$$Lu = \Sigma a_{ij}(x, u, u_x) D_i D_j u + \ldots = f .$$

Hence there are essentially four separate (but related) theories discussed in this book. (There are also chapters on elliptic systems, and on variational problems per se.)

Let $W_2^k(\Omega)$ denote the Sobolev space of real-valued functions on Ω having square-summable generalized derivatives of all orders $\leq k$. Call a subspace W of $W_2^1(\Omega)$ "admissible" for a given class of equations, if the uniqueness theorem for the Dirichlet problem is valid in W "in the small," i.e., if solutions in W of $Lu(x) = f(x)$, $x \in \Omega'$; $u(x) = 0$, $x \in \partial\Omega'$, are unique for small sets $\Omega' \subset \Omega$ The principal results of the book consist of the determination of necessary and sufficient conditions (in terms of integrability properties of the coefficients) in order that an arbitrary admissible solution to a given class of equations possess some additional regularity property, such as boundedness or Holder continuity. Necessary conditions are derived (in the Introduction) by simple considerations; the main part of the book is devoted to proving their sufficiency. It is assumed throughout the book that Ω is a bounded set in n-space, and that the equations studied are uniformly elliptic on Ω. Uniform ellipticity is defined as usual for linear equations, whereas for quasi-linear equations, e.g. in divergence form, the definition is as follows:

$$\nu(|u|)(1+|p|)^{m-2} |\xi|^2 \leqq \Sigma \frac{\partial a_i(x, u, p)}{\partial p_j} \xi_i \xi_j$$

$$\leq \mu(|u|)(1+|p|)^{m-2} |\xi|^2$$

for all real n-vectors ξ, where $m > 1$ is a fixed constant, and where ν and μ are constants depending only on $|u|$.

The authors have attempted to give a self-contained and complete solution to the problems posed; many of the results are published here for the first time. Granted a few basic results of functional analysis, the arguments are elementary. Most of them are also difficult - Chapter 3, for example, gives a complete derivation of the famous a priori estimates of Schauder, which though frequently referred to, are seldom proved in textbooks.

No applications are given, neither to physical nor mathematical topics. For far-reaching mathematical applications of related results, we refer to Multiple Integrals in the Calculus of Variations, by C.B. Morrey, Grundlehren der Math. Wiss. Vol. 130, Springer, Berlin (1966).

The translation from the 1964 Russian edition was edited by L. Ehrenpreis, and is completely adequate.

Colin Clark, University of British Columbia