THE JOURNAL OF SYMBOLIC LOGIC
Volume 25, Number 1, March 1960

TWENTY-FIFTH ANNUAL MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

The twenty-fifth annual meeting of the Association for Symbolic Logic was held on Tuesday, January 24, 1961 at the Willard Hotel in Washington, D.C. in conjunction with the meetings of the American Mathematical Society and the Mathematical Association of America.

At the morning session Professor Leon Henkin and Professor Theodore Hailperin presided. Four twenty minute papers were presented. An invited address on *Non-Standard Arithmetics and Non-Standard Analysis* was given by Professor Abraham Robinson of Hebrew University and Princeton University.

At the afternoon session Professor S. C. Kleene of the University of Wisconsin delivered an invited address on *Foundations of Intuitionistic Mathematics*. Professor Haskell B. Curry presided.

The Council of the Association met at luncheon.

Abstracts follow. The last three papers were presented by title. DAVID NELSON

REED C. LAWLOR. *Logical theory of United States patent claims.*

This paper deals with the application of symbolic logic to the analysis of patent claims. Such claims define classes. By converting into propositions the class descriptions of various parts, the properties of various parts, the relationships of various parts, and the relationships between any of the foregoing, the scope of a patent can be defined as a sum of products. Such an equation can then be used to determine questions of literal infringement and literal anticipation. They may also be used to define classes of non-infringing devices. (Received October 22, 1960.)

CLIFFORD JOSEPH MALONEY. *Contribution to the foundation of logic and information retrieval.*

It is usual to express the foundation of logic, mathematics, and the theory of information in terms of the results of class and class membership. This paper proposes a different foundation which seeks to avoid the difficulties associated with the class theory approach and to provide for those areas such as semantic information and information retrieval which have so far not been adequately accounted for.

The objects of study are called elementary sequences. Each elementary sequence consists of an infinite succession of 4-valued symbols. One of the 4-valued symbols, which may be taken as $+1$ and written $+$, can be thought of intuitively as indicating that a certain property pertains to the referent of the particular elementary sequence, another -1, written $-$, that it does not. A "universal" symbol, u, would indicate that the elementary sequence pertains to a class containing both members, or both sub-classes, or to a proposition inclusive of both, etc. Finally the null symbol, n, indicates that the signification of the given position is irrelevant to that of the elementary sequence.

To apply the system to information retrieval, the possibility that the appropriate 4-values symbol in a given position is unspecified or unknown is allowed. Each position is assigned a "weight", the same for all of the 4-valued symbols, but zero if the value of the position is unknown. From this, a non-probabilistic measure of semantic information is derived. (Received September 30, 1960.)

HUBERT H. SCHNEIDER. *A syntactical characterization of the predicate calculus with identity and universal validity.*

In an earlier paper (***Portugaliae Mathematica***, vol. 17 (1958), 85–96) we investigated the semantics of the predicate calculus with identity, IPC, when the empty individual domain is included in the consideration. It was shown, in particular, that any open formula and any Λ-closed formula of the IPC is valid in the empty individual domain, while any V-closed formula of IPC is invalid in the empty individual domain. Mostowski XVI 107, Hailperin XVIII 197, and Quine XIX 177 have exhibited systems in order to describe *syntactically* all formulas of the predicate calculus which are valid in all individual domains. As we pointed out, the system of Mostowski lacks the principle of extensionality, since he treats vacuous quantifiers as non-existing in the evaluation of formulas. The systems of Hailperin and Quine are restricted so as to have as theorems only closed formulas. The following system, M, does not have these limitations. Moreover, both the universal and the existential quantifiers are used as primitive symbols, and the identity symbol $\equiv$ is included among the primitive symbols.

The *axioms* of M are: (i) if H is tautologous then $\vdash H$; (ii) $\vdash \Lambda x(x \equiv x)$; (iii) $\Lambda x \Lambda y(x \equiv y \rightarrow (H \rightarrow H[x/y]))$, where $H[x/y]$ is the result of replacing each free occurrence of x in H by a free occurrence of y. The *rules of inference* of M are: (i) from $\vdash H$ and $\vdash H \rightarrow \Theta$ to infer $\vdash \Theta$, provided H is closed or Θ is open; (ii) from $\vdash H$ to infer $\vdash H[x/y]$; (iii) from $\vdash H \rightarrow \Theta$ to infer $\vdash \Lambda xH \rightarrow \Theta$, provided Θ is open or Λ-closed; (iv) from $\vdash H \rightarrow \Theta$ to infer $\vdash H \rightarrow \Lambda x\Theta$, provided x is not free in H; (v) from $\vdash H \rightarrow \Theta$ to infer $\vdash \mathrm{V}xH \rightarrow \Theta$, provided x is not free in Θ; (vi) from $\vdash H \rightarrow \Theta$ to infer $\vdash H \rightarrow \mathrm{V}x\Theta$, provided H is open or V-closed.

This system M is a modification of a system M_1 developed by Hermes and Scholz (***Mathematische Logik***, Enzyklop. d. math. Wissensch. Il. 1, Leipzig 1952). The system M differs from M_1 by the restrictions put on the rules of inference (i), (iv), and (vi). These restrictions are made so as to obtain the following result: The system M is sound and complete in the sense that a formula of the IPC is provable in M if and only if it is valid in *all* individual domains. (Received November 8, 1960).

MARTIN DAVIS and HILARY PUTNAM. *Diophantine sets over polynomial rings.*

Let $\mathfrak{R}$ be a ring containing a subring $\mathfrak{J}$ isomorphic to (and here identified with) the integers. A set of S of positive integers is called Diophantine over $\mathfrak{R}$ if there is a polynomial form $P(\xi_0, \xi_1, \ldots, \xi_n)$ over $\mathfrak{R}$ such that:

$$S = \{x \in J \mid x > 0 \wedge \bigvee_{\alpha_1, \cdots, \alpha_n \in R} P(x, \alpha_1, \ldots, \alpha_n) = 0\}.$$

It is shown, using the Pell equation, that a set S is diophantine over $\mathfrak{J}[\xi]$ (the ring of polynomials in one indeterminate over $\mathfrak{J}$) if and only if it is recursively enumerable. If $\mathfrak{R}$ is computable (in the sense of Rabin), it is easy to see that the existence of a non-recursive set diophantine over $\mathfrak{R}$, implies the unsolvability of the problem of determining of a given diophantine equation over $\mathfrak{R}$, whether or not it has a solution in $\mathfrak{R}$. Hence, this latter problem is unsolvable over $\mathfrak{J}[\xi]$. (Received November 28, 1960)

ANDRZEJ MOSTOWSKI. *Completeness theorems for some many-valued functional calculi.*

Let L be the language of the first order functional calculus with two quantifiers and an arbitrary number N of propositional connectives with $p_1, \ldots, p_N$ arguments. Consider the following interpretation of L: the set of truth values is an ordered complete set Z; the designated truth values are elements of a subset D of Z; quantifiers are the l.u.b. and g.l.b. operations on subsets of Z; propositional connectives are mappings f_i of Z^{p_i} into Z, $i = 1, 2, \ldots, N$; functional variables are mappings of suitable Cartesian powers of a set I into Z. The set V of valid formulas is defined in the usual way.

Problem: for what $Z, D, f_1, \ldots, f_N$ is V recursively enumerable?

https://doi.org/10.1017/S0022481200126301 Published online by Cambridge University Press

It is shown that the answer is positive in the following cases: I. Z (considered as a topological space with the interval topology) is separable, D is open and the f_i are continuous in this topology; the elementary theory of relations

$$(*) \qquad x \in D,\ x \leqq y,\ x = f_i(x_1, \ldots, x_{p_i}), \quad i = 1, 2, \ldots, N$$

is decidable.

II. Z is the set of zero-one sequences, D is an open rational interval of Z, $f_1, \ldots, f_N$ are recursive functionals.

III. Z is the set of ordinals $\leqq \alpha$ (α arbitrary), D is an initial segment of Z, $f_1, \ldots, f_N$ are continuous and the elementary theory of relations (*) is decidable.

Tichonov's theorem is the principal tool used in the proofs. (Received November 2, 1960.)

H. E. Rose. *Independence of induction schema in recursive arithmetic.*

We consider a particular formalisation of primitive recursive arithmetic, known as Ternary Recursive Arithmetic (see my paper to appear in the 1960 ***Annales de L'Ecole Normale***, Paris). The consistency of primitive recursive arithmetic is provable in a system S consisting of Ternary Recursive Arithmetic with one doubly recursive function ϕ (ϕ is effectively a function enumerating the terms of the original system) (see also my paper to appear in the 1961 ***Zeitschr. für Math. Logik und Grundlagen der Mathematik.***) The system S′ consists of Ternary Recursive Arithmetic and the following induction schema K (where the functions $e(x)$, $e'(x)$, $f(m, x, y)$, $g(m, x, y, z)$ and $h(m, x)$ are primitive recursive)

$$A(0, x, e(x))$$
$$A(m, e'(x), y) \rightarrow A(m + 1, 0, h(m, y))$$
$$A(m + 1, x, y) \rightarrow [A(m, f(m, x, y), z) \rightarrow A(m + 1, x + 1, g(m, x, y, z))]$$
$$A(m, x, y) \rightarrow B(m, x)$$
$$\vdash B(m, x)$$

has the property that any theorem of S, whose statement does not contain ϕ, is also a theorem of S′. Hence, as we can prove that the statement of consistency for primitive recursive arithmetic is undecidable in this system alone, K is not reducible to the simple rule of mathematical induction in primitive recursive arithmetic. We can show further that S′ does not yield all the theorems of double recursive arithmetic, in particular the consistency of S′ is provable in this system. (Received January 23, 1961)

A. A. Mullin. *Two simple-minded unsolvable algebraic problems.*

Lemma: It is algorithmically unsolvable to determine whether or not an arbitrarily given finite sequence of inner automorphisms of an arbitrary group, say G, from among the set of all inner automorphisms of G, carries an arbitrary element of G onto itself, and a fortiori, an arbitrary subset of G onto itself.

Proof: Trivially, the set A of all inner automorphisms of G is a group. Hence among other things, we have to decide whether or not an arbitrary word of A is equal to the identity of A. But, by the *Boone-Britton-Novikov* theorem (see W. W. Boone, *The word problem*, ***Annals of Math.***, vol. 70 (1959), pp. 207–265; J. L. Britton, *The word problem for groups*, ***Proc. London Math. Soc.***, Third series, vol. 8 (1958), pp. 493–506; P. S. Novikov, *On the algorithmic unsolvability of the word problem in group theory* (in Russian), ***Trudy Math. Inst. Steklov***, no. 44, Moscow, 1955.) the word problem for finitely generated groups is recursively unsolvable.

Corollary: It is algorithmically unsolvable to determine whether or not an arbitrarily given finite sequence of endomorphisms of an arbitrary group, say G, from among the set of all endomorphisms of G carries an arbitrary element of G onto itself, and a fortiori, an arbitrary subset of G onto itself.

Proof: Note that the set of all endomorphisms of G is a monoid. The problem is unsolvable, for otherwise, the problem stated in the previous lemma would be solvable, thereby contradicting the *Boone-Britton-Novikov* theorem. (Received November 8, 1960)

The ASSOCIATION FOR SYMBOLIC LOGIC announces the following elections, each for a period of three years beginning January 1, 1961.

As members of the Executive Committee, Professor Hartly Rogers, Jr. of the Massachusetts Institute of Technology, and Professor Patrick Suppes of Stanford University.

As members of the Council, Professor Ernst Specker of the Swiss Federal School of Technology, and Professor K. J. J. Hintikka of the University of Helsinki.

As Treasurer, Professor Joshua Barlaz, of Rutgers — The State Univerity.

The amendment separating the offices of Secretary and Treasurer was passed.

A meeting of the Association for Symbolic Logic will be held at the Chalfonte-Haddon Hall hotel, Atlantic City, New Jersey, December 27, 1961 in conjuction with a meeting of the Eastern Division of the American Philosophical Association. Members desiring to present papers will please submit abstracts by October 31, 1961, in duplicate, and not longer than three hundred words each, to Nuel D. Belnap, Jr., Department of Philosophy, Yale University, New Haven, Connecticut. Every effort will be made to accomodate late papers.

In addition to the regular meeting of the Association for Symbolic Logic, there will be a joint symposium with the American Philosophical Association on Modal Logic.