = 2$			$\gamma = 4$			$\gamma = 6$		
		$\psi = \frac{1}{6}$	$\psi = \frac{1}{4}$	$\psi = \frac{1}{2}$	$\psi = \frac{1}{6}$	$\psi = \frac{1}{4}$	$\psi = \frac{1}{2}$	$\psi = \frac{1}{6}$	$\psi = \frac{1}{4}$	$\psi = \frac{1}{2}$
Panel A. Optimal Percentage of Wealth Annuitized at Retirement										
0% cost										
Rational	$\xi = 0.5$	47.8	56.5	82.8	85.6	85.8	90.8	87.1	89.7	91.8
Rational	$\xi = 2$	29.8	37.3	66.3	72.5	76.8	86.8	78.8	82.6	89.8
Rational	$\xi = 5$	12.6	17.4	40.9	54.3	59.0	77.2	64.1	68.3	82.4
Procrast	$\xi = 2$	6.7	3.1	1.4	54.0	50.0	46.8	65.8	64.1	61.9
20% cost										
Rational	$\xi = 0.5$	26.2	35.0	69.8	83.7	87.3	91.2	88.5	89.7	92.2
Rational	$\xi = 2$	3.2	8.3	42.4	68.2	75.0	87.8	80.7	84.1	89.5
Rational	$\xi = 5$	0.0	0.0	6.2	45.3	51.5	73.1	62.5	67.3	80.9
Procrast	$\xi = 2$	0.0	0.0	0.0	48.1	44.6	40.0	65.4	62.8	57.1
Panel B. Percentage Welfare Gain from Annuitization										
0% cost										
Rational	$\xi = 0.5$	0.47	0.68	1.67	4.36	4.49	4.74	6.58	6.44	6.02
Rational	$\xi = 2$	0.17	0.28	1.00	3.28	3.61	4.50	5.63	5.89	5.93
Rational	$\xi = 5$	0.04	0.07	0.37	1.26	1.80	3.50	1.87	3.11	5.36
Procrast	$\xi = 2$	0.42	0.25	0.15	11.38	13.03	10.68	15.29	19.81	18.89
20% cost										
Rational	$\xi = 0.5$	0.11	0.20	0.69	2.73	2.83	2.97	4.32	4.19	3.85
Rational	$\xi = 2$	0.00	0.02	0.26	1.86	2.14	2.78	3.61	3.75	3.78
Rational	$\xi = 5$	0.00	0.00	0.01	0.58	0.87	1.97	1.13	1.88	3.32
Procrast	$\xi = 2$	0.00	0.00	0.00	6.82	7.97	6.72	10.29	13.31	13.24
Panel C. Wealth–Income Ratio at Age 60 (20% Cost)										
Rational	$\xi = 0.5$	5.1	5.2	5.7	9.8	9.6	9.3	12.4	12.0	11.1
Rational	$\xi = 2$	6.3	6.4	6.3	10.5	10.2	9.5	12.9	12.4	11.3
Rational	$\xi = 5$	8.4	8.3	7.7	12.3	11.8	10.2	14.3	13.6	11.8
Procrast	$\xi = 2$	2.4	1.7	0.7	3.9	2.5	0.7	5.5	3.4	0.9

The saving rate shown is defined as after-tax income less consumption, $(1-\tau_Y)Y_t - c_t\tilde{F}_t$, relative to after-tax income, $(1-\tau_Y)Y_t$.

First, focus on the rational individuals without annuity access (solid blue lines). Graph A of Figure 3 shows that expected consumption has the hump shape typically produced by life-cycle models with consumption peaking near retirement. Graph B shows that the individual saves a relatively large fraction of income early in life to build a wealth buffer against bad times, and the expected saving rate exceeds 10% up to age 50 after which it gradually declines and turns negative around age 60, meaning that consumption exceeds after-tax income. To smooth consumption over time and states, the optimal saving rate depends on the individual's income and accumulated wealth and can deviate considerably from the expected value shown. Graph C documents that wealth is steadily built up until retirement, after which wealth decumulates (at first rather slowly due to the risk of significant medical expenses). Without annuities, expected wealth peaks at around $527,000 at retirement, and at age 60, the accumulated wealth is 11.2 times the annual after-tax income. With access to annuities at 20% costs, expected wealth is $463,000 at retirement and the age 60 expected wealth–income ratio is 10.2.[19] Until around age 55, all financial wealth is held in stocks, but then bonds enter the portfolio and the stock weight declines steadily toward 50% late in life (cf. Graph D). Consistent with the existing life-cycle literature, this glidepath pattern emerges because of the gradual decline in the bond-like human capital relative to financial wealth.

Panels A and B of Table 2 report that with access to an annuity with 0% [20%] cost, the rational individual chooses to annuitize 76.8% [75.0%] of wealth at retirement, which increases the individual's welfare by 3.61% [2.14%] or about $31,000 [$18,500] in present value terms. Annuitization allows significantly larger consumption late in life and also slightly larger consumption in the working years as less wealth has to be accumulated for retirement. The annuity has 50% invested in stocks, but the individual prefers a higher stock weight early in retirement and therefore tilts the private portfolio more toward stocks.

Next, turn to the procrastinator. Figure 3 shows that expected consumption is again hump-shaped but peaks already around age 50. Relative to the rational individual, the procrastinator consumes more and saves less early in life and ends up with very low consumption near and in retirement. Without annuitization, expected wealth peaks at $185,000 (35% of the rational individual's maximum wealth) and the wealth–income ratio at age 60 is only 2.6. With access to annuities at 20% cost, the procrastinator's expected wealth peaks at $165,000 and the age 60 wealth–income ratio is 2.5. The procrastinator's wealth is tilted toward the bond-like human capital throughout life, which generates a higher portfolio share of stocks. Furthermore, the procrastinator appreciates annuity access and, with 20% costs, chooses to annuitize 44.6% of wealth with a welfare gain of 7.97% (cf. Table 2). In dollar terms the annuity bought generates only a modest income, and late-life consumption remains low. With optimal annuitization at 20% costs, the procrastinator's welfare is 25.5% lower than for the corresponding rational individual or roughly $220,000 in present value terms. A mandatory pension plan can

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[19] With a zero-cost annuity, the numbers are $442,000 and 9.5, respectively.

reduce this welfare loss substantially by bringing the life-cycle patterns closer to that of the rational ideal.

B. Implications for Pension Plan Design

The saving rate of 10%–15% through most of working life shown in Graph B of Figure 3 does not translate directly into preferred contribution rates in a mandatory pension plan. The pension savings are illiquid, and the buffer saving motive can only be met by saving in a private account. This suggests that the contribution rate should be somewhat lower than the saving rate in the no-pension case. On the other hand, the tax benefits of pension returns can motivate larger savings. It is also important to note that the saving rate varies considerably with preference parameters. As an indication, Panel C of Table 2 reports the expected wealth–income ratio at age 60. With an RRA of $\gamma = 2$, this ratio is around 6 compared with 10 for $\gamma = 4$ and 12 for $\gamma = 6$. The saving rate is increasing in the bequest motive and varies somewhat with the EIS parameter ψ. A high contribution rate could significantly reduce the welfare of rational, risk-tolerant individuals.

Graph D of Figure 3 suggests that a mandatory pension fund should employ a fairly aggressive (i.e., stock-heavy) glidepath strategy.[20] The optimal stock weight is generally decreasing in the RRA, but relatively insensitive to other preference parameters. With a moderate contribution rate, high-RRA individuals will likely maintain significant private savings which could be invested primarily in bonds to counter-balance a high stock weight of the pension fund. Low-RRA individuals prefer a high stock weight throughout life. At the same time, they save less, so they would dislike maintaining a large private stock position to balance off the lower stock weight applied by the fund. Therefore, while a fairly aggressive glidepath strategy seems like a good default choice for a mandatory pension fund, an option to choose an even more aggressive strategy is probably highly valued by low-RRA individuals.

Table 2 reports that annuitization is particularly valuable to individuals with a medium-to-high risk aversion and a modest bequest preference. In spite of the presence of annuity-style Social Security benefits, possibly large medical expenses, and a 20% cost rate, these individuals benefit substantially from annuitizing a large share of savings. A reduction in the costs from 20% to 0% improves welfare by 1–2 percentage points for many such rational individuals and by more for procrastinators. In contrast, low-RRA individuals worry less about outliving their wealth and thus annuitize a much lower fraction of their wealth, which improves welfare only marginally. These results suggest that a broad mandatory pension plan should feature automatic annuitization of a significant default share of retirement savings, but also allow individuals to select a lower degree of annuitization.

The design of a broad mandatory pension plan should acknowledge the welfare of individual members who are heterogeneous in terms of preferences, rationality, wealth, and income. Clearly, the distribution of individuals along such dimensions is important. For our analysis, the share of procrastinators in the economy is a central factor, but difficult to assess. An individual might save very

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[20] Such a strategy would also benefit the many individuals holding no or only few stocks.

little as a deliberate, active choice based on the discount rate β, the RRA γ, and the bequest weight ξ being low. The observed saving behavior is only informative about the procrastinator share if we fix other preference parameters. However, within our modeling framework, the very low savings accumulated by many U.S. households can only be an optimal, rational choice for preference parameter values that are generally considered extreme in the economics literature.

In the age group 55–64 in the 2019 SCF, the median net worth is \$212,500 and the median before-tax income is \$63,600 (source: https://www.federalreserve.gov/econres/scfindex.htm, accessed on Mar. 11, 2021). Deducting 30% income tax, this leads to a wealth–income ratio of 4.8. Panel C of Table 2 reports that, for rational individuals, the optimal wealth–income ratio at age 60 in our model is larger for all preference parameter combinations considered; the ratio seems fairly insensitive to initial income and wealth. Our model can generate an average wealth–income ratio similar to that derived from the SCF if a significant share of individuals are procrastinators. As a back-of-the-envelope calculation, suppose that the share s of individuals are rational and the rest procrastinators. The rational individuals are split evenly among RRA 2, 4, and 6, and similarly for the procrastinators. All individuals have the baseline values for other parameters. Then s must be around 0.32 to generate an average wealth–income ratio at age 60 equal to the SCF-motivated value of 4.8. Roughly two thirds of the population would then be procrastinators. The mirror image of the observed low saving rate is a front-loaded consumption profile. In fact, the data show a hump-shaped consumption pattern peaking already at age 45 (Thurow (1969), Gourinchas and Parker (2002)). Comparing with Graph A of Figure 3, this is also supportive of a large share of procrastinators in the economy.

IV. Designing a Mandatory Pension Plan

This section explores how mandatory pension plans affect procrastinators and rational individuals. For each pension design, we calculate an individual's utility gain using equation (13) relative to the case without a mandatory pension plan but with optimal choice of annuitization at a 20% cost. Given that few individuals buy annuities, the calculated welfare gains are probably understating the true gains. For the no-pension benchmark, we assume that all returns are taxed at $\tau_F = 20\%$. The gains thus reflect how a tax-reduced mandatory pension plan affects individual welfare compared to a case with no subsidies to retirement savings. We discuss the role of the tax differential later. A mandatory plan instituted by the government to help irrational individuals should preferably not harm a large number of rational individuals relative to a situation with no government interventions at all. The mandatory plan covers individuals with different preferences and sophistication and thus affects their welfare differently. To streamline the presentation and keep the numerical calculations tractable, we first focus on differences in the RRA across individuals since the optimal consumption, investment, and annuitization decisions vary substantially with RRA and less so with the EIS and the bequest weight, as Table 2 also reports. Subsequently, we discuss how the results vary with the other preference parameters.

A. Identifying a Good Plan

Table 3 illustrates the percentage welfare gains of plans with mandatory contributions from age 25, 30, or 35 until retirement for a selection of contribution rates. We label the plan with 7% contributions from age 25 as “7%/25y,” and so forth. We consider rational individuals in Panel A and procrastinators in Panel B. In each group, we consider an RRA of 2, 4, and 6, and use the baseline values of other preference parameters. To identify a good default design, we determine for each individual the best choice of investment policy (out of IP1, …, IP5) and the best choice of the solidarity factor I (out of 0.0, 0.1, …, 1.0). For example, with the

TABLE 3

Welfare Implications of Selected Pension Plans

In Table 3, the plans have contribution rates and periods shown in the column headings, and the default is investment strategy IP3 and full solidarity ($I = 1$). Panel A considers rational individuals and Panel B considers procrastinators, in each case for RRA 2, 4, and 6. For each individual, we report the welfare gain from the default plan as well as the optimal IP and I and the corresponding gain. Panel C reports the average gains across the six participant types, both an equal-weighted average and a weighted average assuming that two thirds are procrastinators and one third rational and with 25% of each having RRA 2, 50% RRA 4, and 25% RRA 6. Each average is calculated, assuming either that all take the default choice or that either all or only the rational optimally chooses IP and I. Losses are shown in red. The maximum in each row is shown in blue. Except for the RRA, the baseline parameter values from Table 1 are used. *Please see online version for correct colors.*

	25–66 Years				30–66 Years				35–66 Years			
	7%	8%	9%	10%	9%	10%	11%	12%	12%	13%	14%	15%
Panel A. Rational												
RRA 2												
Default	−2.39	−3.12	−3.91	−4.75	−1.43	−1.89	−2.39	−2.93	−1.06	−1.37	−1.71	−2.07
With IP/SF option	−0.15	−0.67	−1.26	−1.84	0.13	−0.21	−0.62	−1.08	0.32	0.07	−0.23	−0.55
Best IP/SF	5/0.1	5/0.2	5/0.2	5/0.0	5/0.3	5/0.3	5/0.2	5/0.2	5/0.4	5/0.4	5/0.4	5/0.3
RRA 4												
Default	1.40	1.13	0.94	0.13	2.08	2.03	1.85	1.58	2.18	2.18	2.11	1.97
With IP/SF option	1.44	1.27	0.69	0.47	2.08	2.06	1.97	1.78	2.18	2.18	2.19	2.12
Best IP/SF	3/0.9	3/0.9	3/0.9	3/0.9	3/1.0	3/0.9	3/0.9	3/0.9	3/1.0	3/0.9	3/0.9	3/0.9
RRA 6												
Default	1.98	2.21	2.25	2.09	2.15	2.50	2.73	2.84	2.08	2.39	2.63	2.80
With IP/SF option	1.98	2.21	2.25	2.09	2.15	2.50	2.73	2.84	2.08	2.39	2.63	2.80
Best IP/SF	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0
Panel B. Procrastinator												
RRA 2												
Default	6.74	6.17	5.51	4.74	7.49	7.13	6.70	6.19	7.49	7.21	6.86	6.47
With IP/SF option	12.98	12.77	12.49	11.96	12.42	12.32	12.06	11.69	11.72	11.58	11.34	11.02
Best IP/SF	5/0.0	5/0.1	5/0.0	5/0.0	5/0.1	5/0.1	5/0.1	5/0.1	5/0.1	5/0.1	5/0.1	5/0.1
RRA 4												
Default	29.62	29.65	29.32	28.75	30.22	30.34	30.19	29.85	29.73	29.75	29.60	29.32
With IP/SF option	29.62	29.65	29.32	28.75	30.22	30.34	30.19	29.85	29.73	29.75	29.60	29.32
Best IP/SF	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0
RRA 6												
Default	44.82	46.27	46.95	47.10	44.45	45.69	46.40	46.68	43.03	43.91	44.44	44.68
With IP/SF option	44.82	46.27	46.95	47.10	44.45	45.69	46.40	46.68	43.03	43.91	44.44	44.68
Best IP/SF	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0
Panel C. Average Gains												
Equal weights												
Default	13.70	13.72	13.51	13.01	14.16	14.30	14.25	14.04	13.91	14.01	13.99	13.86
All optimal IP/SF	15.12	15.25	15.08	14.75	15.24	15.45	15.46	15.29	14.84	14.98	15.00	14.90
Rational opt. IP/SF	14.08	14.15	13.91	13.55	14.42	14.58	14.56	14.38	14.14	14.25	14.25	14.14
Nonequal weights												
Default	18.67	18.74	18.54	18.02	19.14	19.30	19.25	19.02	18.78	18.88	18.85	18.69
All optimal IP/SF	19.90	20.06	19.88	19.52	20.09	20.31	20.31	20.12	19.60	19.73	19.73	19.60
Rational opt. IP/SF	18.86	18.96	18.72	18.32	19.27	19.45	19.42	19.21	18.89	19.00	18.98	18.84

10%/30y plan, the rational individual with RRA 4 prefers IP3 and $I = 0.9$, which gives a welfare gain of 2.06%. The procrastinator with RRA 4 also prefers IP3 but $I = 1.0$, leading to a welfare gain of 30.34%. Looking across the table, we see that individuals with RRA 4 or 6 prefer IP3 and $I = 1.0$ or $I = 0.9$. Individuals with RRA 2 prefer the more aggressive IP5 and a lower I. This is true for any combination of the contribution rate and enrollment age covered by the table. Given the (limited) literature on the cross section of preferences, an RRA around 2 (or lower) does not appear dominating, so we choose IP3 and $I = 1.0$ to be the default plan choice.

We see that procrastinators benefit substantially from a mandatory plan, in particular procrastinators with RRA 4 or 6 for which the welfare gains are up to 47%, corresponding to about $406,000 in present value terms. The procrastinators with RRA 4 or 6 are happy with the default plan. Procrastinators with RRA 2 also benefit from the default plan, but would increase their welfare gains notably by actively choosing IP5 and a low value of I. Rational individuals with RRA 4 or 6 benefit marginally from all the plans reported in Table 3, but would lose, for example, if the plan mandates contributions from age 25 above a certain level. Rational individuals with RRA 2 dislike most of the mandatory plans considered, but if they can choose IP and I themselves even these individuals benefit from plans with low-to-modest contributions not starting very early.

For each contribution rate and enrollment age, we calculate the average percentage welfare gain across the six participant types considered in the table, that is, rational individuals and procrastinators with RRA 2, 4, and 6. Panel C of Table 3 reports both an equal-weighted average gain and a weighted average assuming that i) two thirds are procrastinators and one third rational as suggested by our previous discussion and ii) 25% of individuals have RRA 2, 50% have RRA 4, and 25% have RRA 6. We assume either that all individuals stick to the default IP and I, that all individuals optimize over IP and I, or that only the rational individuals optimize. In any case, the largest average gain is obtained either by the 10%/30y plan or the 11%/30y plan, which indicates some robustness toward how different individual types are weighted, although extreme weights clearly lead to other plans having the highest average gain. Henceforth, we focus on the 10%/30y plan since this has a slightly higher average gain than the 11%/30y plan for most of the averages considered and because the 10%/30y plan is less harsh on the rational RRA 2 individuals with a loss of 0.21% instead of 0.62% with optimal choice of IP and SF. The 0.21% loss corresponds to about $1,800 in present value terms. Below we discuss the introduction of additional options that may eliminate the loss. Assuming that all individuals optimally self-select the investment strategy and the solidarity factor, the weighted average gain for the 10%/30y plan is 20.31% or about $175,000 per participant.

Table 4 further illustrates how the investment policy and the solidarity factor affect the welfare implications of the 10%/30y plan. The investment policy is only marginally important for the rational individual with RRA 4 who can adjust the private investments accordingly, but the policies IP2–IP4 with a glidepath element are preferred. The rational RRA 2 individual prefers the full-stock policy IP5. Given the sizeable mandatory pension savings, this individual holds little private savings and thus cannot make up for a conservative fund strategy by investing all private savings in stocks. The solidarity factor has at least as large a welfare effect as the

TABLE 4

The Role of the Investment Policy and the Solidarity Factor

Table 4 reports percentage welfare gains when individuals are enrolled in a mandatory pension plan with 10% contributions from age 30. The gray background indicates the default choice of IP3 and $I = 1.0$. The numbers in blue indicate the best combination of IP and I for the individual. *Please see online version for correct colors.*

	RRA 2					RRA 4				
	IP1	IP2	IP3	IP4	IP5	IP1	IP2	IP3	IP4	IP5
Panel A. Rational										
0.0	−1.50	−1.22	−1.28	−0.67	−0.34	−2.32	−1.92	−1.86	−1.68	−1.86
0.1	−1.34	−1.05	−1.11	−0.54	−0.26	−1.76	−1.31	−1.25	−1.12	−1.41
0.2	−1.23	−0.93	−0.99	−0.46	−0.22	−1.23	−0.75	−0.68	−0.60	−1.00
0.3	−1.16	−0.86	−0.91	−0.42	−0.21	−0.74	−0.23	−0.16	−0.13	−0.61
0.4	−1.14	−0.84	−0.88	−0.42	−0.24	−0.29	0.24	0.31	0.30	−0.25
0.5	−1.17	−0.87	−0.90	−0.46	−0.30	0.13	0.67	0.74	0.69	0.09
0.6	−1.24	−0.94	−0.97	−0.55	−0.40	0.51	1.04	1.11	1.04	0.40
0.7	−1.36	−1.06	−1.09	−0.69	−0.54	0.85	1.37	1.44	1.35	0.69
0.8	−1.52	−1.24	−1.27	−0.88	−0.72	1.15	1.64	1.71	1.62	0.96
0.9	−1.75	−1.50	−1.52	−1.15	−0.99	1.38	1.84	1.90	1.83	1.19
0.0	−2.08	−1.87	−1.89	−1.56	−1.41	1.43	1.82	1.87	1.82	1.27
Panel B. Procrastinator										
0.0	10.65	11.03	10.87	11.83	12.31	18.31	19.66	19.58	19.38	15.16
0.1	10.83	11.23	11.08	11.94	12.32	20.25	21.56	21.50	21.10	16.69
0.2	10.89	11.30	11.17	11.94	12.24	21.87	23.13	23.09	22.54	18.04
0.3	10.82	11.25	11.13	11.83	12.06	23.22	24.42	24.41	23.74	19.26
0.4	10.59	11.04	10.94	11.59	11.77	24.32	25.48	25.48	24.75	20.40
0.5	10.19	10.67	10.58	11.20	11.33	25.23	26.32	26.35	25.58	21.50
0.6	9.54	10.07	9.99	10.60	10.72	25.97	27.00	27.05	26.31	22.60
0.7	8.58	9.17	9.11	9.75	9.90	26.67	27.61	27.68	27.02	23.80
0.8	7.51	8.10	8.06	8.74	8.95	27.56	28.38	28.44	27.92	25.23
0.9	6.99	7.41	7.35	7.99	8.24	28.78	29.51	29.55	29.19	26.91
0.0	6.84	7.20	7.13	7.69	7.91	29.60	30.28	30.34	30.07	28.19

investment policy. Overall, the IP and I options reduce the welfare loss of the risk-tolerant rational individual from 1.89% to 0.21%. Furthermore, the risk-tolerant procrastinator benefits substantially from the options that increase the welfare gain from 7.13% to 12.32%. A pension plan with full solidarity resembles a variable life annuity, so the finding that low RRA individuals dislike full solidarity is consistent with the results on optimal annuitization in Table 2. Of course, the optionality also opens up for welfare-reducing, suboptimal decisions, and for the procrastinator with RRA 4, the welfare gain of 30.34% with the default choice drops to 15.16% with the worst choice of IP and I. Intuitively, individuals procrastinating on saving decisions might be more prone to stick to the default choices than rational individuals, and it is difficult to see why procrastinators should actively pick a nondefault choice of investment policy and solidarity factor as these quantities have no effect on their consumption potential early in life.

Table 5 provides the welfare implications of the 10%/30y plan for a broader set of preference parameters. The welfare gain is sizeable for all procrastinators considered and increases with the RRA and the EIS and decreases with the bequest weight. The rational individuals with RRA 4 or 6 gain moderately from the plan with the gain increasing in the EIS. However, rational individuals with RRA 2 and a relatively low bequest weight and a relatively low EIS (and thus a preference for late resolution of uncertainty) incur a small loss. As reported in Table 2, these individuals are not interested in building up substantial savings and gain little from annuitization. Panel B of Table 5 confirms that investment policy IP3 and full

TABLE 5

Welfare Implications of the 10%/30y Plan

For a range of combinations of the preference parameters γ (RRA), ψ (EIS), and ξ (bequest weight), Panel A of Table 5 reports the percentage welfare gain of imposing the plan with 10% mandatory contributions from age 30 and the option to choose the investment policy and the solidarity factor. Panel B provides the optimal choice of the investment policy and the solidarity factor.

	RRA = 2			RRA = 4			RRA = 6		
	$\psi=\frac{1}{6}$	$\psi=\frac{1}{4}$	$\psi=\frac{1}{2}$	$\psi=\frac{1}{6}$	$\psi=\frac{1}{4}$	$\psi=\frac{1}{2}$	$\psi=\frac{1}{6}$	$\psi=\frac{1}{4}$	$\psi=\frac{1}{2}$
Panel A. Welfare Gains									
Rational									
$\xi=0.5$	−1.35	−0.72	1.28	1.93	2.10	2.46	2.28	2.38	2.59
$\xi=2$	−0.53	−0.21	1.25	1.78	2.06	2.54	2.37	2.50	2.65
$\xi=5$	0.25	0.37	0.95	0.65	1.09	2.32	0.83	1.52	2.67
Procrastinator									
$\xi=0.5$	11.92	12.65	13.20	28.06	34.48	46.29	29.35	47.49	71.62
$\xi=2$	11.57	12.32	11.10	24.07	30.34	28.73	31.46	45.69	43.14
$\xi=5$	10.38	12.37	10.21	14.29	21.07	27.99	17.69	32.89	42.11
Panel B. Optimal IP/I									
Rational									
$\xi=0.5$	5/0.5	5/0.7	5/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0
$\xi=2$	5/0.1	5/0.3	5/0.8	3/0.9	3/0.9	3/1.0	3/1.0	3/1.0	3/1.0
$\xi=5$	5/0.0	5/0.0	5/0.4	3/0.6	4/0.8	3/1.0	3/1.0	3/1.0	3/1.0
Procrastinator									
$\xi=0.5$	5/0.3	5/0.5	5/0.9	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0
$\xi=2$	5/0.0	5/0.1	5/0.5	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0
$\xi=5$	5/0.0	5/0.0	5/0.1	4/1.0	4/1.0	3/1.0	3/1.0	3/1.0	3/1.0

solidarity ($I=1$) is a good default choice, but also that risk-tolerant individuals prefer a riskier investment strategy and less-than-full solidarity.

B. Effects on Life-Cycle Decisions

In order to better understand why and how the mandatory pension plan identified above improves welfare, Figure 4 investigates how the introduction of the plan changes the consumption and investment decisions of both the rational individual and the procrastinator with baseline preference parameters. The dashed curves are for the case with no mandatory savings but access to annuitization with a 20% cost, as in Figure 3. The solid curves are for the case with the mandatory plan.

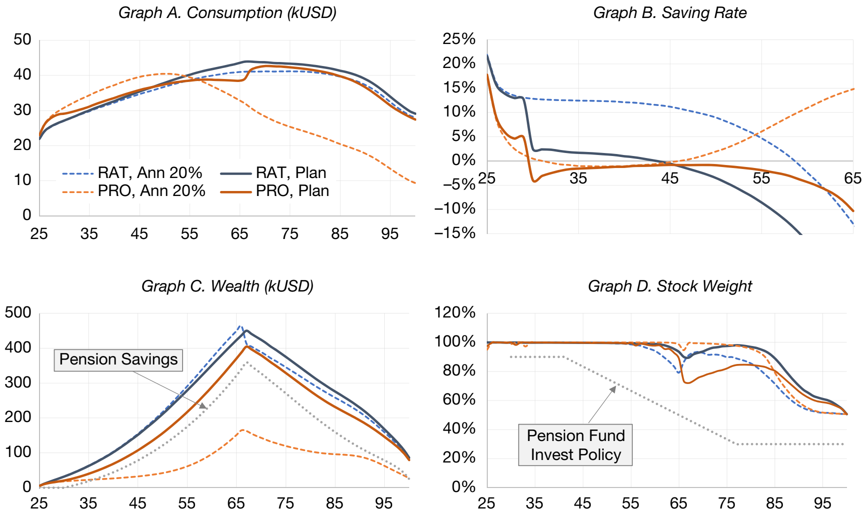
Graph A of Figure 4 shows that the expected consumption of the procrastinator moves close to that of the rational individual when imposing the plan. The mandatory savings reduce the procrastinator's excessive early-life consumption and boost retirement consumption. The consumption pattern is close to what the procrastinator prefers, but is unable to generate without the mandatory saving plan. The rational individual expects to consume slightly more at all ages with the mandatory plan, which is possible because of the lower return taxation and the better annuitization terms.

As shown in Graph B of Figure 4, the rational individual lowers the private savings to compensate for the mandatory savings, but still builds a liquid wealth buffer in the early years. The procrastinator accumulates some liquid wealth early on, but then saves, on average, virtually nothing from age 35 to 50, whether or not the plan is imposed. Without the plan, the procrastinator saves considerably in the

FIGURE 4

Life with a Pension Plan

In Figure 4, the horizontal axis shows the individual's age. The blue lines refer to a rational individual, and the orange lines refer to a procrastinator. The solid lines are for the case with a mandatory pension plan featuring 10% contributions from age 30, where the individual can choose the solidarity factor l and the investment policy from the menu IP1–IP5. The rational individual chooses IP3 and $l = 0.9$, and the procrastinator chooses IP3 and $l = 1$. The dashed lines are for the case with access to variable life annuities (50% stocks) with 20% cost. All the lines show means across 10,000 simulated paths. Graph A shows the annual consumption. Graph B displays the private saving rate, that is, the fraction of income after taxes and mandatory contributions that the individual chooses to save on a private account. Graph C shows the total wealth, which includes either the value of the annuity portfolio after cost or the after-tax pension wealth $(1-\tau_Y)A_t$. The dotted gray curve shows the after-tax pension wealth with IP3 and $l = 1$. Graph D illustrates the fraction of private wealth invested in the stock market. Here, the dotted gray curve depicts the fraction of pension wealth invested in stocks under the investment policy IP3. The baseline parameter values listed in Table 1 are used. *Please see online version for correct colors.*



Graph A. Consumption (kUSD)

Graph B. Saving Rate

Graph C. Wealth (kUSD)

Graph D. Stock Weight

years leading up to retirement, partly to make use of the annuitization option. With the plan, the procrastinator does not have to do that as the annuitization is built into the plan.

The rational individual's total wealth pattern is roughly the same with the mandatory plan as without (cf. Graph C of Figure 4), so the mandatory pension savings are replacing voluntary, private savings. As indicated by the difference between the solid blue curve and the dotted gray curve, the rational individual maintains a sizeable liquid wealth, which is desirable to have for buffering against shocks. The total wealth of the procrastinator is obviously much larger with the mandatory plan as the large mandatory savings by far outweigh the small reduction in private savings.

Graph D of Figure 4 shows the optimal fraction of private wealth invested in stocks together with the stock weight of the pension savings. Note that the private portfolio is tilted more toward stocks than the pension portfolio. While this could indicate that individuals would like the pension fund to take more risk, even late in life, we find that IP3 is preferred to the more aggressive IP4 and IP5 for individuals with RRA 4 or 6. By having the larger stock weight in the private, liquid portfolio, the individuals can quickly reduce their stock exposure if hit by large income or medical shocks or very poor returns, whereas the asset allocation mix of the pension savings is inflexible.

C. Sources and Robustness of Welfare Gains

With the baseline parameter values, the welfare gain is 2.06% for a rational individual and 30.34% for a procrastinator when the 10%/30y pension plan with individual choice of investment policy and solidarity factor is imposed. Table 6 reports how these welfare gains are impacted by variations in some assumptions and plan features.

First, we have measured welfare gains relative to the case with optimal individual annuitization assuming a 20% cost on annuities. Even when comparing to the case with fair annuities, the welfare gain is still large for the procrastinator (24.49%) and stays positive for the rational individual (0.61%). The gains generated by the mandatory plan are significantly larger (40.73% and 4.25%, respectively) when we compare with the case without individual annuitization, which seems a relevant benchmark for most individuals given the low observed annuitization rates.

Second, the gains are partly due to pension fund returns being tax-exempt. However, if all returns are taxed at 20%, the welfare gain is only marginally reduced to 1.56% for the rational individual and 29.56% for the procrastinator. If all returns are untaxed, the gains are slightly smaller, 1.36% and 28.44%. With $\tau_A = 10\%$ and $\tau_F = 20\%$, the gains are 1.79% and 29.89%. Some tax advantage may be important, though, for the acceptance of a mandatory pension scheme by individuals with low RRA and EIS. A lower, but positive, tax rate on pension returns may, in fact, lead to similar or larger total return tax revenues due to the significantly larger investments (and thus more taxable returns) with a mandatory pension plan. For example, with $\tau_A = 10\%, \tau_F = 20\%$, and baseline preferences, the present value of lifetime return taxes drops from about $92,000 to $68,000 for the rational individual, but increases from about $30,000 to $48,000 for the procrastinator.[21] With two thirds of individuals being procrastinators, total revenues increase.

Third, our baseline assumption is that both the private investor and the fund manager face a risk-free rate of 1% and a risky asset portfolio with an expected excess return of 4% and a volatility of 15.7%. Due to economies of scale, access to

TABLE 6

Robustness of Welfare Gains

Table 6 lists percentage welfare gains from imposing a mandatory pension plan with 10% contributions from age 30 and a choice of the investment policy and the solidarity factor. The baseline parameter values listed in Table 1 are used. In particular, $\gamma = 4$, $\psi = 1/4$, and $\xi = 2$.

	Rational	Procrastinator
Baseline parameters and plan	2.06	30.34
Gain relative to fair-annuity case	0.61	24.49
Gain relative to no-annuity case	4.25	40.73
No tax advantage, $\tau_A = \tau_F = 20\%$	1.56	29.56
No tax advantage, $\tau_A = \tau_F = 0\%$	1.36	28.44
Smaller tax advantage, $\tau_A = 10\%, \tau_F = 20\%$	1.79	29.89
Larger exp. return in fund (+0.5 pct pts)	2.87	31.32
Lower volatility in fund (−1 pct pt)	2.47	30.84
Medical costs (h, H) reduced by 1/3	2.46	31.00
No medical costs	3.27	32.24

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[21] We discount expected return tax payments from simulations to age 25 using the risk-free rate.

more asset classes, or simply better information and skills, pension fund managers may be better investors than private investors, even after accounting for management fees. Table 6 reports that if the fund manager can raise the expected excess return to 4.5% for the same volatility, the welfare gains increase moderately to 2.87% and 31.32% for the two investor types. Conversely, if the fund manager can obtain the expected excess return of 4% with a volatility of only 14.7%, the gains are 2.47% and 30.84%.

Finally, our model includes the risk of substantial out-of-pocket medical expenses late in life (a key concern of U.S. households). Such costs affect lifetime utility both with and without a pension scheme. Since medical costs cannot be paid out of illiquid pension savings, a mandatory pension scheme is less appreciated when medical costs are large, but Table 6 reports that gains are relatively insensitive to the assumed costs.

D. Sweetening Options

Rational individuals with low RRA (and low-to-modest EIS and bequest weight) dislike the plan with 10% contributions from age 30, even if they can choose the full-stock strategy IP5 and their favorite solidarity factor. While they would benefit from contributions at a lower rate or starting later, such a change would reduce the welfare gains for procrastinators and more risk-averse rational individuals. An alternative is to maintain the 10%/30y plan, but add options that are valuable primarily to the low-RRA rational individuals and do not allow procrastinators to reduce their savings substantially in the early part of the accumulation phase. For computational tractability, the options have to respect the separation (equation (10)).

By choosing IP5 instead of the default IP3, the rational RRA 2 individuals obtain a 19% larger expected pension wealth at age 60 (using $I = 1$). Hence, we consider a "payout option," allowing individuals turning 60 to pay out up to 19% of their pension savings, taxed at the income tax rate but without any penalty for the premature payout. By adding this option, the welfare loss can be transformed into a gain for some risk-tolerant rational individuals. For example, with RRA 2, EIS 1/4, and bequest weight 2, the loss of -0.21% turns into a gain of 0.38%. However, with RRA 2, EIS 1/6, and bequest weight 1/2, the option cannot generate a gain, but the loss is reduced from 1.35% to 0.61%; such an individual needs an option to pay out 35% at age 50, for example, to reach a gain. On average, rational individuals with RRA 2 choose to pay out 13%–16% of their savings with the exact number depending on the EIS and the bequest weight. The "19% at 60" payout option even leads to a slightly larger gain for procrastinators who would fully exploit the option in most states. However, a higher payout percentage or earlier payout age can reduce the gains for procrastinators. Overall, an option to pay out 19% at age 60 seems valuable to a broad range of individuals and particularly to the risk-tolerant rational individuals. The Supplementary Material provides additional numerical analysis of payout options.

An alternative is a "halt option," allowing members to end contributions at some specified age before retirement. As the payout option, individuals would exercise the halt option if their pension wealth is a large share of total wealth or if income is low relative to total wealth. However, ending contributions, say, at age 60 reduces the accumulated savings at retirement rather modestly, which results in smaller welfare improvements than found for the payout option. For example, a rational individual with RRA 2, EIS 1/4, and bequest weight 2 chooses to halt contributions at age 60 in 31% of the states, which reduces the loss from 0.21% to 0.11%.

E. Reduction in Social Security Benefits

The above analysis is calibrated to the current level of Social Security benefits paid to retirees. However, in a 2021 Gallup poll, 73% of respondents worry a great deal or a fair amount about the Social Security system (source: https://news.gallup.com/poll/1693/social-security.aspx, accessed on Apr. 23, 2021). Such concerns seem justified. The Social Security and Medicare Boards of Trustees conclude in their Apr. 2020 report that the system will show net cash outflows from 2021, and by 2035 benefits have to be cut by 24% to make the system sustainable (source: https://www.ssa.gov/OACT/TR/2020/index.html, accessed on Aug. 7, 2020).

We thus consider the implications of a 24% reduction in Social Security benefits from 45% to 34.2% of preretirement income. Since we have modeled the possible out-of-pocket medical expenses as fractions of Social Security benefits, we multiply these fractions by $1/(1-0.24)$ to keep expenses fixed in dollar terms. Repeating the analysis from Section IV.A with the new parameter values, we find that individuals appreciate a given pension plan more when Social Security is cut. The optimal contribution rate increases. Based on the weighted average gains, the best plans still have contributions from age 30, but now contribution rates of 11% or 12% provide the largest average gain of about 22.4% (or $193,000), whereas the largest average gain of around 20.3% ($175,000) in the base case was obtained with a contribution rate of 10% and 11%. As in the base case, the lower of the two contribution rates (now 11%) has the benefit of causing only marginal losses for the most risk-tolerant rational individuals, losses that can be eliminated or reduced by adding a payout option as discussed above. Hence, with lower Social Security benefits, a well-designed mandatory pension scheme is even more appreciated, and the optimal contribution rate increases from 10% to 11%. Details can be found in the Supplementary Material.

V. Discussion

Our analysis supports the introduction of a broad, well-designed, mandatory pension scheme. The presumably large group of individuals procrastinating on savings make significant welfare gains from such a scheme. Many rational individuals also experience gains, although smaller in magnitude, but some rational individuals with low RRA, low EIS, and a small bequest motive suffer a small welfare loss. We have given an example of an optional feature that primarily benefits these individuals. Within our framework, further improvements can be made by incorporating other options, additional fund investment policies to choose from, a more flexible contribution scheme, or pension payouts that are not constant in expectation through retirement. A downside of such features is the increased complexity of the plan. We acknowledge that, most likely, any pension plan with significant mandatory saving reduces the welfare of individuals who rationally prefer little saving due to "extreme preferences" such as a very low discount factor β, but presumably only few, if any, individuals have such preferences.

In reality, individuals have different degrees of procrastination, but it is unclear how this distribution can be estimated. Due to the concavity of the utility function, the gain from the more balanced consumption induced by a mandatory pension plan is an increasing convex function of the degree of procrastination. Our assumption of a single degree of procrastination (decision beta 0.85 vs. evaluation beta 0.96) thus leads to a lower average welfare gain from the mandatory plan than if we allowed for a distribution with the same average degree of procrastination. To illustrate this, recall that with a decision beta of 0.85 (and baseline parameters), the expected wealth–income ratio at age 60 is 2.5 in the absence of a mandatory plan and the welfare gain from our "favorite" plan is around 30%. A stronger procrastinator with a decision beta of 0.8 has an expected wealth–income ratio at 60 of 1.5 and has a 65% gain from the plan. A weaker procrastinator with decision beta 0.878 has an expected wealth–income ratio of 3.5 and a welfare gain of 17% from the plan. With an equal mix of the two types, the average degree of procrastination as measured by the wealth–income ratio at 60 is the same as before, but the average gain of 41% exceeds the 30% gain with our single type.

While our life-cycle model is rich, a number of relevant extensions easily come to mind. First, as the few other pension design articles, we ignore housing. Adding housing consumption and investments would complicate the numerical optimization method considerably and make our thorough analysis practically impossible. As broad mandatory plans embrace both renters and owners, the model should ideally endogenize the renting/owning decision, and is likely to result in renters preferring a very different pension plan than owners, among other things due to the fact that homeowners often build substantial home equity that can partly replace retirement savings. Young, prospective homeowners favor private saving for a down payment to illiquid pension savings, which backs our recommendation not to require pension contributions from an early age. We leave the very challenging pension design problem with both owners and renters to future research. Second, we assume illiquid pension savings, but it would be interesting to allow hardship withdrawals, maybe at some penalty rate, in case labor income drops considerably or medical shocks occur. Other extensions would be to allow for more elaborate labor income dynamics (see, e.g., Guvenen, Karahan, Ozkan, and Song (2021)), for nonproportional taxes and Social Security benefits, and for time-varying investment opportunities. Note, however, that any of these extensions induces an additional state variable which further complicates and prolongs the numerical solution.

Our welfare analysis ignores some potential general equilibrium effects. As pensions savings crowd out some private savings, a zero tax on pension returns lowers the total taxes collected from financial returns. However, we have shown that a mandatory pension scheme can be welfare improving even with no or only a modest tax advantage to pension savings, and with the increase in total savings the government could then collect an even larger return tax revenue compared with the case without mandatory saving. Moreover, with the higher retirement income generated by a broad mandatory plan, fewer retirees need tax-financed welfare payments or other forms of support. Our welfare calculations assume that the level of the risk-free rate and the expected excess return and volatility of the stock market do not change upon the introduction of the mandatory scheme. Any such changes could also have ramifications for firms' cost of capital and thus overall production and labor market conditions. We leave a complete general equilibrium analysis to future research.

VI. Conclusion

Our quantitative life-cycle analysis shows that a well-designed mandatory pension scheme substantially improves the welfare of individuals who procrastinate on retirement saving. At the same time, the scheme slightly improves the welfare of rational individuals, except those with a low risk aversion, a low elasticity of intertemporal substitution, and a low bequest motive. Calibrated to U.S. data, our model suggests a scheme with i) mandatory contributions of 10% of income from age 30 until retirement, ii) a glidepath investment policy resembling existing target-date funds, iii) tax-exempt returns on pension savings, iv) automatic lifelong annuitization, and v) an option to choose a different investment policy and a different annuity solidarity factor. Across individuals with different levels of risk aversion and sophistication, such a scheme delivers an average welfare gain corresponding to $175,000 per person in present value terms. The welfare gains come from reducing the under-saving problem of procrastinators, from ensuring annuitization of retirement wealth at fair terms, and (to a smaller extent) from eliminating taxes on returns on pension savings. With the projected cut in Social Security benefits, the contribution rate should be raised to 11%, and the mandatory savings plan is then even more appreciated with the average gain increasing to $193,000.

Supplementary Material

To view supplementary material for this article, please visit http://doi.org/10.1017/S0022109023000029.

References

Arias, E., and J. Xu. "United States Life Tables, 2017." *National Vital Statistics Reports*, 68 (2019), 1–66.

Barber, B. M., and T. Odean. "Trading Is Hazardous to Your Wealth: The Common Stock Investment Performance of Individual Investors." *Journal of Finance*, 55 (2000), 773–806.

Benartzi, S., and R. H. Thaler. "Heuristics and Biases in Retirement Savings Behavior." *Journal of Economic Perspectives*, 21 (2007), 81–104.

Benzoni, L.; P. Collin-Dufresne; and R. S. Goldstein. "Portfolio Choice over the Life-Cycle when the Stock and Labor Markets Are Cointegrated." *Journal of Finance*, 62 (2007), 2123–2167.

Bernheim, B. D.; J. Skinner; and S. Weinberg. "What Accounts for the Variation in Retirement Wealth Among U.S. Households?" *American Economic Review*, 91 (2001), 832–857.

Beshears, J.; J. J. Choi; D. Laibson; and B. C. Madrian. "Behavioral Household Finance." In *Handbook of Behavioral Economics: Foundations and Applications*, Chap. 3, B. D. Bernheim, S. DellaVigna, and D. Laibson, eds. Amsterdam, The Netherlands: Elsevier (2018), 177–276.

Bhamra, H. S., and R. Uppal. "Does Household Finance Matter? Small Financial Errors with Large Social Costs." *American Economic Review*, 109 (2019), 1116–1154.

Blake, D.; D. Wright; and Y. Zhang. “Age-Dependent Investing: Optimal Funding and Investment Strategies in Defined Contribution Pension Plans when Members Are Rational Life Cycle Financial Planners.” *Journal of Economic Dynamics and Control*, 38 (2014), 105–124.

Bodie, Z.; R. C. Merton; and W. F. Samuelson. “Labor Supply Flexibility and Portfolio Choice in a Life Cycle Model.” *Journal of Economic Dynamics and Control*, 16 (1992), 427–449.

Bonaparte, Y., and F. J. Fabozzi. “Estimating the Elasticity of Intertemporal Substitution Accounting for Stockholder-Specific Portfolios.” *Applied Economics Letters*, 24 (2017), 923–927.

Branger, N.; L. S. Larsen; and C. Munk. “Hedging Recessions.” *Journal of Economic Dynamics and Control*, 107 (2019), Article no. 103715.

Bricker, J.; L. J. Dettling; A. Henriques; J. W. Hsu; L. Jacobs; K. B. Moore; S. Pack; J. Sabelhaus; J. Thompson; and R. A. Windle. “Changes in U.S. Family Finances from 2013 to 2016: Evidence from the Survey of Consumer Finances.” *Federal Reserve Bulletin*, 103 (2017), 1–42.

Brown, J. R., and A. Previtero. “Saving for Retirement, Annuities, and Procrastination.” Working Paper, University of Illinois and Indiana University, available at http://www.aleprevitero.com/ (2020).

Calvet, L. E.; J. Y. Campbell; F. J. Gomes; and P. Sodini. “The Cross-Section of Household Preferences.” Working Paper, Harvard University, available at https://scholar.harvard.edu/campbell (2021).

Calvet, L. E.; J. Y. Campbell; and P. Sodini. “Down or Out: Assessing the Welfare Costs of Household Investment Mistakes.” *Journal of Political Economy*, 115 (2007), 707–747.

Campbell, J. Y. “Household Finance.” *Journal of Finance*, 61 (2006), 1553–1604.

Campbell, J. Y. “Restoring Rational Choice: The Challenge of Consumer Financial Regulation.” *American Economic Review*, 106 (2016), 1–30.

Campbell, J. Y.; J. F. Cocco; F. Gomes; and P. J. Maenhout. “Investing Retirement Wealth: A Life-Cycle Model.” In *Risk Aspects of Investment-Based Social Security Reform*, Chap. 5, J. Y. Campbell and M. Feldstein, eds. Chicago, IL: University of Chicago Press (2001), 439–473.

Charupat, N., and M. A. Milevsky. “Optimal Asset Allocation in Life Annuities: A Note.” *Insurance: Mathematics and Economics*, 30 (2002), 199–209.

Chiappori, P.-A., and M. Paiella. “Relative Risk Aversion Is Constant: Evidence from Panel Data.” *Journal of the European Economic Association*, 9 (2011), 1021–1052.

Choi, J. J.; D. Laibson; and B. C. Madrian. “$100 Bills on the Sidewalk: Suboptimal Investment in 401 (K) Plans.” *Review of Economics and Statistics*, 93 (2011), 748–763.

Choi, J. J.; D. Laibson; B. C. Madrian; and A. Metrick. “Defined Contribution Pensions: Plan Rules, Participant Choices, and the Path of Least Resistance.” *Tax Policy and the Economy*, 16 (2002), 67–113.

Cocco, J. F. “Portfolio Choice in the Presence of Housing.” *Review of Financial Studies*, 18 (2005), 535–567.

Cocco, J. F.; F. J. Gomes; and P. J. Maenhout. “Consumption and Portfolio Choice over the Life Cycle.” *Review of Financial Studies*, 18 (2005), 491–533.

Dahlquist, M.; O. Setty; and R. Vestman. “On the Asset Allocation of a Default Pension Fund.” *Journal of Finance*, 73 (2018), 1893–1936.

Dammon, R. M.; C. S. Spatt; and H. H. Zhang. “Optimal Asset Location and Allocation with Taxable and Tax-Deferred Investing.” *Journal of Finance*, 59 (2004), 999–1037.

Davidoff, T.; J. R. Brown; and P. Diamond. “Annuities and Individual Welfare.” *American Economic Review*, 95 (2005), 1573–1590.

Davis, S. J., and P. Willen. “Using Financial Assets to Hedge Labor Income Risks: Estimating the Benefits.” Working Paper, University of Chicago and Princeton University (2000).

De Nardi, M.; E. French; and J. B. Jones. “Why Do the Elderly Save? The Role of Medical Expenses.” *Journal of Political Economy*, 118 (2010), 39–75.

De Nardi, M.; E. French; J. B. Jones; and J. McCauley. “Medical Spending of the US Elderly.” *Fiscal Studies*, 37 (2016), 717–747.

Fagereng, A.; C. Gottlieb; and L. Guiso. “Asset Market Participation and Portfolio Choice over the Life-Cycle.” *Journal of Finance*, 72 (2017), 705–750.

Fischer, M., and M. F. Gallmeyer. “Taxable and Tax-Deferred Investing with the Limited Use of Losses.” *Review of Finance*, 21 (2017), 1847–1873.

Ghilarducci, T., and T. James. *Rescuing Retirement: A Plan to Guarantee Retirement Security for All Americans*. New York, NY: Columbia Business School Publishing (2018).

Gomes, F.; K. Hoyem; W.-Y. Hu; and E. Ravina. “Retirement Savings Adequacy in U.S. Defined Contribution Plans.” Available at SSRN: http://ssrn.com/abstract=3294422 (2020).

Gomes, F., and A. Michaelides. “Portfolio Choice with Internal Habit Formation: A Life-Cycle Model with Uninsurable Labor Income Risk.” *Review of Economic Dynamics*, 6 (2003), 729–766.

Gomes, F., and A. Michaelides. “Optimal Life-Cycle Asset Allocation: Understanding the Empirical Evidence.” *Journal of Finance*, 60 (2005), 869–904.

Gomes, F.; A. Michaelides; and V. Polkovnichenko. “Optimal Savings with Taxable and Tax-Deferred Accounts.” *Review of Economic Dynamics*, 12 (2009), 718–735.

Gourinchas, P.-O., and J. A. Parker. “Consumption over the Life Cycle.” *Econometrica*, 70 (2002), 47–89.

Guiso, L., and P. Sodini. “Household Finance: An Emerging Field.” In *Handbook of the Economics of Finance*, Chap. 21, G. M. Constantinides, M. Harris, and R. M. Stulz, eds. Amsterdam, The Netherlands: Elsevier (2013), 1397–1532.

Guvenen, F.; F. Karahan; S. Ozkan; and J. Song. “What Do Data on Millions of U.S. Workers Reveal About Lifecycle Earnings Dynamics?” *Econometrica*, 89 (2021), 2303–2339.

Heimer, R. Z.; K. O. R. Myrseth; and R. S. Schoenle. “YOLO: Mortality Beliefs and Household Finance Puzzles.” *Journal of Finance*, 74 (2019), 2957–2996.

Horneff, W. J.; R. H. Maurer; O. S. Mitchell; and M. Z. Stamos. “Variable Payout Annuities and Dynamic Portfolio Choice in Retirement.” *Journal of Pension Economics and Finance*, 9 (2010), 163–183.

Inkmann, J.; P. Lopes; and A. Michaelides. “How Deep Is the Annuity Market Participation Puzzle?” *Review of Financial Studies*, 24 (2011), 279–319.

Klingsieck, K. B. “Procrastination—When Good Things Don’t Come to Those Who Wait.” *European Psychologist*, 18 (2013), 24–34.

Koijen, R. S. J.; T. E. Nijman; and B. J. M. Werker. “When Can Life-Cycle Investors Benefit from Time-Varying Bond Risk Premia?” *Review of Financial Studies*, 23 (2010), 741–780.

Koijen, R. S. J.; T. E. Nijman; and B. J. M. Werker. “Optimal Annuity Risk Management.” *Review of Finance*, 15 (2011), 799–833.

Koijen, R. S. J.; S. van Nieuwerburgh; and M. Yogo. “Health and Mortality Delta: Assessing the Welfare Cost of Household Insurance Choice.” *Journal of Finance*, 71 (2016), 957–1010.

Laibson, D. “Golden Eggs and Hyperbolic Discounting.” *Quarterly Journal of Economics*, 112 (1997), 443–477.

Lockwood, L. M. “Bequest Motives and the Annuity Puzzle.” *Review of Economic Dynamics*, 15 (2012), 226–243.

Mitchell, O. S.; J. M. Poterba; M. J. Warshawsky; and J. R. Brown. “New Evidence on the Money’s Worth of Individual Annuities.” *American Economic Review*, 89 (1999), 1299–1318.

Paravisini, D.; V. Rappoport; and E. Ravina. “Risk Aversion and Wealth: Evidence from Person-to-Person Lending Portfolios.” *Management Science*, 63 (2017), 279–297.

Schlafmann, K.; O. Setty; and R. Vestman. “Optimal Defined Contribution Pension Plans: One-Size Does Not Fit All.” Discussion paper 8-2020, The Pinhas Sapir Center for Development, Tel Aviv University (2020).

Steel, P. “The Nature of Procrastination.” *Psychological Bulletin*, 133 (2007), 65–94.

Thurow, L. “The Optimum Lifetime Distribution of Consumption Expenditures.” *American Economic Review*, 59 (1969), 324–330.

Viceira, L. M. “Optimal Portfolio Choice for Long-Horizon Investors with Nontradable Labor Income.” *Journal of Finance*, 56 (2001), 433–470.

Vissing-Jørgensen, A. “Limited Asset Market Participation and the Elasticity of Intertemporal Substitution.” *Journal of Political Economy*, 110 (2002), 825–853.

Yaari, M. “Uncertain Lifetime, Life Insurance, and the Thoery of the Consumer.” *Review of Economic Studies*, 32 (1965), 137–150.

Yogo, M. “Portfolio Choice in Retirement: Health Risk and the Demand for Annuities, Housing, and Risky Assets.” *Journal of Monetary Economics*, 80 (2016), 17–34.