own preference, but we think that some of his arguments against the alternative are invalid. He does not allow for the fact that both Einstein's and de Sitter's worlds are simplified types, which the actual universe might approximate to but obviously does not reach. It has been said that Einstein's world contains matter but no motion, whilst de Sitter's world contains motion but no matter. The actual universe must be to some extent a compromise between them, and either model can be made to appear ridiculous if it is treated as other than a mathematical limit.

The moment chosen for writing this book has proved to be rather unfortunate. The theory had been almost stationary for ten years, but it has suddenly developed a considerable advance mainly owing to the work of Lemaître. It is now unnecessary to confine attention to the two limiting cases, and we can deal quite easily with intermediate forms. Einstein's world turns out to be unstable. It seems probable that the universe started as an Einstein world but could not remain in that form owing to instability, and that it is now on its way towards the de Sitter form which lies ahead as the ultimate limit. This point of view is too recent to appear in the book.

Robbed of our infinitude of space we are anxious to know how much room is left us. It is on this point that Dr. Silberstein is most heterodox, insisting on a much smaller radius than that found by other investigators. We do not learn the worst until we reach his Appendix. In the text he takes leave of us imprisoned for ever in a space of 36 million parsecs radius or a little smaller. He returns in the Appendix to cut down our prison to 2 million parsecs radius. This would be a matter of serious concern to astronomers who have been planting out spiral nebulae somewhat beyond the latter distance. Even if we were convinced of the soundness of his method of determining the constant (which differs from the method generally adopted) we should still object that he has chosen very unsuitable data to exercise it on. His reduced radius of space is based on stellar proper motions; these show many systematic features, arising from the dynamics of the galaxy which they form, and the systematic effects of the cosmical term are likely to be masked. The feature which Dr. Silberstein interprets as indicating a 2 million parsecs radius of space is in fact generally attributed by astronomers to the rotation of the galaxy.

A. S. E.

A Course of Analysis. By E. J. Phillips. Pp. viii + 361. 16s. 1930. (Camb. Univ. Press.)

The treatment of inequalities in this book is more adequate than in most English text-books, but the remaining chapters do not come up to the same standard.

H. D. U.

CORRESPONDENCE.

Sir,—On receiving the July number of the *Mathematical Gazette*, I turned at once to the correspondence column to see what people had to say on the important issue raised by Mr. Siddons. It is one which primarily concerns schoolmasters, more a matter of Psychology than of Mathematics. Judge then of my surprise when I found that not a single schoolmaster had anything to say about it at all.

Professor Levy points out that most of the modern exponents of rigour were brought up in a less mathematically ascetic school. This does not of course mean that the ways of our benighted forefathers are good enough for our children; but it is a fact to be considered on its merits as showing that lack of rigour in early training does not necessarily prevent the mind from appreciating rigour later on. Setting this alongside what every mathematical master knows, that an overdose of intricate analysis too early does prevent any appreciation either of rigour or of anything else, the case put by Mr. Siddons seems a strong one.

The problem before the schoolmaster is not how near he can attain to complete rigour in his expositions, but how to make the boy *set up for himself* an ideal of rigour. To achieve this is chiefly a question of starting right and of

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proceeding gently. If we introduce the idea in such a way that it strikes the boy as an ingenious device invented by schoolmasters for making short, easy proofs long and difficult, then he naturally develops a distaste for it. If on the other hand we begin by showing him Lewis Carroll's "proof" that every triangle is isosceles, he is fascinated. We must follow this up by continually providing pointed examples of "things that don't work" (for instance, a consideration of the expression $\frac{1}{\sqrt{n+1}}+\frac{1}{\sqrt{n+2}}+\ldots+\frac{1}{\sqrt{2n}}$, which increases with n although every term in it decreases, is a useful preliminary to a treatment of $\underset{n\to\infty}{\text{Lt}}\left(1+\frac{x}{n}\right)^n$. We shall then find that he is not at all "up against" the idea, and will actually begin to insist on rigour of his own accord.

Nevertheless, although rigorous methods may be made not distasteful, I do not believe that the question ought to be allowed to loom too large at school. It is a big step from a discussion of the expression mentioned above to a formal proof of Tannery's Theorem, and it is questionable whether we should attempt to make boys take it. What are the psychological effects of that type of work? Has anyone ever studied the psychology of mathematicians ? Is it a fact that they are frequently lacking in self-confidence ? Are they nearly always of introvert type ? And if so, is it because of their training in analysis ?

For my own part I do believe that mathematicians are inclined to be self-critical, and that they are not so successful as other people in setting up their mask to face the world with, but I am certain that in the long run, that is gain rather than loss. A sound psychological development requires an honesty with oneself which not everyone attains, and if we can use mathematics for setting up an ideal of honesty and searching criticism, then we are doing well.

But as I said before, the question must not loom too large at school. We must not lose sight of the subject itself. Professor Neville shows us the right way when he reminds us that the series for the sine is one of the delightful surprises of Mathematics. That is the chief thing about it as far as we schoolmasters are concerned, and if we forget that point of view, our teaching will be dead. E. H. LOCKWOOD.

Felsted School, 22nd July, 1930.

DEAR SIR,—In response to criticism based on a misunderstanding, I wish to emphasise that the sole object of my article on differential equations which appeared on pp. 99-102 of the *Gazette*, May 1930, was to point out difficulties which attend the use of a general formula given by Forsyth. I should not for a moment advocate this as the practical method [indicated in equation (B) on p. 100]; indeed I said so explicitly in the opening sentence of the last paragraph of my article.—Yours sincerely, F. UNDERWOOD.

University College, Nottingham,
21st July, 1930.

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787. The student of mathematics who in the course of a single introductory lecture on the calculus completes the differentiation of the function x^n would be a good deal soothed to know that he has covered in an hour a problem which took the generation of Barrow, Newton, and Leibniz about forty years to clear up.—L. Hogben, *The Realist*, Dec. 1929. [Per Prof. E. H. Neville.]

788. Mathematics in its prime, the mathematics of Newton and Lagrange and Laplace, advanced our knowledge like the mental work of a man in his prime : mathematics dealing with imaginary nonentities is like the unintelligible fancies of a dreaming dotard who *has been* learned and profound, but in his old age lets idle imaginations take possession of him.—R. A. Proctor, *Gentleman's Mag.* Jan. 1884 [on Cayley's *Brit. Assoc. Address*, 1883; and v. *Cambridge Review*, Feb. 20, 1884].