SPECTRAL OPERATORS AND WEAKLY COMPACT HOMOMORPHISMS IN A CLASS OF BANACH SPACES

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The purpose of this note is to present certain aspects of the theory of spectral operators in Grothendieck spaces with the Dunford–Pettis property, briefly, GDP-spaces, thereby elaborating on the recent note **[10]**.

For example, the sum and product of commuting spectral operators in such spaces are again spectral operators (cf. Proposition 2.1) and a continuous linear operator is spectral if and only if it has finite spectrum (cf. Proposition 2.2). Accordingly, if a spectral operator is of finite type, then its spectrum consists entirely of eigenvalues. Furthermore, it turns out that there are no unbounded spectral operators in such spaces (cf. Proposition 2.4). As a simple application of these results we are able to determine which multiplication operators in certain function spaces are spectral operators.

One approach to the theory of operator algebras generated by σ-complete Boolean algebras of projections in a Banach space X is via weakly compact homomorphisms with domain a $C(\Omega)$-space, where Ω is a compact Hausdorff space, and mapping into the space of continuous linear operators on X (i.e. operational calculi). Such weakly compact homomorphisms, in the particular case when the underlying Banach space X is a *GDP*-space, are considered in §3. It turns out that such homomorphisms are of a very restricted type and exhibit some rather strong properties (cf. Proposition 3.2).

1. Preliminaries and notation. Let X be a Banach space and $L(X)$ the vector space of all continuous linear operators of X into X. Then $L_s(X)$ and $L_u(X)$ will denote the space $L(X)$ equipped with the strong and uniform operator topologies, respectively. Of course, $L_u(X)$ is then a Banach space and $L_s(X)$ a quasicomplete locally convex space, [**11**, p. 85 Corollary].

A continuous linear operator S in a Banach space X is a scalar-type operator, briefly, a scalar operator, if $S = \int_{\mathbb{C}} z\, dP(z)$ where $P: \mathcal{B} \to L_s(X)$ is a spectral measure defined on the Borel sets $\mathcal{B}$ of the complex plane $\mathbb{C}$; see [**3**, Chapters XV, XVII], for example. Of course, to say that P is a spectral measure means that P is σ-additive in the strong operator topology, $P(E \cap F) = P(E)P(F)$ for each $E \in \mathcal{B}$ and $F \in \mathcal{B}$, and $P(\mathbb{C}) = I$, the identity operator on X. An operator $T \in L(X)$ is said to be a spectral operator if there exist a scalar operator $S \in L(X)$ and a quasinilpotent operator $N \in L(X)$ such that $NS = SN$ and $T = S + N$. This agrees with the original definition due to N. Dunford, [**3**, XV Theorem 4.5].

A Banach space X is called a Grothendieck space if every sequence $\{x_n'\}$ in the continuous dual space, X', of X, which converges for the weak-star topology to zero converges weakly to zero. Since X' is quasicomplete for the weak-star topology, [**11**, IV Proposition 6.1], it follows that X' is weakly sequentially complete whenever X is a Grothendieck space. A Banach space X is said to have the Dunford–Pettis property if

$\lim_{n\to\infty} \langle x_n, x'_n\rangle = 0$ whenever $\{x_n\} \subseteq X$ tends weakly to zero and $\{x'_n\} \subseteq X'$ tends weakly to zero. Well known examples of GDP-spaces include L^∞-spaces, $H^\infty(\mathbb{D})$, injective Banach spaces and certain $C(\Omega)$-spaces; see [**9**], for example.

Lemma 1. (cf. [**10**]). *Let X be a GDP-space and $P:\Sigma \to L_s(X)$ be a spectral measure with domain a σ-algebra Σ. Then there exist finitely many disjoint commuting projections $P_1, \ldots, P_n$, in the range of P, each one an atom, such that $I = \sum_{i=1}^{n} P_i$ and each operator $P(E)$, $E \in \Sigma$, is a partial sum of $\{P_1, \ldots, P_n\}$. In particular, P assumes only finitely many values and is σ-additive with respect to the uniform operator topology. Furthermore, if $S \in L(X)$ is a scalar-type spectral operator, then $\sigma(S)$ is a finite set.*

Remark. It is well known that any Bade σ-complete Boolean algebra of projections (cf. [**3**, p. 2195] for the definition) in a separable Banach space is necessarily a Bade complete Boolean algebra, [**3**, XVII Lemma 3.21]. It follows from Lemma 1.1 and [**3**, XVII Corollary 3.10] that there are other classes of Banach spaces, not necessarily separable, for which the same statement is true; any GDP-space is such a space.

2. Spectral operators. There are many classes of Banach spaces, including Hilbert spaces, L^p-spaces for $1 < p < \infty$ and complemented subspaces of L^p-spaces, $1 \le p \le \infty$, for example, with the property that the sum and product of commuting spectral operators are again spectral; an example due to S. Kakutani shows that this is not the case for all Banach spaces (see [**3**, pp. 2098–2101] for a more detailed discussion of these remarks, including the relevant references). A further result in this direction is the following

Proposition 2.1. *The sum and product of commuting spectral operators in a GDP space are again spectral operators.*

Proof. It suffices to establish the result for commuting scalar operators; see the remark on p. 64 of [**5**]. But, then Lemma 1 implies that the hypotheses of Theorem 8 in [**8**] are satisfied, from which the result follows.

Proposition 2.2. *Let X be a GDP-space and $T \in L(X)$. Then T is a spectral operator if and only if its spectrum, $\sigma(T)$, is a finite set.*

Proof. Suppose $\sigma(T) = \{\lambda_1, \ldots, \lambda_k\}$. It follows from Theorem 5.6.1 and its Corollary in [**7**] that

$$T = \sum_{i=1}^{k} N_i + \sum_{i=1}^{k} \lambda_i E_i$$

where the operators E_i, $1 \le i \le k$, are pairwise disjoint projections commuting with T such that $I = \sum_{i=1}^{k} E_i$ and the pairwise commuting operators $N_i = E_i(T - \lambda_i I)$, $1 \le i \le k$, are quasinilpotent and satisfy $E_i N_j = N_j E_i = \delta_{ij} N_j$. Since $\sigma(N_i) = \{0\}$ for each $1 \le i \le k$ it follows that $N=\sum_{i=1}^{k} N_i$ also satisfies $\sigma(N)=\{0\}$, [**3**, XV Lemma 4.4], and hence, is quasinilpotent. Since $S=\sum_{i=1}^{k} \lambda_i E_i$ is a scalar operator commuting with N the operator $T=S+N$ is spectral.

Conversely, suppose that T is spectral, say $T=S+N$, where N is a quasinilpotent operator commuting with S. Lemma 1 implies that $\sigma(S)$ is finite (cf. its proof in [**10**]) and hence, also $\sigma(T)=\sigma(S)$ is finite, [**3**, XV Lemma 4.4].

Remark. The proof of the finiteness of $\sigma(T)$ implying the spectrality of T does not require X to be a GDP-space; it is valid in any Banach space.

Proposition 2.2 shows that the classification of spectral operators in GDP-spaces is very close to that of linear operators in finite dimensional spaces. Of course, in finite dimensional spaces the spectrum of any linear operator (necessarily a spectral operator by the Jordan decomposition theorem) consists entirely of eigenvalues. It is natural to ask whether this is also the case for spectral operators in GDP-spaces. For spectral operators of finite type (cf. [**3**, pp. 1943–1944] for the definition) this is indeed the case.

Proposition 2.3. *Let X be a GDP-space and $T \in L(X)$ be a spectral operator of finite type. Then $\sigma(T)$ is finite and consists entirely of eigenvalues.*

Proof. Suppose that $\sigma(T)=\{\lambda_1,\ldots,\lambda_k\}$; see Proposition 2.2. If P is the resolution of the identity for T, then $P(\{\lambda_i\}) \neq 0$ for each $1 \leq i \leq k$, [**3**, p. 2076, Ex. 15], and hence, each λ_i, $1 \leq i \leq k$, is an eigenvalue of T, [**3**, XV Theorem 8.3].

It is always the case in any Banach space X that if $T \in L(X)$ is a spectral operator of finite type, then its residual spectrum is empty, [**3**, XV Theorem 8.3]. If X is a GDP-space, then Proposition 2.3 shows that the continuous spectrum of T is also void. So, the natural question to ask is whether there exist spectral operators in GDP-spaces which are not of finite type and if so, whether they have spectral points other than eigenvalues? The answer is affirmative for both questions.

Example 1. Let $X=L^\infty([0,1])$ and $T \in L(X)$ be the operator defined by $Tf=g$, $f \in X$, where

$$g(t)=\int_0^t f(s)\,ds, \qquad t \in [0,1].$$

It can be verified that T is quasinilpotent (and hence, is a spectral operator, which is clearly not of finite type) and that zero belongs to the residual spectrum of T.

Let X be a Banach space and T be a spectral operator, possibly unbounded, in the sense of N. Dunford, [**3**, Chapter XVIII], with resolution of the identity $P:\mathcal{B}\to L_s(X)$, necessarily unique, [**3**, XVIII Theorem 2.5]. It is a consequence of the definition of spectral operator (cf. [**3**, p. 2228]) and the Closed Graph Theorem that if the support of the measure P is a bounded subset of $\mathbb{C}$, then necessarily $T \in L(X)$. This is precisely what happens if X is a GDP-space. For, in this case it follows from Lemma 1 applied to the resolution of the identity $P:\mathscr{B}\to L_s(X)$ that the support of P is necessarily a finite subset of $\mathbb{C}$. This establishes the following

PROPOSITION 2.4. *There are no unbounded spectral operators in GDP-spaces.*

We conclude this section by determining the spectrality of some familiar operators in certain function spaces.

EXAMPLE 2. Let G be a locally compact abelian group and for each $g \in G$, let T_g denote the continuous translation operator in $L^\infty(G)$ defined by $(T_g f)(y) = f(y+g)$ where $f \in L^\infty(G)$ and $y \in G$ locally a.e. Then T_g is spectral if and only if g has finite order.

If g has finite order, say n, then $T_g^n = I$ and so the spectral mapping theorem implies that $\sigma(T_g)$ consists of at most the nth roots of unity. It follows from Proposition 2.2 that T_g is spectral. Conversely, if g has infinite order, then $\sigma(T_g)$ is the entire unit circle, [**6**, Theorem 1], and so Proposition 2.2 shows that T_g cannot be spectral.

EXAMPLE 3. Let $\mu:\Sigma\to[0,\infty]$ be a localizable measure, $f\in L^\infty(\mu)$ and T_f denote the continuous operator in $L^\infty(\mu)$ of multiplication by f. Then T_f is a scalar operator if and only if f is a Σ-simple function.

For, if f is a Σ-simple function, then it is clear that T_f is a scalar operator. Conversely, suppose that T_f is scalar. Let S_f denote the continuous operator in $L^1(\mu)$ of multiplication by f. Then S_f is a scalar operator and $S_f = \int f\,dP$ where $P:\Sigma\to L_s(L^1(\mu))$ is the spectral measure of multiplication by characteristic functions of elements of Σ, [**1**, Theorem 4]. Since P and μ have the same null sets it follows from [**3**, XVII Corollary 2.11(ii)] that

$$\sigma(S_f) = \bigcap\{\overline{f(E)};\, E\in\Sigma,\, P(E)=I\} = \bigcap\{\overline{f(E)};\, E\in\Sigma,\, \mu(E^c)=0\}, \tag{1}$$

where E^c denotes the complement of the set $E\in\Sigma$. As T_f is the adjoint operator of S_f (the localizability of μ ensures that $(L^1(\mu))' = L^\infty(\mu)$) it is known that $\sigma(T_f)=\sigma(S_f)$. Since $\sigma(T_f)$ is finite (cf. Proposition 2.2) it follows that the right-hand side of (1) is finite and, hence, that f is a Σ-simple function.

EXAMPLE 4. Let $f\in H^\infty(\mathbb{D})$ and T_f denote the continuous operator in $H^\infty(\mathbb{D})$ of pointwise multiplication by f. Then T_f is spectral if and only if f is a constant function.

If f is constant, then T_f is a multiple of I and so is a scalar operator. Suppose now that T_f is spectral, in which case $\sigma(T_f)$ is finite by Proposition 2.2. But, it is easily shown that $\sigma(T_f)$ is precisely the closure in $\mathbb{C}$ of $f(\mathbb{D}) = \{f(z);\, z\in\mathbb{D}\}$ and, hence, $f(\mathbb{D})$ is a finite set. Since f is analytic it follows that f is a constant function.

If Ω is a compact Hausdorff space and $f\in C(\Omega)$, then T_f denotes the operator in $C(\Omega)$ of pointwise multiplication by f. It was shown by U. Fixman, [**4**, Example 2.2], that a necessary condition for spectrality of T_f is that f be constant on the connected components of Ω. However, this condition is clearly not sufficient. For example, if Ω is the one-point compactification of the natural numbers, in which case the connected components of Ω are just the singleton subsets of Ω, and $f\in C(\Omega)$ is the function given by $f(\infty)=1$ and $f(n)=n^{-1}+1$, $n=1,2,\ldots,$ then f is constant on the connected components of Ω, but T_f is not spectral. The problem is that f assumes too many values on the connected components. If f is replaced by any element of $C(\Omega)$ which is constant in some neighbourhood of infinity, in which case it assumes only finitely many values, then it is clear that T_f is scalar. For GDP-spaces $C(\Omega)$, it turns out that this is the only way that T_f can be spectral.

EXAMPLE 5. Let Ω be a compact Hausdorff space for which $C(\Omega)$ is a GDP-space and $f \in C(\Omega)$. Then T_f is spectral if and only if f has finite range.

Indeed, if T_f is spectral, then $\sigma(T_f) = \overline{f(\Omega)}$ is finite by Proposition 2.2 and, hence, f assumes only finitely many values. Conversely, suppose that $f(\Omega) = \{\lambda_1, \ldots, \lambda_n\}$. It follows from the continuity of f that each set $E_i = f^{-1}(\{\lambda_i\})$, $1 \le i \le n$, is both open and closed in Ω and, hence, $\chi_{E_i} \in C(\Omega)$. If Σ denotes the (finite) σ-algebra generated by the disjoint sets $\{E_i; 1 \le i \le n\}$, then the mapping $P: \Sigma \to L_s(C(\Omega))$ of pointwise multiplication by characteristic functions of elements of Σ is a spectral measure such that $T_f = \int_\Omega f\,dP$. Accordingly, T_f is scalar, [**3**, XVII Lemma 2.9].

3. Weakly compact homomorphisms. It is well known that there is an intimate connection between weakly compact homomorphisms from $C(\Omega)$-spaces into $L_s(X)$ and the theory of algebras of commuting scalar operators in $L(X)$; see [**3**, Chapter XVII], [**12**] and [**13**], for example. This connection is explicitly formulated in Proposition 3.1 below. The purpose of this section is to investigate the nature of such homomorphisms in the particular case when X is a GDP-space.

Let Y be a Banach space and Z a locally convex Hausdorff space. A linear operator $T: Y \to Z$ is said to be compact (weakly compact) if $\{Ty; \|y\| \le 1\}$ is relatively compact (relatively weakly compact) in Z. Such operators are necessarily continuous.

If Ω is a compact Hausdorff space, then $\mathcal{B}(\Omega)$ denotes the σ-algebra of Borel subsets of Ω. A linear mapping $\Phi: C(\Omega) \to L(X)$ is a homomorphism if $\Phi(fg) = \Phi(f)\Phi(g)$, for each $f, g \in C(\Omega)$, and $\Phi(1) = I$.

PROPOSITION 3.1. *Let Ω be a compact Hausdorff space and X be a Banach space.*

(i) *If $\Phi: C(\Omega) \to L_s(X)$ is a weakly compact operator, then there exists a unique (regular) operator-valued measure $P: \mathcal{B}(\Omega) \to L_s(X)$ such that*

$$\Phi f = \int_\Omega f\,dP, \qquad f \in C(\Omega). \tag{2}$$

If, in addition, Φ is a homomorphism, then P is a spectral measure.

(ii) *If $P: \mathcal{B}(\Omega) \to L_s(X)$ is a (regular) operator-valued measure, then the linear mapping of $C(\Omega)$ into $L_s(X)$ defined by* (2) *is weakly compact. If, in addition, P is a spectral measure, then this mapping is a homomorphism.*

REMARK. Regularity of operator-valued measures is defined by analogy with scalar measures, using the topology of the image space $L_s(X)$.

A continuous linear operator between Banach spaces is said to be absolutely summing if it maps weakly unconditionally Cauchy series into absolutely convergent series.

Proposition 3.2. *Let Ω be compact Hausdorff space, X be a GDP-space and $\Phi: C(\Omega) \to L_s(X)$ be a weakly compact homomorphism. Then,*

(i) *$\Phi: C(\Omega) \to L_u(X)$ is a compact homomorphism,*
(ii) *$\Phi: C(\Omega) \to L_s(X)$ is a compact homomorphism and*
(iii) *$\Phi: C(\Omega) \to L_u(X)$ is an absolutely summing operator.*

Proof. (i) Let $P: \mathscr{B}(\Omega) \to L_s(X)$ be the unique spectral measure given by Proposition 3.1(i). Since $P: \mathscr{B}(\Omega) \to L_u(X)$ is then also σ-additive (cf. Lemma 1), it follows that each $f \in C(\Omega)$ is P-integrable in $L_u(X)$. Of course, the integrals $\int_\Omega f\,dP$, $f \in C(\Omega)$, are the same operators in $L(X)$ irrespective of whether we consider P as an $L_s(X)$-valued or $L_u(X)$-valued measure. Now $\Phi: C(\Omega) \to L_u(X)$ is actually continuous (this follows easily from [**3**, XVII Theorem 2.10]) and its representing measure $P: \mathscr{B}(\Omega) \to L_u(X)$, having finite range (cf. Lemma 1), certainly has relative compact range in $L_u(X)$. It follows that $\Phi: C(\Omega) \to L_u(X)$ is compact, [**2**, VI Theorem 2.18].

(ii) If $\mathscr{A}$ denotes the closure of $\{\Phi f;\ \|f\|_\infty \le 1\}$ in $L_u(X)$, then $\mathscr{A}$ is compact by (i). Since the identity mapping of $L_u(X)$ into $L_s(X)$ is continuous it follows that $\mathscr{A}$ is also compact as a subset of $L_s(X)$ and, hence, $\Phi: C(\Omega) \to L_s(X)$ is compact.

(iii) Since $P: \mathscr{B}(\Omega) \to L_u(X)$ is the representing measure of $\Phi: C(\Omega) \to L_u(X)$, to show that Φ is absolutely summing it suffices to show that P has finite variation, [**2**, VI Theorem 3.3].

Let $\{P_1, \ldots, P_n\}$ be the atoms in the range of P as given by Lemma 1. If $\mathbb{T} = \{A_1, \ldots, A_r\}$ is a partition of Ω, then each operator $P(A_i)$, $1 \le i \le r$, is a partial sum of $\{P_k\}_{k=1}^n$ and it is not difficult to show, using the disjointness of $\{P_k\}_{k=1}^n$, that if $i \neq j$, then there are no elements of $\{P_k\}_{k=1}^n$ forming the partial sum for $P(A_i)$ in common with those forming the partial sum for $P(A_j)$. Then $P(A_i) = \sum_{j=1}^{k_i} P_{i,j}$, for each $1 \le i \le r$, where all the projections $P_{i,j}$, for $1 \le i \le r$ and $1 \le j \le k_i$, belong to $\{P_k\}_{k=1}^n$ and there are no repetitions. In particular, $\left(\sum_{i=1}^{r} k_i\right) \le n$. It follows that $\|P(A_i)\| \le \sum_{j=1}^{k_i} \|P_{i,j}\| \le \alpha k_i$, for each $1 \le i \le r$, where α is a bound for $\{\|P(E)\|;\ E \in \mathscr{B}(\Omega)\}$, and hence, $\sum_{i=1}^{r} \|P(A_i)\| \le \alpha n$. Since $\mathbb{T}$ was an arbitrary partition of Ω it follows that P has finite variation in $L_u(X)$. This completes the proof of the proposition.

Let X be a Banach space and $\Phi: C(\Omega) \to L(X)$ a linear mapping. For $f \in C(\Omega)$ fixed, the operator $\Phi(f)$ belongs to $L(X)$ and so possesses an adjoint operator $\Phi(f)' \in L(X')$. We denote by $\Phi': C(\Omega) \to L(X')$ the linear mapping $f \to \Phi(f)'$, $f \in C(\Omega)$.

Proposition 3.3. *Let Ω be a compact Hausdorff space and X be a Grothendieck space. If $\Phi: C(\Omega) \to L_s(X)$ is a weakly compact operator, then the mapping $\Phi': C(\Omega) \to L_s(X')$ is also weakly compact. If, in addition, X has the Dunford–Pettis property and Φ is a homomorphism, then*

(i) *$\Phi': C(\Omega) \to L_u(X')$ is a compact homomorphism,*
(ii) *$\Phi': C(\Omega) \to L_s(X')$ is a compact homomorphism and*
(iii) *$\Phi': C(\Omega) \to L_u(X')$ is an absolutely summing operator.*

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Proof. By Proposition 3.1(i), there exists a regular operator-valued measure $P:\mathcal{B}(\Omega)\to L_s(X)$ such that Φ is given by (2). Let $P':\mathcal{B}(\Omega)\to L_s(X')$ denote the set function defined by $P'(E)=(P(E))'$, $E\in\mathcal{B}(\Omega)$. Then P' is σ-additive, that is, the X'-valued measure $P'x':E\to P'(E)x'$, $E\in\mathcal{B}(\Omega)$, is σ-additive with respect to the *norm* topology of X', for each $x'\in X'$. Indeed, if $x'\in X'$ is fixed, then the identities

$$\langle x, P'(E)x'\rangle = \langle P(E)x, x'\rangle, \qquad E\in\mathcal{B}(\Omega),$$

valid for each $x\in X$, shows that $P'x'$ is σ-additive for the weak-star topology. Since X' is weakly sequentially complete (cf. §1) it cannot contain a copy of l^∞ and, hence, $P'x'$ is σ-additive with respect to the norm topology of X', [**2**, I Corollary 4.7]. Hence, $P':\mathcal{B}(\Omega)\to L_s(X')$ is an operator-valued measure and so by Proposition 3.1(ii) the mapping of $C(\Omega)$ into $L_s(X')$ given by

$$f\to\int_\Omega f\,dP', \qquad f\in C(\Omega),$$

is weakly compact. But, this mapping is precisely Φ' as can be seen by the identities

$$\langle(\Phi f)x, x'\rangle = \int_\Omega f(w)\,d\langle P(w)x, x'\rangle = \int_\Omega f(w)\,d\langle x, P'(w)x'\rangle = \left\langle x, \left(\int_\Omega f\,dP'\right)x'\right\rangle,$$

valid for each $f\in C(\Omega)$ and each $x\in X$ and $x'\in X'$. Accordingly, Φ' is weakly compact.

If, in addition, X has the Dunford–Pettis property and Φ is a homomorphism, then the operator-valued measure $P:\mathcal{B}(\Omega)\to L_s(X)$ is a spectral measure with finite range (cf. proof of Proposition 3.2). But, then the representing (spectral) measure P' for Φ' has the same properties as the measure P in the proof of Proposition 3.2. Accordingly, (i), (ii) and (iii) can be proved in the same way as the corresponding statements of Proposition 3.2 by simply replacing P with P'.

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