the one that you buy." Tristan Needham ends his Preface to the new edition with the toast, "Cheers! I raise my glass to the *next* 25 years!". This will take us to the golden anniversary of VCA and, rather presciently, the parcel containing my review copy weighed in at a golden 1.618 kg!

10.1017/mag.2024.142 NICK LORD
Published by Cambridge University Press
on behalf of The Mathematical Association

Mastering calculus through practice by Barbara de Holanda, Maia Teixeira and Edmundo Capelas de Oliveira, pp 348, EUR 53.49 (paper), ISBN 978-3-030-95823-7, Springer Verlag (2023)

Calculus is understood in this book as watered-down mathematical analysis, not putting much value on rigour. To master calculus properly would mean to go much further in the direction of analysis and to be able to solve more difficult problems. This book contains solved exercises from the standard course in calculus, with topics functions, limits, derivatives and integrals in Chapters 2 to 5 respectively. The first chapter, entitled 'Preliminaries', is the largest, comprising one third of the book; it consists of high school mathematics. The authors put much focus here on elementary geometric exercises, unrelated to themes from calculus, which might be more suitable for a pre-calculus book. Chapter 6, 'Brief Recap', reviews the preceding material. The last chapter contains the solutions.

With few exceptions, most of the exercises could be found in standard calculus textbooks; they are simple and straightforward, rather than challenging and thought-provoking tough nuts to be cracked.

The solutions are not always the best, and some of them are actually wrong. For instance, exercise 1.31 is: 'Let $n \in \mathbb{N}$. Show that, for $n > 2$, the inequality $n^3 > 3n + 5$ is valid'. This is 'proved' by the 'method of exhaustion in contrapositive statement' so that the inequality $n^3 \leqslant 3n + 5$ is checked for $n = 0$, 1, 2. And that's it. The authors claim that the result is proved since direct statement and contrapositive statement are logically equivalent. It is elementary logic that if you want to use the indirect method of proof you must show that $n^3 \leqslant 3n + 5 \Rightarrow n \leqslant 2$.

Some solutions are baffling. For instance, exercise 6.5 asks for the area A of the region bounded by the curve $y(x) = \sqrt{4x + 1}$ and the straight lines $x = 0$, $y = 0$ and $x = 12$. The worked solution uses the trapezium rule with division of the interval $[0, 12]$ in three subintervals with the points $x_0 = 0$, $x_1 = 2$, $x_2 = 6$ and $x_3 = 12$ to calculate the sum of the areas of the three trapeziums, which yields 56 as the approximation of $A = \int_0^{12} \sqrt{4x + 1}dx = 57$. But then the readers will be perplexed by the false claim that considering 12 trapeziums, all having the same width, would give the inequality $56 \leqslant A \leqslant 58$.

In the preface the authors write that they avoid the traditional method of ordering the exercises in their order of difficulty, but they don't seem to offer any alternative.

That there is no logical order in their selection is shown by exercise 4.42 which is to determine the equation of the tangent to the curve $y = 9\sqrt{x - 1}$ at its inflection point. Then, after two pages, exercise 4.53 asks how to define an inflection point. In fact Exercise 4.42 is clearly a misprint; the given solution $x = 1$ is an inflection point for the function $y = 9\sqrt[3]{x - 1}$.

There is unnecessary repetition. Exercise 2.30 asks for the minimum value of $z = x^2 + y^2$ for real x and y given that $2x + 3y = 16$. Then 2.38 is the same exercise for $2x + 3y = 12$ with only slightly changed wording.

The translation of the text could also have been more precise: 'confrontation theorem' should be 'squeeze theorem', 'counterpositive sentence' should be 'contrapositive statement', and so on.

The book could be used for learning basic calculus but no more. It might possibly aid pre-university courses in countries where calculus is not met at school, but under systems such as that in the UK, where calculus techniques (without rigorous definitions) are often met at GCSE and are central to A-level, it has little to offer compared with standard textbooks. 'Mastering' in the title is an exaggeration; those students who want to master calculus in a more rigorous sense should look elsewhere.

10.1017/mag.2024.143 © The Authors, 2024
Published by Cambridge University Press
on behalf of
The Mathematical Association

MARTIN LUKAREVSKI
Department of Mathematics and Statistics,
University "Goce Delcev", Stip, North Macedonia
e-mail: *martin.lukarevski@ugd.edu.mk*

Calculus by Amber Habib, pp 391, £49.99 (paper), ISBN 978-1-00915-969-2, Cambridge University Press (2022)

The one-word title brings to mind books such as Spivak's *Calculus* and this indeed gives an accurate flavour of the style and content of the book under review. It is conceived as a first-year undergraduate bridging course between the calculus techniques met at school and either more advanced analysis courses or more sophisticated applications to science or economics. As well as supplying rigorous foundations for familiar techniques and an introduction to tools for future use, a related aim is to build confidence in writing out formal proofs. The author's style is impressively clear and crisp, with every topic fully motivated and proofs deftly handled with a minimum of fuss. As the detailed bibliography attests, full advantage is taken of recent thinking about such courses and there are some intriguing choices made of organisation and content: keep your eye on the order of chapters described below!

Chapter 1 deals with background material on functions and the real numbers. The completeness axiom used is Tarski's modification of Dedekind's: if A, B are non-empty subsets of $\mathbb{R}$ such that $a \leqslant b$ for all $a \in A$, $b \in B$ then there is a real number m such that $a \leqslant m \leqslant b$ for all $a \in A$, $b \in B$—notice that it is *not* assumed that $A \cup B = \mathbb{R}$. This axiom has the advantage of being well adapted both to handling lower and upper sums in integration and to using nested intervals to provide unified proofs that continuous functions on closed, bounded intervals are bounded, attain their bounds, have the intermediate value property (IVP), and are uniformly continuous.

Chapter 2 introduces Riemann integration via lower and upper step functions. The focus is on proving properties of the integral, including the integrability of monotone functions: this enables the algebraic properties of $\ln x$ (defined as $\int_1^x \frac{1}{t} dt$) and $\exp(x)$ (its inverse) to be readily established. Chapter 3 covers limits and continuity: as well as the usual results in this area, there is a quick review of trigonometric functions and a proof that continuous functions are integrable. The author's approach to differentiation in Chapter 4 emphasises 'local linearity' as a prelude to the standard results on combining derivatives: it was also good to see Darboux's theorem (that derivatives have the IVP) proved and later used and, in an excellent section on extrema and curve sketching, it was striking that the mean value theorem is not invoked.

Chapter 5 has a brisk résumé of techniques of integration squarely focused on anti-derivatives: two notable features here are a full account of the integration of