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Some investigations of combinatorial integer matrices using cyclotomy

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This thesis is mainly concerned with the combinatorial matrix equation

(1) $$AA^T = sI + tJ\ , \quad s,\ t \text{ integers,}$$

where I is the identity matrix, J the square matrix each entry of which is 1, and A is a square matrix of order n. We investigate different cases where s and t satisfy certain conditions and study their application to combinatorial theory.

In the case that A is a $(1, -1)$ matrix and $s = n$, $t = 0$, one is dealing with Hadamard matrices. By the application of (1) and other facets of combinatorial theory we have constructed new Hadamard matrices.

If $s = r - \lambda$, $t = \lambda$, and A is a $(0, 1)$ matrix, we have the usual Balanced Incomplete Block Designs which are studied here through Supplementary Difference Sets.

Then more generally we consider the construction of integer matrices satisfying (1); this construction is of significance because of questions asked by Ryser [7] and recent work on orthogonal designs [6].

In finding new Hadamard arrays (1) has been generalised to allow s to be a function of commuting variables A, B, C, D with $s = A^2 + B^2 + C^2 + D^2$ and $t = 0$. This is an important special case in the study of orthogonal designs, being the case of four variables each repeated the same number of times.

The main results of the thesis are contained in [1], [2], [3], [4],

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and [5].

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