Each section of each chapter ends with a substantial collection of exercises, of which roughly half are given solutions at the back of the book. Further exercises are available online, and as many of these have randomised numerical inputs there will be no shortage of examples to use.

A particular strength of the book is its copious use of geometrical illustrations. Most learners will find their intuition greatly helped by this (*O si sic omnes!*), and it hardly matters that some of the diagrams are not very clear.

The writing is accessible, indeed very readable (the phrase 'as a sanity check' is used repeatedly), but it does not lack rigour. Although mention is made of the computational advantages of block matrices and sparse matrices, very little use is made of computational aids:

> There is actually an explicit formula for the inverse of a matrix of any size, which we derive in Section 3.A.1. However, it is hideous for matrices of size $3 \times 3$ and larger and thus the method of computing the inverse based on Gauss-Jordan elimination is typically much easier to use.

(I can't resist mentioning that when as an undergraduate I shared a room with a very much better mathematician, my mathematical contribution to the partnership was to invert his matrices.) The deferring of the definition of determinants allows a non-recursive definition to be given, while the more familiar definition met at school level emerges as a consequence. The frequent use of commentary and explanation in the margins allows the main thread of the arguments to be clearly seen while appropriate 'hand-holding' is available at once.

One might take issue with certain bits of the exposition—I could imagine a clearer introduction to matrix multiplication. On the other hand I particularly liked features such as the explanation of the simplex algorithm in the context of solving simultaneous equations, the discussion of Vandermonde matrices, the use of a 74-dimensional matrix to describe the 'look-and-say' sequence, and the extension of the result $(U\Lambda U^{-1})^n = U\Lambda^n U^{-1}$ to negative and fractional $n$.

It will be apparent that this book covers relatively limited and elementary ground in considerable detail. (There is a second volume, *Advanced linear and matrix algebra*, which I have not seen.) It seems to me an excellent first course for undergraduates without a strong mathematical background who are taking subjects where the technicalities of linear algebra are needed, for instance computer animation or some courses in economics or financial modelling, while it also gives splendid enrichment for those taking, say, further mathematics A level. International Centres such as those taking more ambitious CIE examinations would certainly find it useful. Above all, any schools where further mathematics is taught would be well advised to have a copy for departmental use; teachers who read it would feel more confident in their teaching, and I am sure they would enjoy the experience.

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**Counterexamples in measure and integration** by René L. Schilling and Franziska Kühn, pp. 399, £34.99 (paper), ISBN 978-1-00900-162-5, Cambridge University Press (2021)

A young mathematician's view of the role played by counterexamples might well take one of the two following forms. Firstly, we picture a PhD student or postdoc working late into the night, fuelled, inevitably, by coffee. Then cut to the following morning: bursting into their advisor's office, a frantic scramble at a blackboard. "Look! Professor X's theorem is *wrong*!" In our second vision, we find ourselves answering a question of the form "find the minimal value of $K$ such that..." and after the bulk of the argument, we require a coda saying, for example, "...and the following small example shows that three colours are not always sufficient".

The first vision corresponds to a blockbuster impression of mathematics as intellectual combat that (happily, in most opinions) lies far from reality. The second might seem a recurring and important component of school-level competition problems, but later becomes just another building block of logical proofs to be taken for granted when the result has this form.

In general, counterexamples come into play when discussing statements of the form "If we have property X, then we also have property Y", and whether the converse holds. In this context, a counterexample might consist of a scenario S where Y holds but X does not. So this would certainly 'disprove' the converse statement that property Y implies property X, but, more importantly, it acts as test case for two properties to be considered in future work. One only understands 'uniform convergence' of functions when one has a clear idea of what it means to converge pointwise but not uniformly. Thus the ideal counterexample is general, but also streamlined, so that in future work, when considering whether to impose condition X or condition Y, you think about whether it is important to include S in your analysis.

Such counterexamples come up in university mathematics courses essentially from the beginning (unique factorisation modulo a composite number; sequences which don't have limits) but continue into more exotic settings. The classical text *Counterexamples in Analysis* by Gelbaum and Olmsted has been in print since the 1960s, and many working mathematicians have a copy on their shelf. I also have in my office a copy of the newer *Counterexamples in Probability* by Stoyanov, whose low price atones for the rather dense formatting in the Dover edition. One summary of the text under review is that I plan to keep it there on the same shelf, and I expect to consult all three occasionally and gladly over the years to come.

The new Schilling and Kühn text is slightly less dense technically than Gelbaum and Olmsted, but inevitably there are many sections that are of specialist interest only. An effort is made to give definitions of most phenomena under consideration, and these are stated with an elegant conciseness helpful to anyone with prior expertise. Students taking a first course in measure theory would be advised to look elsewhere for guidance. However, their lecturer might well find various chapters to be a fruitful source of inspiration for problem sheets and class discussion. I'm sure many researchers with an occasional need to dabble in analysis may end up folding over some page corners, for example Table 11.1, which offers a $7 \times 7$ matrix of the logical relationships between various modes of functional convergence in general measure spaces, and links to the various counterexamples.

Compendia of counterexamples remain useful and thought-provoking resources, and this text is a high-quality example in an analytic direction.

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