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Understanding wind farm power densities

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Kirby *et al.* (*J. Fluid Mech.*, vol. 953, 2022, A39) adapted the *two-scale momentum theory* (Nishino & Dunstan, *J. Fluid Mech.*, vol. 894, A2) to large finite-sized farms. They demonstrated that analytical estimates agree excellently with large eddy simulations, and that the model provides a good upper limit of the power production for a given array density. Crucially, they introduced the concepts of farm-scale losses, caused by the atmospheric response to the whole farm, and turbine-scale losses, owing to internal flow interactions in the wind farm. These two new theoretical concepts offer a novel way to analyse the performance of extended wind farms. For large offshore wind farms, losses at the wind-farm scale are typically twice as high as at the turbine scale. This demonstrates that there is limited potential for layout optimizations of extended arrays. Instead, optimization strategies should focus on developing methods to increase the energy entrainment into the wind farm. This work provides an exciting roadmap for analysing the effective efficiency of large wind farms.

Key words: turbulence modelling, atmospheric flows, general fluid mechanics

1. Introduction

Wind energy is one of the leading renewable energy technologies and is key to the renewable energy transition. When wind turbines are placed together in a wind farm, they produce less energy than when placed in isolation. The harvesting of wind energy leads to the formation of wind turbine wakes, a region with reduced wind speed, behind each turbine. These wind turbine wakes affect the performance of downstream turbines in the farm. In addition, large wind farms act as additional resistance to the atmospheric boundary layer (ABL). This reduces the wind speed upstream and inside the farm, which affects the power production of the wind farm compared with the ideal situation where the upstream wind speed is not affected (Nishino & Dunstan 2020). This effect is known as farm blockage (Bleeg *et al.* 2018; Segalini & Dahlberg 2020) and its importance has

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only been recognised recently as large wind farms began to operate. Understanding the interactions between wind farms and flow in the ABL flow is one of the grand challenges the wind energy community faces (Veers *et al.* 2019, 2022), and is a prime example of a 21st century fluid dynamics problem for which more fundamental insight needs to be developed.

Two main analytical approaches are employed to model wind farm performance, which is crucial to optimise their design. The first approach is based on modelling wind turbine wakes and can be considered a *bottom-up* or wake modelling approach (Jensen 1983; Bastankhah & Porté-Agel 2014). Wake models generally work well for the entrance region of the wind farm and are commonly employed to optimise wind farm design. However, field and wind tunnel data comparisons have demonstrated that wake models have limitations in capturing wake–wake and wind farm–ABL interactions.

The second approach is known as the *top-down* or single-column approach, in which momentum analysis and horizontal averaging are used to estimate the flow inside the wind farm. Seminal works (Newman 1977; Frandsen 1992; Frandsen *et al.* 2006) show that including a resistive force at the turbine height and the resistive force at the ground allows for crucial insights into the vertical velocity profile inside the wind farm. Calaf, Meneveau & Meyers (2010) developed an improved top-down model by including an additional wake layer with enhanced turbulence levels. Later works have extended this approach to finite-length wind farms (Meneveau 2012), and included atmospheric stratification effects (Abkar & Porté-Agel 2013; Peña & Rathmann 2014; Sescu & Meneveau 2015; Li *et al.* 2022). Although top-down models include the response of an idealised ABL to a large wind farm, this approach cannot account for layout effects.

Efforts at combining top-down and bottom-up approaches include the work of Frandsen *et al.* (2006). This work developed a novel coupling between wake and top-down models, which considers a single column of turbines in a farm. However, this limits the applicability of the model to regular arrays. The coupled wake boundary layer (CWBL) model (Stevens, Gayme & Meneveau 2016) widens the applicability of coupled models by introducing a two-way coupling between a wake model and a top-down model. This approach agrees well with large eddy simulations (LES) and field observations and has been extended to general wind farm layouts (Starke *et al.* 2021).

An important question is whether there is a theoretically derivable maximum power density for large wind farms. For single turbines, the German physicist Albert Betz showed, using the principles of mass and momentum conservation of the airflow through an idealised actuator disc, that no turbine can capture more than 59.3 % of the kinetic energy in the wind. In reality, most turbines are placed within large wind farm clusters. For wind farms, no fundamental performance limit has been derived yet as the underlying mechanism depends on *turbulence*. For extended wind farms the primary source of kinetic energy is the flow energy entrained from the geostrophic wind above the farm. Hence, the turbulence entrainment sets a physical limit to power output density that can be obtained with wind farms. This limit should not be confused by the optimal wind turbine spacing, which results from the balance between economical considerations and the wind farm fluid dynamics (Meyers & Meneveau 2012; Stevens *et al.* 2017).

Although deriving a fundamental limit for the performance of wind energy clusters from first principles has remained elusive, various modelling approaches have been explored. As top-down models estimate the energy flux from above they can be used to estimate the maximum wind farm power density (Meneveau 2019). Luzzatto-Fegiz & Caulfield (2018) developed a two-interface entrainment model for fully developed wind farms to analyse the power output density of wind farms. Their main result is that the wind farm power density is proportional to the rate at which energy is entrained from the ABL. The predicted power output agrees well with field measurements when tailoring the model to reflect current wind farm designs. Furthermore, they showed that the performance of a wind farm can be about an order of magnitude higher in the idealised situation where the boundary layer mixes perfectly with the flow inside the wind farm. Antonini & Caldeira (2021) showed with mesoscale simulations and model calculations that the maximum achievable power output density of wind farms is primarily determined by the strength of the geostrophic wind that drives the boundary layer flow. Kirby, Nishino & Dunstan (2022) introduced a combined theoretical and computational approach to analyse fluid mechanics processes that determine the optimal performance of extended wind farms. This approach provides new estimates for optimal wind power density and a novel way to study the effective efficiency of extended wind turbine arrays.

2. Overview

Kirby *et al.* (2022) employed the *two-scale momentum theory* introduced by Nishino & Dunstan (2020) to estimate the power production of large wind farms. This theory splits the multi-scale flow into *external* and *internal* subproblems. The external farm-scale determines the amount of momentum available to the bottom resistance of the ABL. The internal turbine scale describes this resistance in terms of wind turbine drag and land/sea surface friction. The two subproblems are coupled to each other through a non-dimensional parameter known as the *farm induction factor*. Using LES, they demonstrate that the model accurately predicts the power output of infinite wind turbine arrays, even though the theory does not account for the wind farm layout. The observation that the layout does not affect the performance of extended wind farm arrays is in line with previous LES (Stevens & Meneveau 2017). An exciting contribution is the extension of the two-scale momentum theory to estimate the optimal power density of large finite-sized farms with the same layout. They demonstrated that the power production of such farms depends on both the array density and turbine layout and that the analytical model provides a good upper limit of the power production for a given array density.

Crucially, Kirby *et al.* (2022) provided a novel analysis of the underlying fluid dynamics. They introduced the concept of turbine-scale losses Π_T, due to internal flow interactions in the wind farm, and wind-farm-scale losses Π_F, which indicate the overall performance loss due to the interaction between the wind farm with the ABL. Farm-scale losses estimate the optimal wind farm performance that can be expected for a given array density. Turbine-scale losses indicate the losses that can be prevented by optimising the wind farm layout. This novel approach introduces an exciting new concept to assess the effective efficiency of extended wind turbine arrays, i.e. how well does a wind farm perform compared with the estimated optimal performance for the given array density.

Kirby *et al.* (2022) demonstrate that for large offshore wind farms, losses at the wind-farm scale are typically twice as high as the losses at the turbine scale. Furthermore, the ratio between the farm- and turbine-scale losses increases with wind farm size. Figure 1 shows that these turbine-scale losses are smaller than traditional wake losses, which also include effects induced by the response of the ABL to the flow resistance imposed by the wind farm. This novel analysis demonstrates that the performance of large turbine arrays is mostly determined by the overall interaction between the wind farm and the ABL rather than by direct interactions among the turbines. This convincingly shows that the potential for layout optimisations of extended arrays is limited. The two-scale momentum theory thus correctly captures the effect of the energy entrainment from higher atmospheric layers

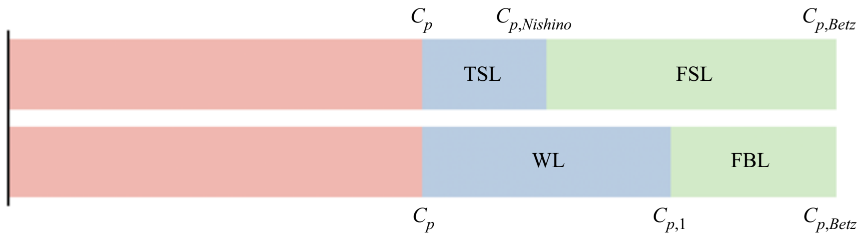

Figure 1. Comparison of turbine-scale loss (TSL) and farm-scale loss (FSL) with what is known as wake loss (WL) and farm blockage loss (FBL). Here $C_{p,1}$ is the power coefficient recorded by a farm's first row of turbines. Adapted from Kirby *et al.* (2022).

into the wind farm. Furthermore, the analysis indicates that the turbine-scale losses depend on the strength of the large-scale atmospheric response, i.e. how much momentum can be extracted from the higher atmospheric layers, and the wind farm size. Additional insight into these dependencies will be crucial to assess the effective efficiency of large wind farms.

3. Future

The insights of Kirby *et al.* (2022) provide an exciting roadmap to study the performance of extended wind farms. The present study focused on neutral atmospheric conditions and statistical stationary situations. Given that the model relies on momentum theory, the general observations are expected to extend to a wider range of atmospheric stability conditions. However, more investigation will be required to quantify how momentum extraction depends on atmospheric conditions and wind farm layout and size. In addition, the momentum entrainment into the wind farm may be affected by the dynamic changes in geostrophic forcing that drive the ABL. Numerical weather prediction models provide an excellent avenue to study this, although it should be realised that an accurate representation of wind farms in such models is challenging (Fischereit *et al.* 2022). Furthermore, the insight that layout optimisations will not be effective in larger wind farm arrays confirms that other optimisation strategies urgently need to be pursued. A key question, for example, is whether dynamic wind farm control strategies (Meyers *et al.* 2022) can increase the wind farm's overall momentum entrainment.

Declaration of interests. The author reports no conflict of interest.

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