A Mechanical Construction for the Quartic Trisectrix.

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Figure 46.

The model consists of a circular template, of radius a, hinged at O, a point on its circumference, to a bar OA, where $OA = a$. P is the centre of the circle, and E is the middle point of OP. C is a point on the circumference such that $OP = OC = a$, so that CE is perpendicular to OP.

From E two arms EX and EY radiate, and are so arranged by a linkage that the angles YEC and XEC are equal.

To trisect an angle with the instrument, place OA along one side, make the arm EX fall on A, and open or close AOC, EX running along A, until the point where the arm EY cuts the circumference of the template lies on the other side of the angle. Let this point be called B.

Then OC is one of the trisectors of the angle AOB.
Since $OP = OC = OA = a$,
∴ A, C, P are on the circumference of a circle equal to OCB, which cuts it at C, and has O for its centre.

Now CE is perpendicular to OP, which joins the centres.
∴ arcCB = arcCA.
∴ angles CPB and COA are equal.

But angle CPB = 2 . angle COB,
∴ angle COA = 2 . angle COB,
or OC is one of the trisectors of the angle AOB.

Figure 47.

To draw the curve, let OA be fixed, and let the arm EX always run along A: then the point B describes the Quartic Trisectrix.

https://doi.org/10.1017/S0013091500034581 Published online by Cambridge University Press

Let B be any point on the curve.

With centre O, and radius OA describe a circle ACP.

Join BA, cutting the circle ACP at Q : join OQ.
Then, obviously, OQ = OA, and the angles OQA and OAQ are equal. Also, from the congruence of the triangles BPE and AOE, we see that PBAO is a trapezium.

Hence the angles OQA, OAQ, and PBQ are equal.

Hence OQ is equal and parallel to PB.
Therefore QBPO is a parallelogram, and QB = OP and is constant.

It follows that the locus of B is that particular kind of Limaçon which is called the Quartic Trisectrix.

FIGURE 48.

Another mechanical method of drawing the Limaçon is given by the linkage shown in the figure.

Here $AB = OC = a$, $OA = CB = DE = 2a$, $CD = BE = 4a$, and, in the case of the trisectrix, DP = OD. O and A are fixed, and P describes the curve.

From the arrangement it will be seen that $\angle AOC = \angle CDE$.

$$\therefore \quad \angle AOD = \angle CDP.$$

Thus P moves so that $OD = DP$ and $\angle AOD = \angle ODP$, thus describing the trisectrix.

Other Limaçons may also be drawn, the eccentricity depending on the length of DP.

https://doi.org/10.1017/S0013091500034581 Published online by Cambridge University Press